SPREAD-SPECTRUM MODULATION

This chapter introduces a modulation technique called spread-spectrum modulation, which is radically different from the modulation techniques that are covered in preceding chapters. In spread-spectrum modulation, channel bandwidth and transmit power are sacrificed for the sake of secure communications.

Specifically, we cover the following topics:

- *Spreading sequences in the form of pseudo-noise sequences, their properties, and methods of generation.*
- *The basic notion of spread-spectrum modulation.*
- *The two commonly used types of spread-spectrum modulation: direct sequence and frequency hopping.*

The material presented in this chapter is basic to wireless communications using code-division multiple access, which is covered in Chapter 8.

7.1 *Introduction*

A major issue of concern in the study of digital communications as considered in Chapters 4, 5, and 6 is that of providing for the efficient use of bandwidth and power. Notwithstanding the importance of these two primary communication resources, there are situations where it is necessary to sacrifice this efficiency in order to meet certain other design objectives. For example, the system may be required to provide a form of *secure* communication in a *hostile* environment such that the transmitted signal is not easily detected or recognized by unwanted listeners. This requirement is catered to by a class of signaling techniques known collectively as *spread-spectrum modulation.*

The primary advantage of a spread-spectrum communication system is its ability to reject *interference* whether it be the *unintentional* interference by another user simultaneously attempting to transmit through the channel, or the *intentional* interference by a hostile transmitter attempting to jam the transmission.

The definition of spread-spectrum modulation[1] may be stated in two parts:

1. Spread spectrum is a means of transmission in which the data sequence occupies a bandwidth in excess of the minimum bandwidth necessary to send it.
2. The spectrum spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The same code is used in the receiver

(operating in synchronism with the transmitter) to despread the received signal so that the original data sequence may be recovered.

Although standard modulation techniques such as frequency modulation and pulse-code modulation do satisfy part 1 of this definition, they are not spread-spectrum techniques because they do not satisfy part 2 of the definition.

Spread-spectrum modulation was originally developed for military applications, where resistance to jamming (interference) is of major concern. However, there are civilian applications that also benefit from the unique characteristics of spread-spectrum modulation. For example, it can be used to provide *multipath rejection* in a ground-based mobile radio environment. Yet another application is in *multiple-access* communications in which a number of independent users are required to share a common channel without an external synchronizing mechanism; here, for example, we may mention a ground-based radio environment involving mobile vehicles that must communicate with a central station. More is said about this latter application in Chapter 8.

In this chapter, we discuss principles of spread-spectrum modulation, with emphasis on direct-sequence and frequency-hopping techniques. In a *direct-sequence spread-spectrum* technique, two stages of modulation are used. First, the incoming data sequence is used to modulate a wideband code. This code transforms the narrowband data sequence into a noiselike wideband signal. The resulting wideband signal undergoes a second modulation using a phase-shift keying technique. In a *frequency-hop spread-spectrum* technique, on the other hand, the spectrum of a data-modulated carrier is widened by changing the carrier frequency in a pseudo-random manner. For their operation, both of these techniques rely on the availability of a noiselike spreading code called a *pseudo-random* or *pseudo-noise sequence*. Since such a sequence is basic to the operation of spread-spectrum modulation, it is logical that we begin our study by describing the generation and properties of pseudo-noise sequences.

7.2 *Pseudo-Noise Sequences*

A *pseudo-noise (PN) sequence* is a periodic binary sequence with a noiselike waveform that is usually generated by means of a *feedback shift register*, a general block diagram of which is shown in Figure 7.1. A feedback shift register consists of an ordinary *shift register* made up of m flip-flops (two-state memory stages) and a *logic circuit* that are interconnected to form a multiloop *feedback* circuit. The flip-flops in the shift register are regulated by a single timing *clock*. At each pulse (tick) of the clock, the *state* of each flip-flop is shifted to the next one down the line. With each clock pulse the logic circuit computes a

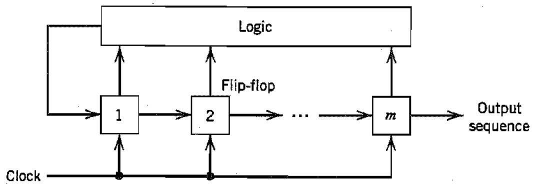

FIGURE 7.1 Feedback shift register.

