We thus see from Equation (7.11) that the data signal $b(t)$ is reproduced at the multiplier output in the receiver, except for the effect of the interference represented by the additive term $c(t)i(t)$. Multiplication of the interference $i(t)$ by the locally generated PN signal $c(t)$ means that the spreading code will affect the interference just as it did the original signal at the transmitter. We now observe that the data component $b(t)$ is narrowband, whereas the spurious component $c(t)i(t)$ is wideband. Hence, by applying the multiplier output to a baseband (low-pass) filter with a bandwidth just large enough to accommodate the recovery of the data signal $b(t)$, most of the power in the spurious component $c(t)i(t)$ is filtered out. The effect of the interference $i(t)$ is thus significantly reduced at the receiver output.

In the receiver shown in Figure 7.5*c*, the low-pass filtering action is actually performed by the integrator that evaluates the area under the signal produced at the multiplier output. The integration is carried out for the bit interval $0 \leq t \leq T_b$, providing the sample value v. Finally, a decision is made by the receiver: If v is greater than the threshold of zero, the receiver says that binary symbol 1 of the original data sequence was sent in the interval $0 \leq t \leq T_b$, and if v is less than zero, the receiver says that symbol 0 was sent; if v is exactly zero the receiver makes a random guess in favor of 1 or 0.

In summary, the use of a spreading code (with pseudo-random properties) in the transmitter produces a wideband transmitted signal that appears *noiselike* to a receiver that has *no* knowledge of the spreading code. From the discussion presented in Section 7.2, we recall that (for a prescribed data rate) the longer we make the period of the spreading code, the closer will the transmitted signal be to a truly random binary wave, and the harder it is to detect. Naturally, the price we have to pay for the improved protection against interference is increased transmission bandwidth, system complexity, and processing delay. However, when our primary concern is the security of transmission, these are not unreasonable costs to pay.

7.4 Direct-Sequence Spread Spectrum with Coherent Binary Phase-Shift Keying

The spread-spectrum technique described in the previous section is referred to as *direct-sequence spread spectrum*. The discussion presented there was in the context of baseband transmission. To provide for the use of this technique in passband transmission over a satellite channel, for example, we may incorporate *coherent binary phase-shift keying* (PSK) into the transmitter and receiver, as shown in Figure 7.7. The transmitter of Figure 7.7*a* first converts the incoming binary data sequence $\{b_k\}$ into a polar NRZ waveform $b(t)$, which is followed by two stages of modulation. The first stage consists of a product modulator or multiplier with the data signal $b(t)$ (representing a data sequence) and the PN signal $c(t)$ (representing the PN sequence) as inputs. The second stage consists of a binary PSK modulator. The transmitted signal $x(t)$ is thus a *direct-sequence spread binary phase-shift-keyed* (DS/BPSK) *signal*. The phase modulation $\theta(t)$ of $x(t)$ has one of two values, 0 and π, depending on the polarities of the message signal $b(t)$ and PN signal $c(t)$ at time t in accordance with the truth table of Table 7.3.

Figure 7.8 illustrates the waveforms for the second stage of modulation. Part of the modulated waveform shown in Figure 7.6*c* is reproduced in Figure 7.8*a*; the waveform shown here corresponds to one period of the PN sequence. Figure 7.8*b* shows the waveform of a sinusoidal carrier, and Figure 7.8*c* shows the DS/BPSK waveform that results from the second stage of modulation.

