1-4 The equal –Area stability criterion:

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2}=P_T-P_g=P_a$$

$$P_g=\frac{|E'||V|}{X'_d+X_e}\sin\delta$$

$$2\times\frac{d^2\delta}{dt^2}\times\frac{d\delta}{dt}=\frac{2\pi f}{H}P_a\times\frac{d\delta}{dt}$$

Or

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2=\frac{2\pi f}{H}P_a\times\frac{d\delta}{dt}$$

$$\left(\frac{d\delta}{dt}\right)^2=\frac{2\pi f}{H}\int_{\delta_0}^{\delta_2}P_a\,d\delta$$

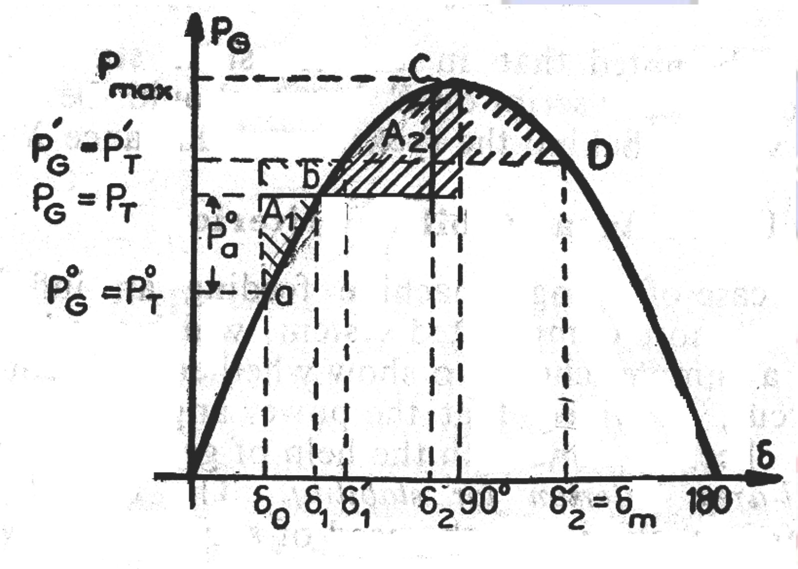
$$\frac{d\delta}{dt}=\omega=\sqrt{\frac{2\pi f}{H}\int_{\delta_0}^{\delta_2}P_a\,d\delta}$$

- Before disturbance, machine is operated with synchronous speed , therefore , $\frac{d\delta}{dt}=0$
- Also , if the system has transient stability , the machine will again operate at synchronous speed after the disturbance i.e. $\frac{d\delta}{dt}=0$

- Therefore , if the system has transient stability it fulfils the condition :

$$\sqrt{\frac{2\pi f}{H}\int_{\delta_0}^{\delta_2} P_a \, d\delta} = 0 = \int_{\delta_0}^{\delta_2} P_a \, d\delta$$

This mean , total area under the $P-\delta$ curve between the limits δ_0 and δ_2 should be zero .



A_1 - acceleration area , A_2 - deceleration area

The above criterion is known as the equal area criterion because the two shaded areas are equal in magnitude and opposite in sign . In other words , the above equation is the algebraic sum of the two area A_1 and A_2 , and therefore their sum should be zero .

$$\int_{\delta_0}^{\delta_1} (P_T - P_g)\, d\delta + \int_{\delta 1}^{\delta_2} (P_T - P_g)\, d\delta = 0$$

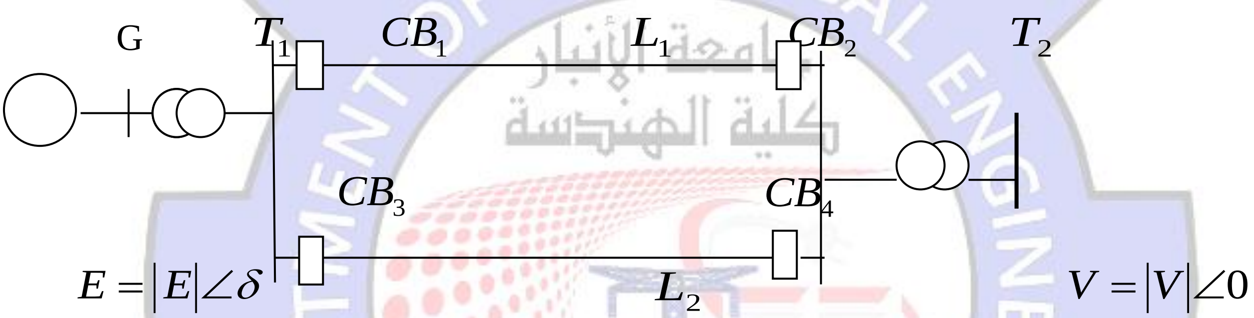
$$\int_{\delta_0}^{\delta_1} (P_T - P_g)\, d\delta - \int_{\delta 1}^{\delta_2} (P_g - P_T)\, d\delta = 0$$

$$\int_{\delta_0}^{\delta_1}(P_T - P_g)\,d\delta = \int_{\delta 1}^{\delta_2}(P_g - P_T)\,d\delta$$