Boolean function of the states of the flip-flops. The result is then fed back as the input to the first flip-flop, thereby preventing the shift register from emptying. The PN sequence so generated is determined by the length m of the shift register, its initial state, and the feedback logic.

Let $s_j(k)$ denote the state of the jth flip-flop after the kth clock pulse; this state may be represented by symbol 0 or 1. The state of the shift register after the kth clock pulse is then defined by the set $\{s_1(k), s_2(k), \ldots, s_m(k)\}$, where $k \geq 0$. For the initial state, k is zero. From the definition of a shift register, we have

$$s_j(k+1) = s_{j-1}(k), \qquad \begin{cases} k \geq 0 \\ 1 \leq j \leq m \end{cases} \tag{7.1}$$

where $s_0(k)$ is the input applied to the first flip-flop after the kth clock pulse. According to the configuration described in Figure 7.1, $s_0(k)$ is a Boolean function of the individual states $s_1(k), s_2(k), \ldots, s_m(k)$. For a specified length m, this Boolean function uniquely determines the subsequent sequence of states and therefore the PN sequence produced at the output of the final flip-flop in the shift register. With a total number of m flip-flops, the number of possible states of the shift register is at most 2^m. It follows therefore that the PN sequence generated by a feedback shift register must eventually become *periodic* with a period of at most 2^m.

A feedback shift register is said to be *linear* when the feedback logic consists entirely of *modulo-2 adders*. In such a case, the *zero state* (e.g., the state for which all the flip-flops are in state 0) is *not* permitted. We say so because for a zero state, the input $s_0(k)$ produced by the feedback logic would be 0, the shift register would then continue to remain in the zero state, and the output would therefore consist entirely of 0s. Consequently, the period of a PN sequence produced by a linear feedback shift register with m flip-flops cannot exceed $2^m - 1$. When the period is exactly $2^m - 1$, the PN sequence is called a *maximal-length-sequence* or simply *m-sequence*.

▶ EXAMPLE 7.1

Consider the linear feedback shift register shown in Figure 7.2, involving three flip-flops. The input s_0 applied to the first flip-flop is equal to the modulo-2 sum of s_1 and s_3. It is assumed that the initial state of the shift register is 100 (reading the contents of the three flip-flops from left to right). Then, the succession of states will be as follows:

100, 110, 111, 011, 101, 010, 001, 100, ...

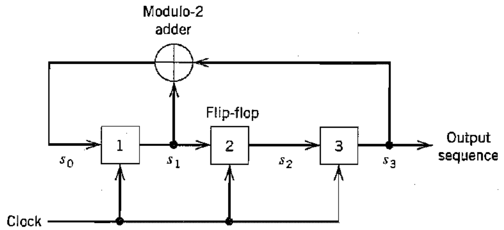

FIGURE 7.2 Maximal-length sequence generator for $m = 3$.

The output sequence (the last position of each state of the shift register) is therefore

$$00111010\ldots$$

which repeats itself with period $2^3 - 1 = 7$.

Note that the choice of 100 as the initial state is arbitrary. Any of the other six permissible states could serve equally well as an initial state. The resulting output sequence would then simply experience a cyclic shift. ◀

▪ PROPERTIES OF MAXIMAL-LENGTH SEQUENCES[2]

Maximal-length sequences have many of the properties possessed by a truly *random binary sequence*. A random binary sequence is a sequence in which the presence of binary symbol 1 or 0 is equally probable. Some properties of maximal-length sequences are as follows:

1. In each period of a maximal-length sequence, the number of 1s is always one more than the number of 0s. This property is called the *balance property*.