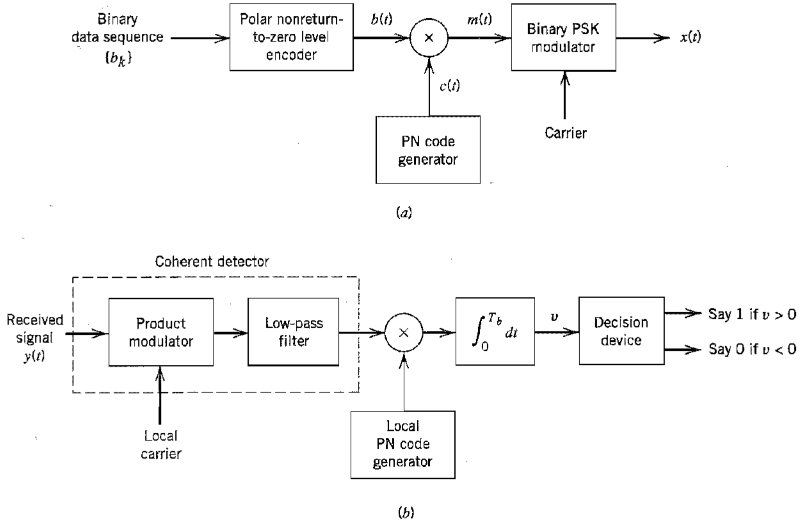

FIGURE 7.7 Direct-sequence spread coherent phase-shift keying. (*a*) Transmitter. (*b*) Receiver.

The receiver, shown in Figure 7.7*b*, consists of two stages of demodulation. In the first stage, the received signal $y(t)$ and a locally generated carrier are applied to a product modulator followed by a low-pass filter whose bandwidth is equal to that of the original message signal $m(t)$. This stage of the demodulation process reverses the phase-shift keying applied to the transmitted signal. The second stage of demodulation performs spectrum despreading by multiplying the low-pass filter output by a locally generated replica of the PN signal $c(t)$, followed by integration over a bit interval $0 \leq t \leq T_b$, and finally decision-making in the manner described in Section 7.3.

MODEL FOR ANALYSIS

In the normal form of the transmitter, shown in Figure 7.7*a*, the spectrum spreading is performed prior to phase modulation. For the purpose of analysis, however, we find it more convenient to interchange the order of these operations, as shown in the model of

TABLE 7.3 ***Truth table for phase modulation $\theta(t)$, radians***

		Polarity of Data Sequence b(t) *at Time* t	
		+	−
Polarity of PN sequence c(t) *at time* t	+	0	π
	−	π	0

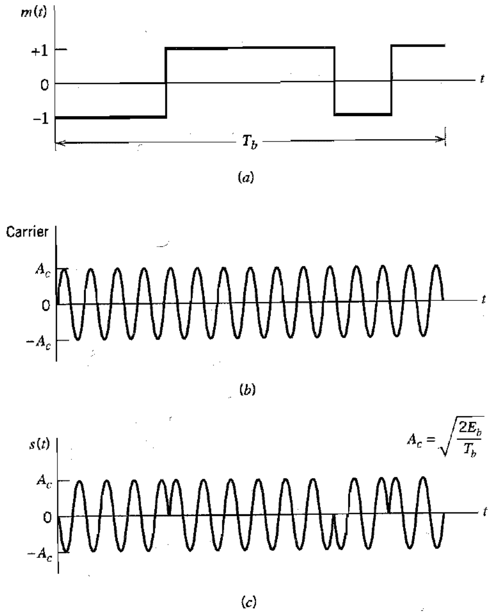

FIGURE 7.8 (*a*) Product signal $m(t) = c(t)b(t)$. (*b*) Sinusoidal carrier. (*c*) DS/BPSK signal.

Figure 7.9. We are permitted to do this because the spectrum spreading and the binary phase-shift keying are both linear operations; likewise for the phase demodulation and spectrum despreading. But for the interchange of operations to be feasible, it is important to synchronize the incoming data sequence and the PN sequence. The model of Figure 7.9 also includes representations of the channel and the receiver. In this model, it is assumed that the interference $j(t)$ limits performance, so that the effect of channel noise may be ignored. Accordingly, the channel output is given by

$$\begin{aligned} y(t) &= x(t) + j(t) \\ &= c(t)s(t) + j(t) \end{aligned} \tag{7.12}$$

