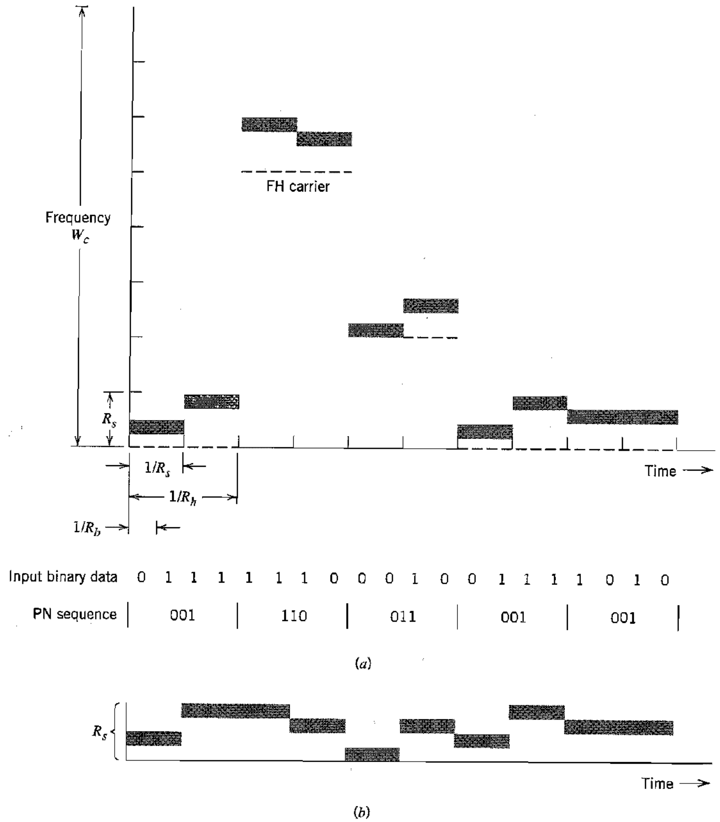

FIGURE 7.11 Illustrating slow-frequency hopping. (*a*) Frequency variation for one complete period of the PN sequence. (*b*) Variation of the dehopped frequency with time.

fast-frequency hopping is used to defeat a smart jammer's tactic that involves two functions: measurements of the spectral content of the transmitted signal, and retuning of the interfering signal to that portion of the frequency band. Clearly, to overcome the jammer, the transmitted signal must be hopped to a new carrier frequency *before* the jammer is able to complete the processing of these two functions.

For data recovery at the receiver, noncoherent detection is used. However, the detection procedure is quite different from that used in a slow FH/MFSK receiver. In particular, two procedures may be considered:

1. For each FH/MFSK symbol, separate decisions are made on the K frequency-hop chips received, and a simple rule based on *majority vote* is used to make an estimate of the dehopped MFSK symbol.
2. For each FH/MFSK symbol, likelihood functions are computed as functions of the total signal received over K chips, and the largest one is selected.

A receiver based on the second procedure is optimum in the sense that it minimizes the average probability of symbol error for a given E_b/N_0.

▶ EXAMPLE 7.5

Figure 7.12*a* illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time. The signal has the following parameters:

Number of bits per MFSK symbol	$K = 2$
Number of MFSK tones	$M = 2^K = 4$
Length of PN segment per hop	$k = 3$
Total number of frequency hops	$2^k = 8$

In this example, each MFSK symbol has the same number of bits and chips; that is, the chip rate R_c is the same as the bit rate R_b. After each chip, the carrier frequency of the transmitted MFSK signal is hopped to a different value, except for few occasions when the k-chip segment of the PN sequence repeats itself.

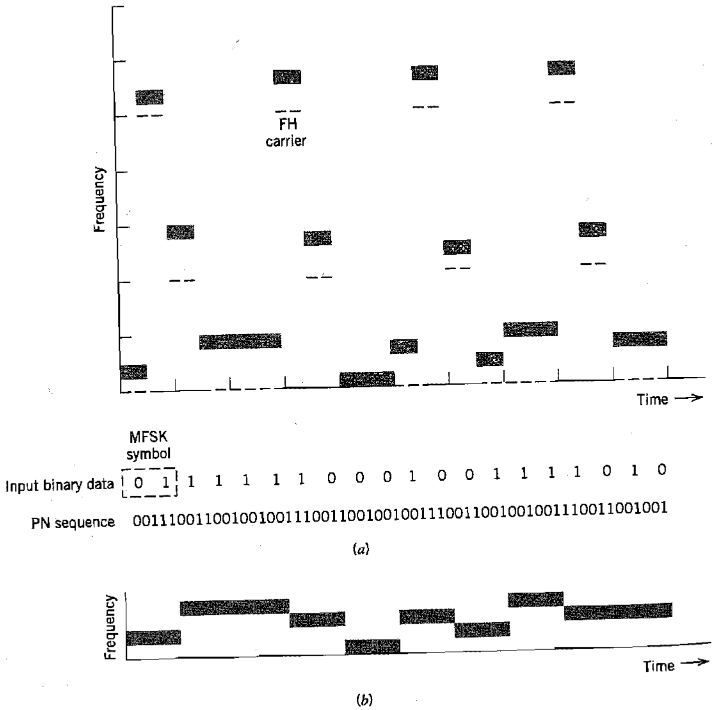

FIGURE 7.12 Illustrating fast-frequency hopping. (*a*) Variation of the transmitter frequency with time. (*b*) Variation of the dehopped frequency with time.