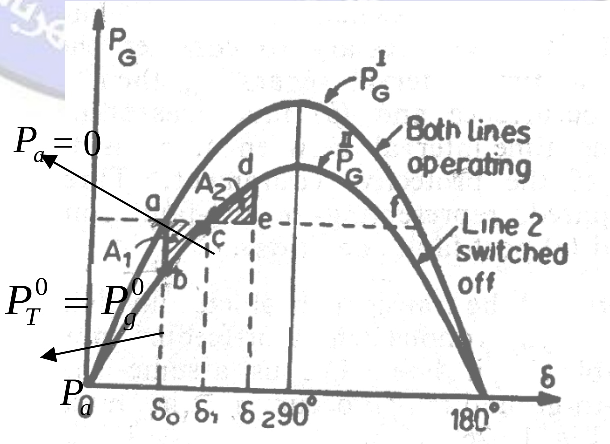
$$\therefore \quad A_1 = A_2$$

Application of equal area criterion for few examples of fault disturbances :

1- *ONE of double lines suddenly switched off* :



Line No. 2 suddenly switched off by opening CB_3 or CB_4



A_1 - acceleration area

A_2 - deceleration area

<u>Before fault :</u>

$$P_g^I = \frac{|E||V|}{X^I} \sin\delta = P_{\max}^I \sin\delta$$

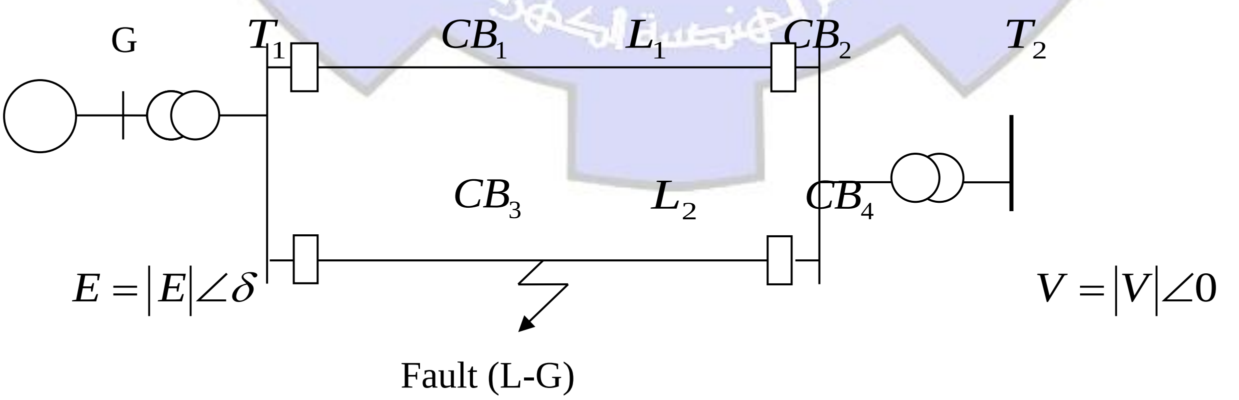
X^I - transfer reactance before switching off line No. 2 .

<u>After fault :</u>

$$P_g^{II} = \frac{|E||V|}{X^{II}} \sin\delta = P_{\max}^{II} \sin\delta$$

X^{II} - transfer reactance after the line N0. 2 is switched off .

2- *System fault and line switching :*



a) Fault occurrence.
b) Fault clearing by the CB.

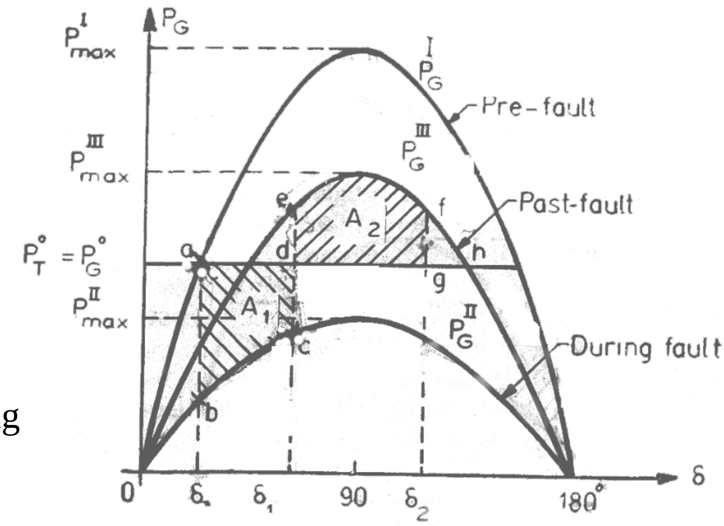
The time interval between these conditions is determined by the delay setting of the protection equipment. Therefore, three $P-\delta$ curves are required representing:

i) pre fault condition

ii) fault condition .

iii) post fault condition .