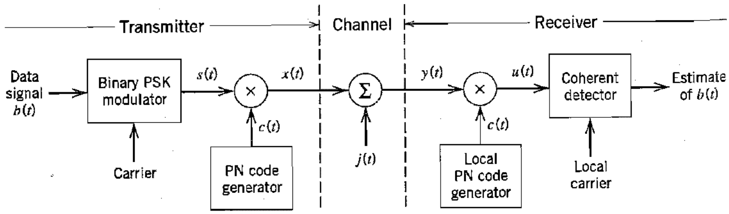

FIGURE 7.9 Model of direct-sequence spread binary PSK system.

where $s(t)$ is the binary PSK signal, and $c(t)$ is the PN signal. In the channel model included in Figure 7.9, the interfering signal is denoted by $j(t)$. This notation is chosen purposely to be different from that used for the interference in Figure 7.5*b*. The channel model in Figure 7.9 is passband in spectral content, whereas that in Figure 7.5*b* is in baseband form.

In the receiver, the received signal $y(t)$ is first multiplied by the PN signal $c(t)$ yielding an output that equals the coherent detector input $u(t)$. Thus,

$$\begin{aligned} u(t) &= c(t)y(t) \\ &= c^2(t)s(t) + c(t)j(t) \\ &= s(t) + c(t)j(t) \end{aligned} \tag{7.13}$$

In the last line of Equation (7.13), we have noted that, by design, the PN signal $c(t)$ satisfies the property described in Equation (7.10), reproduced here for convenience:

$$c^2(t) = 1 \qquad \text{for all } t$$

Equation (7.13) shows that the coherent detector input $u(t)$ consists of a binary PSK signal $s(t)$ embedded in additive code-modulated interference denoted by $c(t)j(t)$. The modulated nature of the latter component forces the interference signal (jammer) to spread its spectrum such that the detection of information bits at the receiver output is afforded increased reliability.

SYNCHRONIZATION

For its proper operation, a spread-spectrum communication system requires that the locally generated PN sequence used in the receiver to despread the received signal be *synchronized* to the PN sequence used to spread the transmitted signal in the transmitter.[4] A solution to the synchronization problem consists of two parts: *acquisition* and *tracking*. In acquisition, or *coarse* synchronization, the two PN codes are aligned to within a fraction of the chip in as short a time as possible. Once the incoming PN code has been acquired, tracking, or *fine* synchronization, takes place. Typically, PN acquisition proceeds in two steps. First, the received signal is multiplied by a locally generated PN code to produce a measure of *correlation* between it and the PN code used in the transmitter. Next, an appropriate *decision-rule and search strategy* is used to process the measure of correlation so obtained to determine whether the two codes are in synchronism and what to do if they are not. As for tracking, it is accomplished using phase-lock techniques very similar to those used for the local generation of coherent carrier references. The principal difference between them lies in the way in which phase discrimination is implemented.

7.5 Signal-Space Dimensionality and Processing Gain

Having developed a conceptual understanding of spread-spectrum modulation and a method for its implementation, we are ready to undertake a detailed mathematical analysis of the technique. The approach we have in mind is based on the signal-space theoretic ideas of Chapter 5. In particular, we develop signal-space representations of the transmitted signal and the interfering signal (jammer).

In this context, consider the set of orthonormal basis functions:

$$\phi_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \cos(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & \text{otherwise} \end{cases} \tag{7.14}$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \sin(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & \text{otherwise} \end{cases} \tag{7.15}$$

$$k = 0, 1, \ldots, N-1$$

where T_c is the *chip duration*, and N is the number of chips per bit. Accordingly, we may describe the transmitted signal $x(t)$ for the interval of an information bit as follows:

$$\begin{aligned} x(t) &= c(t)s(t) \\ &= \pm\sqrt{\frac{2E_b}{T_b}}\, c(t) \cos(2\pi f_c t) \\ &= \pm\sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \phi_k(t), \qquad 0 \le t \le T_b \end{aligned} \tag{7.16}$$

where E_b is the signal energy per bit; the plus sign corresponds to information bit 1, and the minus sign corresponds to information bit 0. The code sequence $\{c_0, c_1, \ldots, c_{N-1}\}$ denotes the PN sequence, with $c_k = \pm 1$. The transmitted signal $x(t)$ is therefore N-dimensional in that it requires a minimum of N orthonormal functions for its representation.

