Using the definition of Equation (7.38) for the processing gain PG we may reformulate this result as

$$\frac{J}{P} = \frac{\text{PG}}{E_b/N_0} \tag{7.46}$$

The ratio J/P is termed the *jamming margin.* Accordingly, the jamming margin and the processing gain, both expressed in decibels, are related by

$$(\text{Jamming margin})_{\text{dB}} = (\text{Procesing gain})_{\text{dB}} - 10 \log_{10}\left(\frac{E_b}{N_0}\right)_{\text{min}} \tag{7.47}$$

where $(E_b/N_0)_{\text{min}}$ is the minimum value needed to support a prescribed average probability of error.

▶ EXAMPLE 7.3

A spread-spectrum communication system has the following parameters:

$$\text{Information bit duration, } T_b = 4.095 \text{ ms}$$
$$\text{PN chip duration, } T_c = 1\ \mu s$$

Hence, using Equation (7.38) we find that the processing gain is

$$\text{PG} = 4095$$

Correspondingly, the required period of the PN sequence is $N = 4095$, and the shift-register length is $m = 12$.

For a satisfactory reception, we may assume that the average probability of error is not to exceed 10^{-5}. From the formula for a coherent binary PSK receiver, we find that $E_b/N_0 = 10$ yields an average probability of error equal to 0.387×10^{-5}. Hence, using this value for E_b/N_0, and the value calculated for the processing gain, we find from Equation (7.47) that the jamming margin is

$$\begin{aligned}(\text{Jamming margin})_{\text{dB}} &= 10 \log_{10} 4095 - 10 \log_{10}(10)\\ &= 36.1 - 10\\ &= 26.1 \text{ dB}\end{aligned}$$

That is, information bits at the receiver output can be detected reliably even when the noise or interference at the receiver input is up to 409.5 times the received signal power. Clearly, this is a powerful advantage against interference (jamming), which is realized through the clever use of spread-spectrum modulation. ◀

7.7 *Frequency-Hop Spread Spectrum*

In the type of spread-spectrum systems discussed in Section 7.4, the use of a PN sequence to modulate a phase-shift-keyed signal achieves *instantaneous* spreading of the transmission bandwidth. The ability of such a system to combat the effects of jammers is determined by the processing gain of the system, which is a function of the PN sequence period. The processing gain can be made larger by employing a PN sequence with narrow chip duration, which, in turn, permits a greater transmission bandwidth and more chips per bit. However, the capabilities of physical devices used to generate the PN spread-spectrum signals impose a practical limit on the attainable processing gain. Indeed, it may turn out that the processing gain so attained is still not large enough to overcome the effects of some jammers of concern, in which case we have to resort to other methods. One such alternative method is to force the jammer to cover a wider spectrum by *randomly hopping* the data-modulated carrier from one frequency to the next. In effect, the spectrum of the transmitted signal is spread *sequentially* rather than instantaneously; the term "sequentially" refers to the pseudo-random-ordered sequence of frequency hops.

The type of spread spectrum in which the carrier hops randomly from one frequency to another is called *frequency-hop (FH) spread spectrum*. A common modulation format for FH systems is that of *M-ary frequency-shift keying* (MFSK). The combination of these two techniques is referred to simply as FH/MFSK. (A description of M-ary FSK is presented in Chapter 6.)

Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to consider the rate at which the hops occur. In this context, we may identify two basic (technology-independent) characterizations of frequency hopping:

1. *Slow-frequency hopping*, in which the *symbol rate* R_s of the MFSK signal is an integer multiple of the *hop rate* R_h. That is, several symbols are transmitted on each frequency hop.
2. *Fast-frequency hopping*, in which the hop rate R_h is an integer multiple of the MFSK symbol rate R_s. That is, the carrier frequency will change or hop several times during the transmission of one symbol.

Obviously, slow-frequency hopping and fast-frequency hopping are the converse of one another. In the following, these two characterizations of frequency hopping are considered in turn.

■ SLOW-FREQUENCY HOPPING

Figure 7.10*a* shows the block diagram of an FH/MFSK transmitter, which involves *frequency modulation* followed by *mixing*. First, the incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and the output from a digital *frequency synthesizer* are then applied to a mixer that consists of a multiplier followed by a band-pass filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. In particular, successive k-bit segments of a PN sequence drive the frequency synthesizer, which enables the carrier frequency to hop over 2^k distinct values. On a single hop, the bandwidth of the transmitted signal is the same as that resulting from the use of a conventional MFSK with an alphabet of $M = 2^K$ orthogonal signals. However, for a complete range of 2^k frequency hops, the transmitted FH/MFSK signal occupies a much larger bandwidth. Indeed, with present-day technology, FH bandwidths on the order of several GHz are attainable, which is an order of magnitude larger than that achievable with direct-sequence spread spectra. An implication of these large FH bandwidths is that coherent detection is possible only within each hop, because frequency synthesizers are unable to maintain phase coherence over successive hops. Accordingly, most frequency-hop spread-spectrum communication systems use noncoherent M-ary modulation schemes.

In the receiver depicted in Figure 7.10*b*, the frequency hopping is first removed by *mixing* (down-converting) the received signal with the output of a local frequency synthesizer that is synchronously controlled in the same manner as that in the transmitter. The resulting output is then band-pass filtered, and subsequently processed by a *noncoherent* M-ary FSK detector. To implement this M-ary detector, we may use a bank of M noncoherent matched filters, each of which is matched to one of the MFSK tones. (Noncoherent matched filters are described in Chapter 6.) An estimate of the original symbol transmitted is obtained by selecting the largest filter output.

