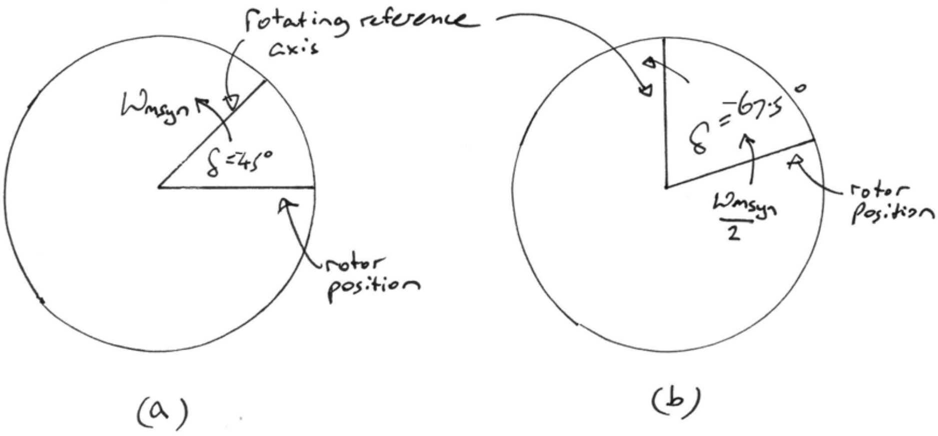
Example: consider the diagram in Figure below. the rotor is rotating at half the synchronous speed $\omega_{msyn}/2$

such that in the time it takes for the reference axis to rotate 45° the rotor only rotates 22.5° and the rotor angular position with reference to the rotating axis changes from -45 ° to -67.5°.



From eq(2) and (4), we see that equation (1) can be written as:

$$J\alpha_m = J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e = T_a \ldots\ldots\ldots\ldots\ldots\ldots 5$$

Since $P = T * \omega_m$

So

$$J\omega_m \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \quad Watt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

$J\omega_m$: angular momentum of the rotor at synchronous speed= M

And called the inertia constant

$$M\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

Let us define (H) constant

$$H = \frac{stored\ kinetic\ energy\ in\ (Mega\ Joules)\ at\ synchronous\ speed}{generator\ MVA\ rating}$$

$$= \frac{\frac{1}{2}J\omega_{msyn}^2}{S_{rated}} = \frac{\frac{1}{2}J\omega_{msyn.}\omega_{msyn\cdot}}{S_{rated}} = \frac{\frac{1}{2}M\omega_{msyn\cdot}}{S_{rated}} \quad MJ/MVA \ldots\ldots\ldots\ldots 8$$

$$\therefore \quad M = \frac{2H}{\omega_{msyn}} S_{rated} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

$$\therefore \quad \frac{2H}{\omega_{msyn}}\frac{d^2\delta_m}{dt^2} = \frac{P_a}{S_{rated}} = \frac{P_m - P_e}{S_{rated}}$$

$$\therefore \quad \frac{2H}{\omega_{msyn}}\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \quad per\ unit$$

When the swing equation is solved we obtain the expression for δ as afunction of time. A graph of a solution is called the swing curve of the machine and inspection of the swing curve of all the machines of the system will show whether the machines remain in synchronism after disturbance.

In a stability study of a power system, only one MVA base common to all parts of the system can be chosen , (usually 100MVA). Then H must be also consistent with the system base

$$H_{system} = H_{machine} * \frac{S_{machine}}{S_{system}}$$

So we can represent the system as one machine just as if their rotors were mechanically coupled and only one swing equation need to be written for them

$$\frac{2H_1}{\omega_{msyn}}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\frac{2H_2}{\omega_{msyn}}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

Adding the two equations

$$\frac{2H}{\omega_{msyn}}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Where

$$H = (H_1 + H_2)$$

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

When a synchronous generator has P poles, the synchronous electrical angular velocity, ω_{syn}, known more correctly as the synchronous electrical radian frequency, can be related to the synchronous mechanical angular velocity by the following relationship.

$$\omega_{syn} = \frac{P}{2}\omega_{msyn}$$

To understand how this relationship arises, consider that the number of mechanical radians in one full revolution of the rotor is 2π. If, for instance, a generator has 4 poles (2 pairs), and there are 2π electrical radians between poles in a pair then the electrical waveform will go through 2*2π=4π electrical radians within the same revolution of the rotor.

In general the number of electrical radians in one revolution is the number of mechanical radians times the number of pole pairs (the number of poles divided by two).

The relationship also holds for the electrical angular acceleration α(t), the electrical radian frequency ω(t), and the electrical power angle δ values.