Consider next the representation of the interfering signal (jammer), $j(t)$. Ideally, the jammer likes to place all of its available energy in exactly the same N-dimensional signal space as the transmitted signal $x(t)$; otherwise, part of its energy goes to waste. However, the best that the jammer can hope to know is the transmitted signal bandwidth. Moreover, there is no way that the jammer can have knowledge of the signal phase. Accordingly, we may represent the jammer by the general form

$$j(t) = \sum_{k=0}^{N-1} j_k \phi_k(t) + \sum_{k=0}^{N-1} \tilde{j}_k \tilde{\phi}_k(t), \qquad 0 \le t \le T_b \tag{7.17}$$

where

$$j_k = \int_0^{T_b} j(t)\phi_k(t)\, dt, \qquad k = 0, 1, \ldots, N-1 \tag{7.18}$$

and

$$\tilde{j}_k = \int_0^{T_b} j(t)\tilde{\phi}_k(t)\, dt, \qquad k = 0, 1, \ldots, N-1 \tag{7.19}$$

Thus the interference $j(t)$ is $2N$-dimensional; that is, it has twice the number of dimensions required for representing the transmitted DS/BPSK signal $x(t)$. In terms of the representation given in Equation (7.17), we may express the average power of the interference $j(t)$ as follows:

$$\begin{aligned} J &= \frac{1}{T_b}\int_0^{T_b} j^2(t)\, dt \\ &= \frac{1}{T_b}\sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b}\sum_{k=0}^{N-1} \tilde{j}_k^2 \end{aligned} \tag{7.20}$$

Moreover, due to lack of knowledge of signal phase, the best strategy a jammer can apply is to place equal energy in the cosine and sine coordinates defined in Equations (7.18) and (7.19); hence, we may safely assume

$$\sum_{k=0}^{N-1} j_k^2 = \sum_{k=0}^{N-1} \tilde{j}_k^2 \tag{7.21}$$

Correspondingly, we may simplify Equation (7.20) as

$$J = \frac{2}{T_b}\sum_{k=0}^{N-1} j_k^2 \tag{7.22}$$

Our aim is to tie these results together by finding the signal-to-noise ratios measured at the input and output of the DS/BPSK receiver in Figure 7.9. To that end, we use Equation (7.13) to express the coherent detector output as

$$\begin{aligned} v &= \sqrt{\frac{2}{T_b}}\int_0^{T_b} u(t)\cos(2\pi f_c t)\, dt \\ &= v_s + v_{cj} \end{aligned} \tag{7.23}$$

where the components v_s and v_{cj} are due to the despread binary PSK signal, $s(t)$, and the spread interference, $c(t)j(t)$, respectively. These two components are defined as follows:

$$v_s = \sqrt{\frac{2}{T_b}}\int_0^{T_b} s(t)\cos(2\pi f_c t)\, dt \tag{7.24}$$

and

$$v_{cj} = \sqrt{\frac{2}{T_b}}\int_0^{T_b} c(t)j(t)\cos(2\pi f_c t)\, dt \tag{7.25}$$

Consider first the component v_s due to the signal. The despread binary PSK signal $s(t)$ equals