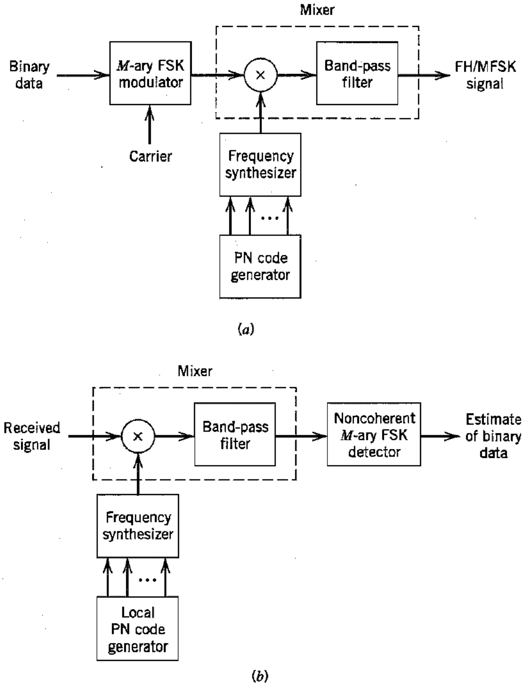

FIGURE 7.10 Frequency-hop spread M-ary frequency-shift keying. (*a*) Transmitter. (*b*) Receiver.

An individual FH/MFSK tone of shortest duration is referred to as a *chip*; this terminology should not be confused with that used in Section 7.4 describing DS/BPSK. The *chip rate*, R_c, for an FH/MFSK system is defined by

$$R_c = \max(R_h, R_s) \tag{7.48}$$

where R_h is the *hop rate*, and R_s is the *symbol rate.*

A slow FH/MFSK signal is characterized by having multiple symbols transmitted per hop. Hence, each symbol of a slow FH/MFSK signal is a chip. Correspondingly, in a slow FH/MFSK system, the bit rate R_b of the incoming binary data, the symbol rate R_s of the MFSK signal, the chip rate R_c, and the hop rate R_h are related by

$$R_c = R_s = \frac{R_b}{K} \geq R_h \tag{7.49}$$

where $K = \log_2 M$.

At each hop, the MFSK tones are separated in frequency by an integer multiple of the chip rate $R_c = R_s$, ensuring their orthogonality. The implication of this condition is that any transmitted symbol will not produce any crosstalk in the other $M - 1$ noncoherent matched filters constituting the MFSK detector of the receiver in Figure 7.10*b*. By "crosstalk" we mean the spillover from one filter output into an adjacent one. The resulting performance of the slow FH/MFSK system is the same as that for the noncoherent detection of conventional (unhopped) MFSK signals in additive white Gaussian noise. Thus the interfering (jamming) signal has an effect on the FH/MFSK receiver, in terms of average probability of symbol error, equivalent to that of additive white Gaussian noise on a conventional noncoherent M-ary FSK receiver experiencing no interference. On the basis of this equivalence, we may use Equation (6.140) for approximate evaluation of the probability of symbol error in the FH/MFSK system.

Assuming that the jammer decides to spread its average power J over the entire frequency-hopped spectrum, the jammer's effect is equivalent to an AWGN with power spectral density $N_0/2$, where $N_0 = J/W_c$ and W_c is the FH bandwidth. The spread-spectrum system is thus characterized by the *symbol energy-to-noise spectral density ratio*:

$$\frac{E}{N_0} = \frac{P/J}{W_c/R_s} \tag{7.50}$$

where the ratio P/J is the reciprocal of the jamming margin. The other ratio in the denominator of Equation (7.50) is the processing gain of the slow FH/MFSK system, which is defined by

$$\begin{aligned} \text{PG} &= \frac{W_c}{R_s} \\ &= 2^k \end{aligned} \tag{7.51}$$

That is, the processing gain (expressed in decibels) is equal to $10 \log_{10} 2^k \simeq 3k$, where k is the length of the PN segment employed to select a frequency hop.

This result assumes that the jammer spreads its power over the entire FH spectrum. However, if the jammer decides to concentrate on just a few of the hopped frequencies, then the processing gain realized by the receiver would be less than $3k$ decibels.

▶ EXAMPLE 7.4

Figure 7.11*a* illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is $2^4 - 1 = 15$. The FH/MFSK signal has the following parameters:

Number of bits per MFSK symbol	$K = 2$
Number of MFSK tones	$M = 2^K = 4$
Length of PN segment per hop	$k = 3$
Total number of frequency hops	$2^k = 8$

In this example, the carrier is hopped to a new frequency after transmitting two symbols or equivalently, four information bits. Figure 7.11*a* also includes the input binary data, and the PN sequence controlling the selection of FH carrier frequency. It is noteworthy that although there are eight distinct frequencies available for hopping, only three of them are utilized by the PN sequence.

Figure 7.11*b* shows the variation of the dehopped frequency with time. This variation is recognized to be the same as that of a conventional MFSK signal produced by the given input data. ◀

■ FAST-FREQUENCY HOPPING

A fast FH/MFSK system differs from a slow FH/MFSK system in that there are multiple hops per M-ary symbol. Hence, in a fast FH/MFSK system, each hop is a chip. In general,