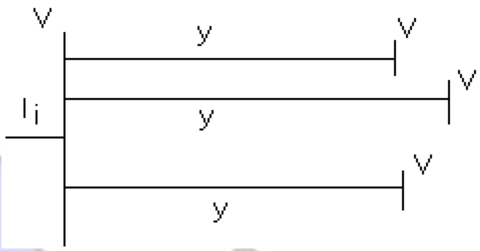
Newton-Raphson Method for Power Flow Solution

The N-R method has rapid convergence independent from system size. Thus, the method usually converges in less than 10 iterations

For the typical node shown



$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_i = \sum_{k=1}^{n} |Y_{ik}|\,|V_k| \angle(\theta_{ik} + \delta_k) \ldots\ldots\ldots\ldots (2)$$

$$S_i^* = P_i - jQ_i = V_i^* I_i \ldots\ldots\ldots\ldots\ldots (3)$$

Substitute (2) in (3)

$$P_i - jQ_i = |V_i| \angle(-\delta_i) \sum_{k=1}^{n} |Y_{ik}||V_k| \angle(\theta_{ik} + \delta_k)$$

$$P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = -\sum_{k=1}^{n} |V_i||V_k||Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$$

If these equations are linearized as before around the point δ_{ik} and $|V_k|$ using the Taylor series expansion around the points we have

$$\begin{bmatrix} \Delta P_2^{(m)} \\ \vdots \\ \Delta P_n^{(m)} \\ - \\ \Delta Q_2^{(m)} \\ \vdots \\ \Delta Q_n^{(m)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(m)}}{\partial \delta_2} & \frac{\partial P_2^{(m)}}{\partial \delta_n} & \frac{\partial P_2^{(m)}}{\partial |V_2|} & \frac{\partial P_2^{(m)}}{\partial |V_n|} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_n^{(m)}}{\partial \delta_2} & \frac{\partial P_n^{(m)}}{\partial \delta_n} & \frac{\partial P_n^{(m)}}{\partial |V_2|} & \frac{\partial P_n^{(m)}}{\partial |V_n|} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial Q_2^{(m)}}{\partial \delta_2} & \frac{\partial Q_2^{(m)}}{\partial \delta_n} & \frac{\partial Q_2^{(m)}}{\partial |V_2|} & \frac{\partial Q_2^{(m)}}{\partial |V_n|} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial Q_n^{(m)}}{\partial \delta_2} & \frac{\partial Q_n^{(m)}}{\partial \delta_n} & \frac{\partial Q_n^{(m)}}{\partial |V_2|} & \frac{\partial Q_n^{(m)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(m)} \\ \vdots \\ \Delta \delta_n^{(m)} \\ - \\ \Delta \left|V_2^{(m)}\right| \\ \vdots \\ \Delta \left|V_n^{(m)}\right| \end{bmatrix}$$

The above matrix can put into the form below

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

or

$$\Delta U = J \cdot \Delta X$$

ΔU: power mismatch

J: Jacobean matrix

ΔX: error correction

P_1, Q_1:- of the slack bus can be computed since for this bus we know V_1 and δ_1.

For every generator bus we know P and $|V|$ for that bus, hence the equation in ΔQ and $\Delta|V|$ for that bus is not needed. Thus we can delete the corresponding row and column from Jacobean matrix. Jacobean matrix is evaluated at every iteration step.

Starting with some initial value for $|V|$ and δ and solve these equations for $\Delta\delta$ and $\Delta|V|$ we can proceed to the next iteration thus:

$$\delta_i^{(m+1)} = \delta_i^m + \Delta\delta_i^{(m)}$$

$$\left|V_i^{(m+1)}\right| = \left|V_i^{(m)}\right| + \Delta\left|V_i^{(m)}\right|$$

We can stop the iteration process when the *power residual* are smaller than a pre-specified value. These are

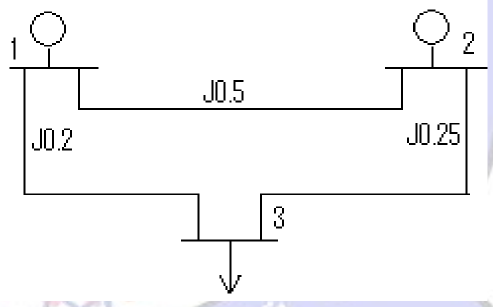
$$\Delta P_i^{(m)} = P_{i(spec)} - P_i^{(m)}$$

$$\Delta Q_i^{(m)} = Q_{i(spec\}} - Q_i^{(m)}$$

These are known as the *power mismatch*.