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t), \qquad 0 \le t \le T_b \tag{7.26}$$

where the plus sign corresponds to information bit 1, and the minus sign corresponds to information bit 0. Hence, assuming that the carrier frequency f_c is an integer multiple of $1/T_b$, we have

$$v_s = \pm\sqrt{E_b} \tag{7.27}$$

Consider next the component v_{cj} due to interference. Expressing the PN signal $c(t)$ in the explicit form of a sequence, $\{c_0, c_1, \ldots, c_{N-1}\}$, we may rewrite Equation (7.25) in the corresponding form

$$v_{cj} = \sqrt{\frac{2}{T_b}} \sum_{k=0}^{N-1} c_k \int_{kT_c}^{(k+1)T_c} j(t) \cos(2\pi f_c t)\, dt \tag{7.28}$$

Using Equation (7.14) for $\phi_k(t)$, and then Equation (7.18) for the coefficient j_k, we may redefine v_{cj} as

$$\begin{aligned} v_{cj} &= \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k \int_0^{T_b} j(t)\phi_k(t)\, dt \\ &= \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k j_k \end{aligned} \tag{7.29}$$

We next approximate the PN sequence as an *independent and identically distributed (i.i.d.) binary sequence*. We emphasize the implication of this approximation by recasting Equation (7.29) in the form

$$V_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} C_k j_k \tag{7.30}$$

where V_{cj} and C_k are random variables with sample values v_{cj} and c_k, respectively. In Equation (7.30), the jammer is assumed to be fixed. With the C_k treated as i.i.d. random variables, we find that the probability of the event $C_k = \pm 1$ is

$$P(C_k = 1) = P(C_k = -1) = \tfrac{1}{2} \tag{7.31}$$

Accordingly, the mean of the random variable V_{cj} is zero since, for fixed k, we have

$$\begin{aligned} E[C_k j_k \mid j_k] &= j_k P(C_k = 1) - j_k P(C_k = -1) \\ &= \tfrac{1}{2} j_k - \tfrac{1}{2} j_k \\ &= 0 \end{aligned} \tag{7.32}$$

For a fixed vector $\mathbf{j}$, representing the set of coefficients $j_0, j_1, \ldots, j_{N-1}$, the variance of V_{cj} is given by

$$\text{var}[V_{cj} \mid \mathbf{j}] = \frac{1}{N} \sum_{k=0}^{N-1} j_k^2 \tag{7.33}$$

Since the *spread factor* $N = T_b/T_c$, we may use Equation (7.22) to express this variance in terms of the average interference power J as

$$\text{var}[V_{cj} \mid \mathbf{j}] = \frac{JT_c}{2} \tag{7.34}$$

Thus the random variable V_{cj} has zero mean and variance $JT_c/2$.

From Equation (7.27), we note that the signal component at the coherent detector output (during each bit interval) equals $\pm\sqrt{E_b}$, where E_b is the signal energy per bit. Hence, the peak instantaneous power of the signal component is E_b. Accordingly, we may define the *output signal-to-noise ratio* as the instantaneous peak power E_b divided by the variance of the equivalent noise component in Equation (7.34). We thus write

$$(\mathrm{SNR})_O = \frac{2E_b}{JT_c} \tag{7.35}$$

The average signal power at the receiver input equals E_b/T_b. We thus define an *input signal-to-noise* ratio as

$$(\mathrm{SNR})_I = \frac{E_b/T_b}{J} \tag{7.36}$$

Hence, eliminating E_b/J between Equations (7.35) and (7.36), we may express the output signal-to-noise ratio in terms of the input signal-to-noise ratio as

$$(\mathrm{SNR})_O = \frac{2T_b}{T_c}(\mathrm{SNR})_I \tag{7.37}$$

It is customary practice to express signal-to-noise ratios in decibels. To that end, we introduce a term called the *processing gain* (PG), which is defined as *the gain in SNR obtained by the use of spread spectrum.* Specifically, we write

$$\mathrm{PG} = \frac{T_b}{T_c} \tag{7.38}$$

which represents the gain achieved by processing a spread-spectrum signal over an unspread signal. We may thus write Equation (7.37) in the equivalent form:

$$10\log_{10}(\mathrm{SNR})_O = 10\log_{10}(\mathrm{SNR})_I + 3 + 10\log_{10}(\mathrm{PG})\ \mathrm{dB} \tag{7.39}$$

