Example 1:

For the 3-bus system in Fig below, the line per-unit reactances are: $x_{12}=0.75$, $x_{13}=0.25$, *and* $x_{23}=0.50$.

Bus 1 is selected as the slack bus, and its voltage set at

$V_1 = 1.0\angle 0\ pu.$

Bus 2 is a generator bus with voltage magnitude $V_2 = 1.02\ pu.$ and real injected power, $P_2=0.6\ pu.$

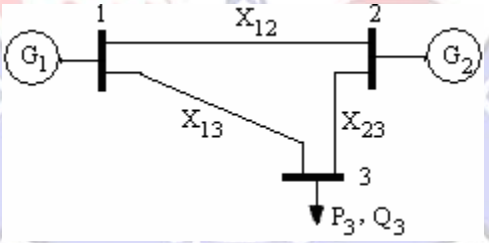
Bus 3 is a load bus with injected power:

$P_3 = -0.8\ pu$ And $Q_3 = -0.6\ pu.$

a) Use the Gauss-Seidel method to determine the power flow solution for power system within a V_3 tolerance of 0.01 pu. For

b) Calculate the power flow through line 1-3 at both ends

c) Calculate the power loss on line 1-3.



Solution

Step 1: formulate Y_{bus}

$Y_{12} = -\frac{1}{jx_{12}} = -\frac{1}{j0.75} = j1.33$ $\qquad$ $Y_{11} = Y_{12} + Y_{13} = -j5.33$

$Y_{13} = -\frac{1}{jx_{13}} = j4$ $\qquad$ $Y_{22}=Y_{12}+Y_{23} =$-j3.33

$Y_{23} = -\frac{1}{jx_{23}} = j2$ $\qquad$ $Y_{33}=Y_{13}+Y_{23} =$-j6

Known quantities:

Bus 1: Slack bus: $V_1 = 1 \angle 0^\circ \, pu$

Bus 2: PV bus: $|V_2| = 1.02 \; pu, \quad P_2 = 0.6pu$

Bus 3: PQ bus: $P_3 = -0.8 \; pu, \quad Q_3 = -0.6pu$

Unknown quantities to be determined

Bus 2: $\delta_2 \quad Q_2$

Bus 3: $\delta_3 \quad |V_3|$

Using a flat start: = $\left|V_3^{(0)}\right| = 1$

$$\delta_2^{(0)} = \delta_3^{(0)} = 0$$

Find Q_i from the equation

$$Q_i^{(m)} = -\mathrm{Im}\, ag[V_i^{*(m)}\{V_i^{(m)}Y_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}V_k^{(m)}\}]$$

Find V_I from the equation

$$V_i^{(m+1)} = \frac{1}{Y_{ii}}[\frac{P_i - jQ_i}{V_i^{(m)*}} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}.V_k^{(\beta)}]$$

β= m k> i

β= m+1 k< i

-the iteration m=0

$$Q_2^{(0)} = -Imag[V_2^{(0)*}\left(V_2^{(0)}Y_{22} + Y_{21}V_1^{(0)} + Y_{23}V_3^{(0)}\right)]$$

$$= -Imag[(1.02\llcorner 0)^*\{(1.02)\llcorner 0(-j3.33) + (j1.33)1\angle 0 + (j2)1\angle 0\}] = 0.07$$

Iteration 1

$$V_2^{(1)} = \frac{1}{Y_{22}}[\frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{21}V_1^{(m+1)} - Y_{23}V_3^{m}]$$

$$= \frac{1}{-j3.33}[\frac{0.6 - j0.07}{(1.02\angle 0)^*} - (j1.33)(1\angle 0) - (j2)1\angle 0]$$

$$= 1.04\angle 9.8$$

$$|V_2| specified \Rightarrow V_2^{(1)} = |V_{2spec}|\angle \delta_{2calcul}$$

$$V_2^{(1)} = 1.02\angle 9.8$$

$$V_3^{(1)} = \frac{1}{Y_{33}}[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31}V_1^{(m+1)} - Y_{32}V_2^{(m+1)}]$$