2. Among the runs of 1s and of 0s in each period of a maximal-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs. This property is called the *run property*. By a "run" we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence. The length of this subsequence is the length of the run. For a maximal-length sequence generated by a linear feedback shift register of length m, the total number of runs is $(N + 1)/2$, where $N = 2^m - 1$.

3. The autocorrelation function of a maximal-length sequence is periodic and binary-valued. This property is called the *correlation property*.

The period of a maximum-length sequence is defined by

$$N = 2^m - 1 \tag{7.2}$$

where m is the length of the shift register. Let binary symbols 0 and 1 of the sequence be denoted by the levels -1 and $+1$, respectively. Let $c(t)$ denote the resulting waveform of the maximal-length sequence, as illustrated in Figure 7.3a for $N = 7$. The period of the waveform $c(t)$ is (based on terminology used in subsequent sections)

$$T_b = NT_c \tag{7.3}$$

where T_c is the duration assigned to symbol 1 or 0 in the maximal-length sequence. By definition, the autocorrelation function of a periodic signal $c(t)$ of period T_b is

$$R_c(\tau) = \frac{1}{T_b}\int_{-T_b/2}^{T_b/2} c(t)c(t - \tau)\, dt \tag{7.4}$$

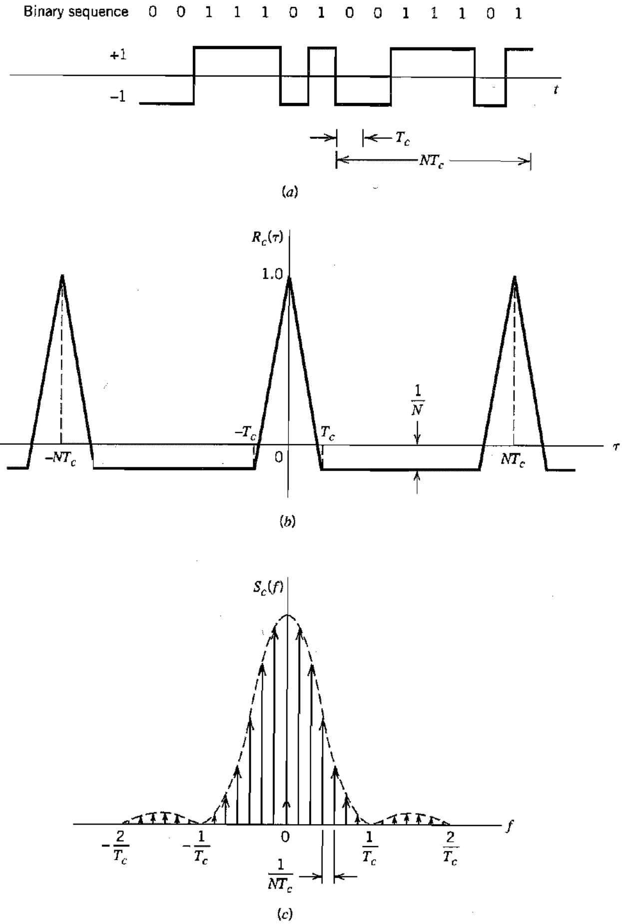

FIGURE 7.3 (*a*) Waveform of maximal-length sequence for length $m = 3$ or period $N = 7$. (*b*) Autocorrelation function. (*c*) Power spectral density. All three parts refer to the output of the feedback shift register of Figure 7.2.

where the lag τ lies in the interval $(-T_b/2, T_b/2)$; Equation (7.4) is a special case of Equation (1.26). Applying this formula to a maximal-length sequence represented by $c(t)$, we get

$$R_c(\tau) = \begin{cases} 1 - \dfrac{N+1}{NT_c}|\tau|, & |\tau| \le T_c \\ -\dfrac{1}{N}, & \text{for the remainder of the period} \end{cases} \tag{7.5}$$

This result is plotted in Figure 7.3*b* for the case of $m = 3$ or $N = 7$.