The 3-dB term on the right-hand side of Equation (7.39) accounts for the gain in SNR that is obtained through the use of coherent detection (which presumes exact knowledge of the signal phase by the receiver). This gain in SNR has nothing to do with the use of spread spectrum. Rather, it is the last term, $10\log_{10}(\mathrm{PG})$, that accounts for the processing gain. Note that both the processing gain PG and the spread factor N (i.e., PN sequence length) equal the ratio T_b/T_c. Thus, the longer we make the PN sequence (or, correspondingly, the smaller the chip time T_c is), the larger will the processing gain be.

7.6 Probability of Error

Let the coherent detector output v in the direct-sequence spread BPSK system of Figure 7.9 denote the sample value of a random variable V. Let the equivalent noise component v_{cj} produced by external interference denote the sample value of a random variable V_{cj}. Then, from Equations (7.23) and (7.27) we deduce that

$$V = \pm\sqrt{E_b} + V_{cj} \tag{7.40}$$

where E_b is the transmitted signal energy per bit. The plus sign refers to sending symbol (information bit) 1, and the minus sign refers to sending symbol 0. The decision rule used by the coherent detector of Figure 7.9 is to declare that the received bit in an interval $(0, T_b)$ is 1 if the detector output exceeds a threshold of zero, and that it is 0 if the detector output is less than the threshold; if the detector output is exactly zero, the receiver makes a random guess in favor of 1 or 0. With both information bits assumed equally likely, we find that (because of the symmetric nature of the problem) the average probability of error P_e is the same as the conditional probability of (say) the receiver making a decision in favor of symbol 1, given that symbol 0 was sent. That is,

$$\begin{aligned} P_e &= P(V > 0 \,|\, \text{symbol 0 was sent}) \\ &= P(V_{cj} > \sqrt{E_b}) \end{aligned} \tag{7.41}$$

Naturally, the probability of error P_e depends on the random variable V_{cj} defined by Equation (7.30). According to this definition, V_{cj} is the sum of N identically distributed random variables. Hence, from the *central limit theorem*, we deduce that for large N, the random variable V_{cj} assumes a Gaussian distribution. Indeed, the spread factor or PN sequence length N is typically large in the direct-sequence spread-spectrum systems encountered in practice, under which condition the application of the central limit theorem is justified.

Earlier we evaluated the mean and variance of V_{cj}; see Equations (7.32) and (7.34). We may therefore state that the equivalent noise component V_{cj} contained in the coherent detector output may be approximated as a Gaussian random variable with zero mean and variance $JT_c/2$, where J is the average interference power and T_c is the chip duration. With this approximation at hand, we may then proceed to calculate the probability of the event $V_{cj} > \sqrt{E_b}$, and thus express the average probability of error in accordance with Equation (7.41) as

$$P_e \simeq \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right) \tag{7.42}$$

This simple formula, which invokes the Gaussian assumption, is appropriate for DS/BPSK binary systems with large spread factor N.

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It is informative to compare Equation (7.42) with the formula for the average probability of error for a coherent binary PSK system reproduced here for convenience of presentation [see Equation (6.20)]

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{7.43}$$

Based on this comparison, we see that insofar as the calculation of bit error rate in a direct-sequence spread binary PSK system is concerned, the interference may be treated as wideband noise of power spectral density $N_0/2$, defined by

$$\frac{N_0}{2} = \frac{JT_c}{2} \tag{7.44}$$

This relation is simply a restatement of an earlier result given in Equation (7.34).

Since the signal energy per bit $E_b = PT_b$, where P is the average signal power and T_b is the bit duration, we may express the signal energy per bit-to-noise spectral density ratio as

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right) \tag{7.45}$$