Example: - a three bus system shown in figure:

$$V_1 = 1.0\angle 0, \quad |V_2| = 1.0, \quad P_2 = 0.6, \quad S_3 = 0.8 + 0.6$$



Solution:

Step 1:

$$Y_{bus} = \begin{bmatrix} -j7 & j2 & j5 \\ j2 & -j6 & j4 \\ j5 & j4 & -j9 \end{bmatrix}$$

Step 2: the initial values

$$\left|V_3^{(0)}\right| = 1, \quad \delta_2^{(0)} = 0, \quad \delta_3^{(0)} = 0$$

Step3:

$$P_2^{(0)} = |V_2||V_1||Y_{21}|\cos(\delta_1 - \delta_2 + \theta_{21}) + |V_2||V_2||Y_{22}|\cos(\delta_2 - \delta_2 + \theta_{22}) + |V_2||V_3||Y_{23}|\cos(\delta_3 - \delta_2 + \theta_{23})$$

$$Q_2^{(0)} = -[|V_2||V_1||Y_{21}|\sin(\delta_1-\delta_2+\theta_{21}) + |V_2||V_2||Y_{22}|\sin(\delta_2-\delta_2+\theta_{22}) + |V_2||V_3||Y_{23}|\sin(\delta_3-\delta_2+\theta_{23})]$$

$$P_3^{(0)} = |V_1||V_3||Y_{31}|\cos(\delta_1-\delta_3+\theta_{31}) + |V_2||V_3||Y_{32}|\cos(\delta_2-\delta_3+\theta_{32}) + |V_3||V_3||Y_{33}|\cos(\delta_3-\delta_3+\theta_{33})$$

$$Q_3^{(0)} = -[|V_1||V_3||Y_{31}|\sin(\delta_1-\delta_3+\theta_{31}) + |V_2||V_3||Y_{32}|\sin(\delta_2-\delta_3+\theta_{32}) + |V_3||V_3||Y_{33}|\sin(\delta_3-\delta_3+\theta_{33})]$$

$P_{2(s)} = 0.6,\quad P_{3(s)} = -0.8,\quad Q_{3(s)} = -0.6$

$$P_2^{(0)} = 1.1.2\cos(0-0+90) + 1.1.6\cos(0-0-90) + 1.1.4\cos(0-0+90) = 0$$

$P_3^{(0)} = 0,\quad Q_3^{(0)} = 0$

$$\Delta P_2{}^{(0)} = \left|P_{2(s)} - P_2^{(0)}\right| = 0.6$$

$$\Delta P_3{}^{(0)} = \left|P_{3(s)} - P_3^{(0)}\right| = -0.8$$

$$\Delta Q_3{}^{(0)} = \left|Q_{3(s)} - Q_3^{(0)}\right| = -0.6$$

Step 4:

$$\frac{\partial P_2{}^{(0)}}{\partial \delta_2} = V_1V_2\,Y_{21}\sin(\delta_1-\delta_2+\theta_{21}) + V_2V_3Y_{23}\sin(\delta_3-\delta_2+\theta_{23}) = 2+4=6$$

$$\frac{\partial P_2{}^{(0)}}{\partial \delta_3} = -V_2V_3Y_{23}\sin(\delta_3-\delta_2+\theta_{23}) = -4$$

$$\frac{\partial P_2{}^{(0)}}{\partial |V_3|} = V_2Y_{23}\cos(\delta_3-\delta_2+\theta_{23}) = 0$$