Figure 7.12*b* depicts the time variation of the frequency of the dehopped MFSK signal, which is the same as that in Example 7.4. ◀

7.8 Computer Experiments: Maximal-Length and Gold Codes

Code-division multiplexing (CDM) provides an alternative to the traditional methods of frequency-division multiplexing (FDM) and time-division multiplexing (TDM). It does not require the bandwidth allocation of FDM (discussed in Chapter 2) nor the time synchronization needed in TDM (discussed in Chapter 3). Rather, users of a common channel are permitted access to the channel through the assignment of a "spreading code" to each individual user under the umbrella of spread-spectrum modulation. The purpose of this computer experiment is to study a certain class of spreading codes for CDM systems that provide a satisfactory performance.

In an ideal CDM system, the cross-correlation between any two users of the system is zero. For this ideal condition to be realized, we require that the cross-correlation function between the spreading codes assigned to any two users of the system be zero for all cyclic shifts. Unfortunately, ordinary PN sequences do not satisfy this requirement because of their relatively poor cross-correlation properties.

As a remedy for this shortcoming of ordinary PN sequences, we may use a special class of PN sequences called *Gold sequences (codes)*,[5] the generation of which is embodied in the following theorem:

> Let $g_1(X)$ and $g_2(X)$ be a preferred pair of primitive polynomials of degree n whose corresponding shift registers generate maximal-length sequences of period $2^n - 1$ and whose cross-correlation function has a magnitude less than or equal to
>
> $$2^{(n+1)/2} + 1 \qquad \text{for } n \text{ odd} \tag{7.52}$$
>
> or
>
> $$2^{(n+2)/2} + 1 \qquad \text{for } n \text{ even and } n \neq 0 \bmod 4 \tag{7.53}$$
>
> Then the shift register corresponding to the product polynomial $g_1(X) \cdot g_2(X)$ will generate $2^n + 1$ different sequences, with each sequence having a period of $2^n - 1$, and the cross-correlation between any pair of such sequences satisfying the preceding condition.

Hereafter, this theorem is referred to as *Gold's theorem*.

To understand Gold's theorem, we need to define what we mean by a primitive polynomial. Consider a polynomial $g(X)$ defined over a *binary field* (i.e., a finite set of two elements, 0 and 1, which is governed by the rules of binary arithmetic). The polynomial $g(X)$ is said to be an *irreducible polynomial* if it cannot be factored using any polynomials from the binary field. An irreducible polynomial $g(X)$ of degree m is said to be a *primitive polynomial* if the smallest integer m for which the polynomial $g(X)$ divides the factor $X^n + 1$ is $n = 2^m - 1$. Further discussion of this topic is deferred to Chapter 8; in particular, see Example 8.3.

Experiment 1. Correlation Properties of PN Sequences

Consider a pair of shift registers for generating two PN sequences of period $2^7 - 1 = 127$. One feedback shift register has the feedback taps [7, 1] and the other one has the feedback taps [7, 6, 5, 4]. Both sequences have the same autocorrelation function shown in Figure 7.13*a*, which follows readily from the definition presented in Equation (7.5).

However, the calculation of the cross-correlation function between PN sequences is a more difficult proposition, particularly for large n. To perform this calculation, we resort to the use of computer simulation for varying cyclic shift τ inside the interval $0 < \tau \leq 2^n - 1$. The results of this computation are presented in Figure 7.13*b*. This figure confirms the poor cross-correlation property of PN sequences compared to their autocorrelation function. The magnitude of the cross-correlation function exceeds 40.