δ_1 - clearing angle depend on clearing time t_1 .



The curves in above figure are drawn from the following equations :

i) for pre fault condition :

$$P_g^{I} = \frac{|E||V|}{X^{I}} \sin\delta = P_{\max}^{I} \sin\delta$$

ii) For fault condition :

$$P_g^{II} = \frac{|E||V|}{X^{II}} \sin\delta = P_{\max}^{II} \sin\delta$$

iii) For post fault :

$$P_g^{III} = \frac{|E||V|}{X^{III}} \sin\delta = P_{\max}^{III} \sin\delta$$

Where :

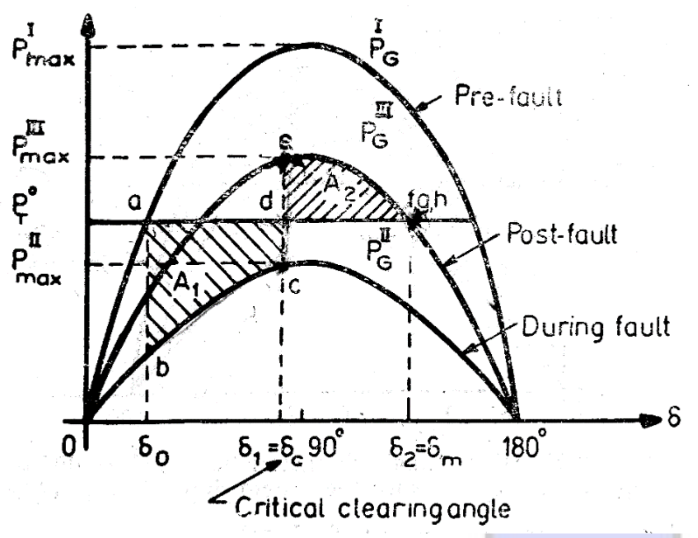
X^{I} , X^{II} , X^{III} - are the transfer reactance corresponding the above three conditions.

Critical clearing angle (δ_C)

From below figure:

$$A_1 = \int_{\delta_0}^{\delta_c} (P_T^0 - P_{\max}^{II} \sin\delta)\, d\delta$$

$$P_T^0 = P_{\max}^{I} \sin\delta$$



$$A_2 = \int_{\delta_c}^{\delta_m} (P_{\max}^{III} \sin\delta - P_T^0)\, d\delta$$

The angle :

$$\delta_0' = \sin^{-1}\left(\frac{P_T^0}{P_{\max}^{III}}\right)$$

And , $\delta_2 = \delta_m = \pi - \delta_0'$

$$= \pi - \sin^{-1}\left(\frac{P_T^0}{P_{\max}^{III}}\right)$$

If , $A_1 = A_2$

$$\int_{\delta_0}^{\delta_c} (P_T^0 - P_{\max}^{II} \sin\delta)\, d\delta = \int_{\delta_c}^{\delta_m} (P_{\max}^{III} \sin\delta - P_T^0)\, d\delta$$

$$\left| P_T^0 \delta + P_{\max}^{II} \cos\delta \right|_{\delta_0}^{\delta_c} = \left| - P_{\max}^{III} \cos\delta - P_T^0 \delta \right|_{\delta_c}^{\delta_m}$$

$$P_T^0(\delta_c - \delta_0) + P_{\max}^{II}(\cos\delta_c - \cos\delta_0) =$$
$$- P_{\max}^{III}(\cos\delta_m - \cos\delta_c) - P_T^0(\delta_m - \delta_c)$$

$$\boxed{\cos\delta_c = \frac{P_T^0(\delta_m - \delta_0) - P_{\max}^{II}\cos\delta_0 + P_{\max}^{III}\cos\delta_m}{P_{\max}^{III} - P_{\max}^{II}}}$$

………(3)

The time (t) corresponding to a given value of the angle(δ) is :

$$t = \sqrt{\frac{2M(\delta - \delta_0)}{P^0}}$$

The maximum time clearing when a short circuit can still be cleared :

$$t_{clear} = \sqrt{\frac{M(\delta_c - \delta_0^I)}{P_{\max}^{II}(\sin\delta_c - \sin\delta_0^I)}} \times \cos^{-1}\left(\frac{\frac{P_0}{P_{\max}^{II}} - \sin\delta_c}{\frac{P_0}{P_{\max}^{III}} - \sin\delta_0} \right)$$