$$\frac{\partial P_3}{\partial \delta_2}^{(0)} = -V_2 V_3 Y_{32} \sin(\delta_2 - \delta_3 + \theta_{32}) = -4$$

$$\frac{\partial P_3}{\partial \delta_3}^{(0)} = V_1 V_3 \, Y_{31} \sin(\delta_1 - \delta_3 + \theta_{31}) + V_2 V_3 Y_{32} \sin(\delta_2 - \delta_3 + \theta_{32}) = 5 + 4 = 9$$

$$\frac{\partial P_3}{\partial |V_3|}^{(0)} = V_1 \, Y_{31} \cos(\delta_1 - \delta_3 + \theta_{31}) + V_2 Y_{32} \cos(\delta_2 - \delta_3 + \theta_{32}) + 2V_3 Y_{33} \cos(\theta_{33}) = 0$$

$$\frac{\partial Q_3}{\partial \delta_2}^{(0)} = -V_2 V_3 \, Y_{32} \cos(\delta_2 - \delta_3 + \theta_{32}) = 0$$

$$\frac{\partial Q_3}{\partial \delta_3}^{(0)} = V_1 V_3 \, Y_{31} \cos(\delta_1 - \delta_3 + \theta_{31}) + V_2 V_3 Y_{32} \cos(\delta_2 - \delta_3 + \theta_{32}) = 0$$

$$\frac{\partial Q_3}{\partial |V_3|}^{(0)} = -V_1 \, Y_{31} \sin(\delta_1 - \delta_3 + \theta_{31}) - V_2 Y_{32} \sin(\delta_2 - \delta_3 + \theta_{32}) - 2V_3 Y_{33} \sin(\theta_{33}) = -5 - 4 + 2(9) = 9$$

$$\begin{bmatrix} 0.6 \\ -0.8 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} \begin{matrix} \textit{divided by } 6 \\ \textit{dividede by } 4 \\ \end{matrix}$$

$$\begin{bmatrix} 0.1 \\ -0.2 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 1 & -0.667 & 0 \\ -1 & 2.25 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} \textit{add this roe to row one}$$

$$\begin{bmatrix} 0.1 \\ -0.1 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 1 & -0.667 & 0 \\ 0 & 1.583 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} \textit{divided by } 1.583$$

$$\begin{bmatrix} 0.1 \\ -0.063 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 1 & -0.667 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

$$\Delta|V_3|^{(0)} = -\frac{0.6}{9} = -0.067$$

$$\Delta\delta_3{}^{(0)} = -0.063$$

$$\Delta\delta_2{}^{(0)} = -0.1 - 0.667 * \Delta\delta_3 = 0.058$$

$$|V_3|^{(1)} = |V_3|^{(0)} + \Delta|V_3|^{(0)} = 1 - 0.067 = 0.933$$

$$\delta_2 = 0 + 0.058$$

$$\delta_3 = 0 - 0.063 = -0.063$$

$$P_2^{(1)} = 1 \cdot 1 \cdot 2\cos(0 - 0.058 + 90) + 1 \cdot 1 \cdot 6\cos(-90) + 1 \cdot 4 \cdot (0.933)\cos(-0.063 - 0.058 + 90) = 0.0099$$

$$P_3^{(1)} = 1 \cdot 5 \cdot (0.933)\cos(90 + 0.063) + 4 \cdot (0.933)\cos(90 + 0.058 + 0.063) + 9(0.933)^2\cos(-90) = -0.013$$

$$Q_3^{(1)} = -[1 \cdot 5 \cdot (0.933)\sin(90 + 0.063) + 4 \cdot (0.933)\sin(90 + 0.058 + 0.063) + 9(0.933)^2\sin(-90)] = -0.562$$

$$\Delta P_2^{(1)} = 0.6 - 0.0099 = 0.59$$

$$\Delta P_3^{(1)} = -0.8 + 0.013 = -0.787$$

$$\Delta Q_3^{(1)} = -0.6 + 0.5625 = -0.0375$$

Continue further iteration