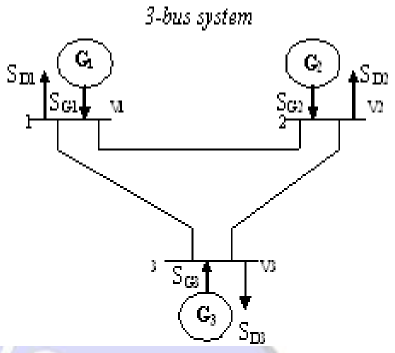
SYSTEM MODEL FOR LOAD FLOW STUDIES

The variables and parameters associated with bus *i* and a neighboring bus *k* are represented in the usual notation as follows:



$$V_i = V_i(\cos\delta_i + j\sin\delta_i) \qquad 1$$

$$Y_{ik} = Y_{ik}(\cos\theta_{ik} + j\sin\theta_{ik}) \qquad 2$$

Complex power

$$S_i = P_i + jQ_i = V_i I_i^* \qquad 3$$

$$S_i^* = P_i - jQ_i = V_i^* I_i \qquad 4$$

Using the indices G and L for generation and load,

$$P_i = P_{G_i} - P_{L_i} = \text{Re}[V_i I_i^*] \qquad 5$$

$$Q_i = Q_{G_i} - Q_{L_i} = \text{Im}[V_i I_i^*] \qquad 6$$

The bus current is given by

$$I_{bus} = Y_{bus}.V_{bus} \qquad 7$$

Hence, from the equations (4) and (5), for an n-bus system:

$$I_i = \frac{P_i - jQ_i}{V_i^*} = Y_{ii}.V_i + \sum_{\substack{k=1 \\ k\neq i}}^{n} y_{ik}.V_k \qquad 8$$

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}.V_k\right] \qquad 9$$

Further, $$P_i + jQ_i = V_i \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik}.V_k$$ 10

i =1,2,…,n

In the polar form

$$P_i + jQ_i = \sum_{k=1}^{n} |V_i.V_k.Y_{ik}| \angle(\delta_i - \delta_k - \theta_{ik})$$ 11

So that

$$P_i = \sum_{k=1}^{n} |V_i.V_k.Y_{ik}| \cos(\delta_i - \delta_k - \theta_{ik})$$ 12

And

$$Q_i = \sum_{k=1}^{n} |V_i.V_k.Y_{ik}| \sin(\delta_i - \delta_k - \theta_{ik})$$ 13

i =1,2,…,n

i ≠ slack bus

The power flow equations (10) are nonlinear. Finally, the powers at the slack bus may be computed from which the losses and all other line flows can be ascertained. Y-matrix interactive methods are based on solution to power flow relations using either current mismatch at a bus given by

$$\Delta I_i = I_i - \sum_{k=1}^{n} Y_{ik}.V_k$$ 14

or by using the voltage form

$$\Delta V_i = \frac{\Delta I_i}{Y_{ii}} \quad 15$$

GAUSS-SEIDEL METHOD FOR POWER-FLOW SOLUTION

In this method, voltages at all buses except at the slack bus are assumed. The voltage at the slack bus is specified and remains fixed at that value. The (n-1) bus voltage relations

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}.V_k \right] \quad 16$$

These equations are solved simultaneously for an improved solution. In order to accelerate the convergence, all newly-computed values of bus voltages are substituted in equation (16). The bus voltage equation for the (m+1)th iteration may then be written as:

$$V_i^{(m+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{(m)*}} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}.V_k^{(m+1)} - \sum_{k=i+1}^{n} Y_{ik}V_k^{(m)}\right] \quad 17$$

The method converges slowly because of the loose mathematical coupling between the buses. The rate of convergence of the process can be increased by using acceleration factors to the solution obtained after each iteration. A fixed acceleration factor α ($1 \leq \alpha \leq 2$) is normally applied to each voltage change:

$$\Delta V_i = \alpha \frac{\Delta S_i^*}{V_i^* Y_{ii}} \quad 18$$

The use of the acceleration factor amounts to a small linear extrapolation of V_i. For a given system, it is quite often found that a near- optimal choice of α exists which is valid over a range of operating conditions. Even though a complex value for α is suggested in literature, it is more convenient to operate with real values given by

$$\left|V_i^{(m)}\right| \angle \delta_i = |\alpha| \left|V_i^{(m)}\right| \angle \delta_i \quad 19$$

Alternatively, different acceleration factors may be used for real and imaginary parts of the voltage.

A non-linear algebraic equation solver iterative steps:

take a function and rearrange it into the form *x* = g(*x*). {there are several possible arrangements}
make an an initial estimate of the variable *x*: *x*[0] = initial value
find an iterative improvement of *x*[*m*], that is: *x*[*m*+1] = g(*x*[*m*])
a solution is reached when the difference between two iterations is
less than a specified accuracy factor: $x^{[m+1]} - x^{[m]} \leq \varepsilon$
acceleration factors: can improve the rate of convergence. the improvement is found as

$$x_{acc}^{m+1} = x_{acc}^{m} + \propto [g(x)^{m} - x_{acc}^{m}]$$

Example

Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$

Step 1. Cast the equation into the $g(x)$ form.

$$9x = -x^3 + 6x^2 + 4$$

$$x = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} = g(x)$$

Step 2. Starting with an initial guess of x[0] = 2, several iterations

are performed.

$$x^{[1]} = g\left(x^{[0]} = 2\right) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.222$$

$$x^{[2]} = g\left(x^{[1]} = 2.2222\right) = -\frac{1}{9}(2.2222)^3 + \frac{6}{9}(2.2222)^2 + \frac{4}{9} = 2.5173$$

$$x^{[3]} = g\left(x^{[2]} = 2.5173\right) = -\frac{1}{9}(2.5173)^3 + \frac{6}{9}(2.5173)^2 + \frac{4}{9} = 2.8966$$

$x^{[4]} = 3.3376$

$x^{[5]} = 3.7398$

$x^{[6]} = 3.9568$

$x^{[7]} = 3.9988$

$x^{[8]} = 4$

Example

Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$ with an acceleration factor of 1.25

Starting with an initial guess of $x[0] = 2$.

$x^{[0]} = 2$

$$g(2) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222$$

$x^{[1]} = 2 + 1.25[2.2222 - 2] = 2.2778$

$$g(2.2778) = -\frac{1}{9}(2.2778)^3 + \frac{6}{9}(2.2778)^2 + \frac{4}{9} = 2.5902$$

$x^{[2]} = 2.2778 + 1.25[2.5902 - 2.2778] = 2.6683$

$x^{[3]} = 3.0801$

$x^{[4]} = 3.1831$

$x^{[5]} = 3.7238$

$x^{[6]} = 4.0084$

$x^{[7]} = 3.9978$

$x^{[8]} = 4.0005$