$$= \frac{1}{-j6}[\frac{-0.8 + j0.6}{(1\angle 0)^*} - (j4)(1\angle 0) - (j2)(1.02\angle 9.8]$$

$$= 0.91\angle -4.8$$

Test for convergence

$$\Delta V_3^{(1)} = |\Delta V_3^{(1)} - \Delta V_3^{(0)}| = 0.09$$

$$Q_2^{(1)} = -Imag[V_2^*(V_2Y_{22} + Y_{21}V_1 + Y_{23}V_3)]$$

$$= -Im[1.02\angle 9.8\{(-j3.33)1.02\angle 9.8 + (j1.33)1\angle 0 + (j2)0.91\angle -4.8 = 0.33$$

Iteration 2

$$V_2^{(2)} = \frac{1}{Y_{22}}[\frac{P_2 - jQ_2}{V_2^{(1)*}} - Y_{21}V_1 - Y_{23}V_3]$$

$$= \frac{1}{-j3.33}[\frac{0.6 - j0.33}{(1.02\angle 9.8)^*} - (j1.33)(1\angle 0) - (j2)0.91\angle -4.8] = 1.02\angle 8.2$$

$$V_3^{(2)} = \frac{1}{Y_{33}}[\frac{P_3 - jQ_3}{V_3^{(1)*}} - Y_{31}V_1 - Y_{32}V_2]$$

$$= \frac{1}{-j6}[\frac{-0.8 + j0.6}{(0.91\angle -4.8)^*} - (j4)(1\angle 0) - (j2)(1.02\angle 8.2]$$

$$= 0.89\angle -5.7$$

$$\Delta V_3^{(2)} = \left|\Delta V_3^{(2)} - \Delta V_3^{(1)}\right| = 0.89 - 0.91 = 0.02$$

$$Q_2^{(2)} = -Im[(1.02\angle 8.2)^* \{(-j3.33)1.02\angle 8.2 + (j1.33) + (j2)0.89\angle -5.7\}] = 0.36$$

Iteration 3

$$V_2^{(3)} = \frac{1}{Y_{22}}[\frac{P_2 - jQ_2}{V_2^{(2)*}} - Y_{21}V_1 - Y_{23}V_3]$$

$$= \frac{1}{-j3.33}[\frac{0.6 - j0.36}{(1.02\angle 8.2)^*} - (j1.33)(1\angle 0) - (j2)0.89\angle -5.7]$$

$$= 1.02\angle 7.7$$

$$V_3^{(3)} = \frac{1}{Y_{33}}[\frac{P_3 - jQ_3}{V_3^{(2)*}} - Y_{31}V_1 - Y_{32}V_2]$$

$$= \frac{1}{-j6}[\frac{0.8 + j0.6}{(0.89\angle -5.7)^*} - (j4) - (j2)(1.02\angle 7.7]$$

$$= 0.88\angle -6$$

$$\Delta V_3^{(3)} = \left|\Delta V_3^{(3)} - \Delta V_3^{(2)}\right| = 0.01$$

$$Q_2^{(3)} = -Imag\left[V_2^{(3)*}(V_2Y_{22} + Y_{21}V_1 + Y_{23}V_3)\right] = 0.38$$

$V_1 = 1\angle 0, \quad V_2 = 1.02\angle 7.7\ , \quad V_3 = 0.88\angle -6$

(2) Power flow in line 1-3

$$S_{13} = V_1\left(\frac{V_1 - V_3}{jX_{13}}\right) = 1\angle 0\left[\frac{1\llcorner 0 - 0.88\angle -6}{j0.25}\right] = 0.62\llcorner 53.5$$

$$S_{31} = V_3\left(\frac{V_3 - V_1}{jX_{13}}\right) = 0.88\angle -6\left[\frac{0.88\llcorner -6 - 1\llcorner 0}{j0.25}\right] = 0.62\angle -126$$

Power loss on line 1-3 = S_{13} +S_{31} =$4.4 \times 10^{-3}\angle -0.0032$