$$\alpha(t) = \frac{P}{2}\alpha_m(t)$$

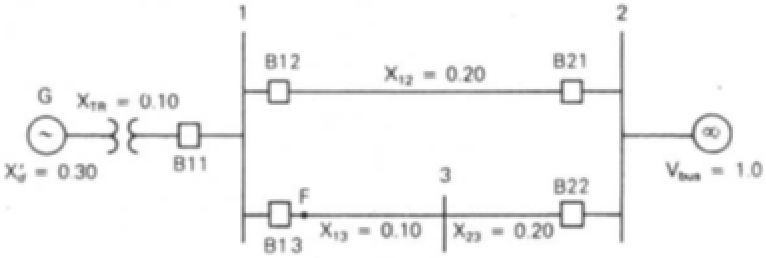
$$\omega(t) = \frac{P}{2}\omega_m(t)$$

$$\delta(t) = \frac{P}{2}\delta_m(t)$$

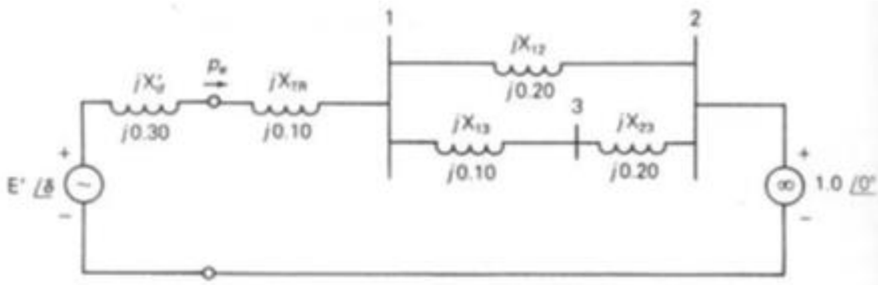
$$P_{a(pu)} = P_{T(pu)} - P_{g(pu)} = \frac{H}{\pi f}\frac{d^2\delta}{dt^2} \quad per\ unit\ MW$$

$$P_{a(pu)} = P_{T(pu)} - P_{g(pu)} = \frac{H}{180 f}\frac{d^2\delta}{dt^2} \quad per\ unit\ MW$$

Example: figure below shows a single line diagram of three phase, 60Hz synchronous generator, connected though a transformer and parallel transmission lines to the ins. All reactances are given in per unit on a common system base. If the infinite bus receive 1.0 per unit real power at 0.85 p.f lagging determine the internal voltage of generator and the equation for the electrical power delivered by the generator virus its power angle δ



Sol.:



$$X_{eq} = X'_d + X_{TR} + X_{12} \| (X_{13} + X_{23})$$
$$= 0.30 + 0.10 + 0.20 \| (0.10 + 0.20)$$
$$= 0.520 \quad \text{per unit}$$

The current into the infinite bus is

$$I = \frac{P}{V_{bus}(\text{p.f.})} \angle -\cos^{-1}(\text{p.f.}) = \frac{(1.0)}{(1.0)(0.95)} \angle -\cos^{-1} 0.95$$
$$= 1.05263 \angle -18.195^\circ \quad \text{per unit}$$

and the machine internal voltage is

$$E' = E' \angle \delta = V_{bus} + jX_{eq} I$$
$$= 1.0 \angle 0^\circ + (j0.520)(1.05263 \angle -18.195^\circ)$$
$$= 1.0 \angle 0^\circ + 0.54737 \angle 71.805^\circ$$
$$= 1.1709 + j0.5200 \quad = 1.2812 \angle 23.946^\circ \quad \text{per unit}$$
$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin\delta = 2.4638 \sin\delta \quad \text{per unit}$$

Example 2: *A 60 Hz, 4 pole turbogenerator rated 100 MVA, 13.8 KV has an inertia constant of 10 MJIMVA.*

(*a*) Find the stored energy in the rotor at synchronous speed.

(b) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.

(c) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.

Solution:

a) Stored energy = GH

G = 100 MVA, H = 10 MJ/MVA.

Stored energy = 100 x 10 = 1000 MJ.

b)

$$P_a = P_i - P_e = 60 - 50 = 10 \text{ MW.}$$

we know, $$M = \frac{GH}{180f} = \frac{100 \times 10}{180 \times 60} = \frac{5}{54} \text{ MJ-Sec/elect deg.}$$

Now $$M.\frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \frac{5}{54}\frac{d^2\delta}{dt^2} = 10$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{10 \times 54}{5} = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \alpha = \text{acceleration} = 108 \text{ elect-deg/Sec}^2$$

c)

$$12 \text{ cycles} = \frac{12}{60} = 0.2 \text{ sec.}$$

$$\text{Change in } \delta = \frac{1}{2}\alpha.(\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2 \text{ elect-degree.}$$

$$= 2.16 \text{ elect-degree.}$$

Now $$\alpha = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \alpha = 60 \times \frac{108}{360^\circ} \text{ rpm/Sec} = 18 \text{ rpm/Sec.}$$

$$\therefore \text{Rotor speed at the end of 12 cycles} = \frac{120f}{P} + \alpha(\Delta t)$$

$$= \left(\frac{120 \times 60}{4} + 18 \times 0.2\right) \text{rpm} = 1803.6 \text{ rpm}$$

Example 3: *A* 100 MVA, two-pole, 50 Hz generator has a moment of inertia 40 x 103 *Kg-m*2. What is the energy stored in the rotor at the rated speed? What is the corresponding angular momentum? Determine the inertia constant *H*.

Solution:

$$\eta_s = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm.}$$

The stored energy is

$$\text{KE (stored)} = \frac{1}{2} J w_m^2 = \frac{1}{2}(40 \times 10^3)\left(\frac{2\pi \times 3000}{50}\right)^2 \text{ MJ}$$

$$= 2842.4 \text{ MJ}$$

Then

$$H = \frac{\text{KE (stored)}}{\text{MVA}} = \frac{2842.4}{100} = 28.424 \text{ MJ/MVA.}$$

$$M = Jw_m = (40 \times 10^3)\left(\frac{2\pi \times 3000}{50}\right)$$

$$\therefore \quad M = 15.07 \text{ MJ-}Sec\text{/mech-radian} \qquad \textbf{Ans.}$$