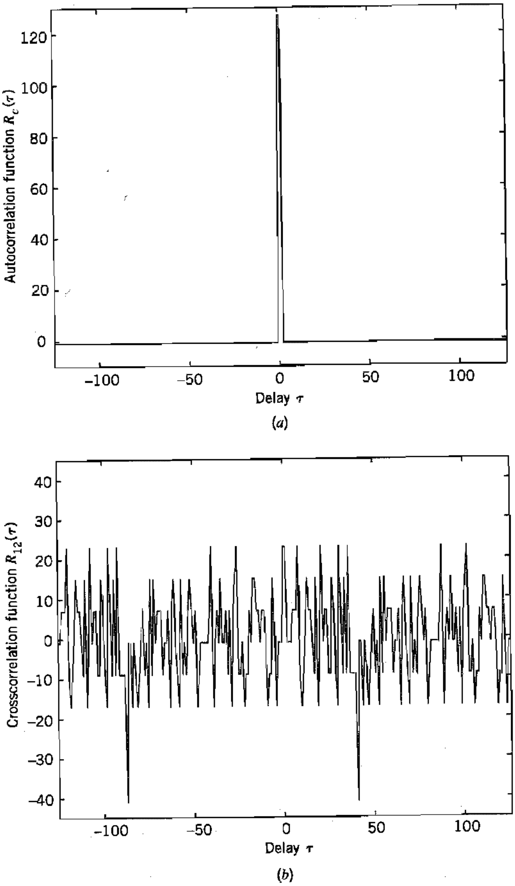

FIGURE 7.13 (*a*) Autocorrelation function $R_c(\tau)$, and (*b*) cross-correlation function $R_{12}(\tau)$ of the two PN sequences [7, 1] and [7, 6, 5, 4].

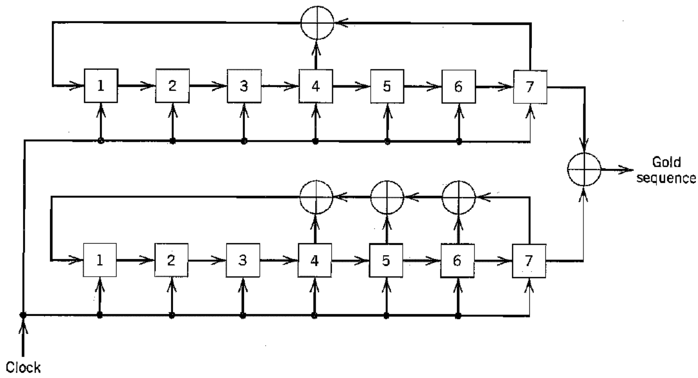

FIGURE 7.14 Generator for a Gold sequence of period $2^7 - 1 = 127$.

Experiment 2. Correlation Properties of Gold Sequences

For our next experiment, we consider Gold sequences with period $2^7 - 1 = 127$. To generate such a sequence for $n = 7$ we need a preferred pair of PN sequences that satisfy Equation (7.52) (n odd), as shown by

$$2^{(n+1)/2} + 1 = 2^4 + 1 = 17$$

This requirement is satisfied by the PN sequences with feedback taps [7, 4] and [7, 6, 5, 4]. The Gold-sequence generator is shown in Figure 7.14 that involves the modulo-2 addition of these two sequences. According to Gold's theorem, there are a total of

$$2^n + 1 = 2^7 + 1 = 129$$

sequences that satisfy Equation (7.52). The cross-correlation between any pair of such sequences is shown in Figure 7.15, which is indeed in full accord with Gold's theorem. In particular, the magnitude of the cross-correlation is less than or equal to 17.

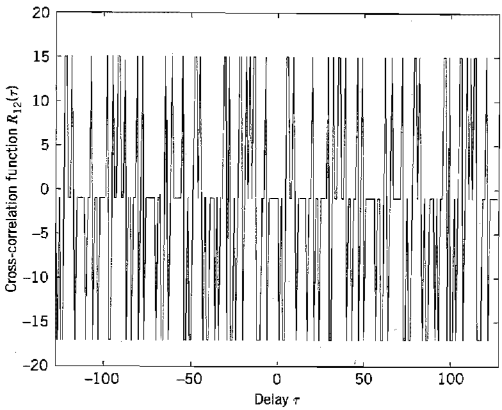

FIGURE 7.15 Cross-correlation function $R_{12}(\tau)$ of a pair of Gold sequences based on the two PN sequences [7, 4] and [7, 6, 5, 4].

7.9 Summary and Discussion

Direct-sequence M-ary phase shift keying (DS/MPSK) and *frequency-hop M-ary frequency shift-keying* (FH/MFSK) represent two principal categories of spread-spectrum communications. Both of them rely on the use of a pseudo-noise (PN) sequence, which is applied differently in the two categories.

In a DS/MPSK system, the PN sequence makes the transmitted signal assume a noiselike appearance by spreading its spectrum over a broad range of frequencies simultaneously. For the phase-shift keying, we may use binary PSK (i.e., $M = 2$) with a single carrier. Alternatively, we may use QPSK (i.e., $M = 4$), in which case the data are transmitted using a pair of carriers in phase quadrature. (Both PSK and QPSK are discussed in Section 6.3.) The usual motivation for using QPSK is to provide for improved bandwidth efficiency. In a spread-spectrum system, bandwidth efficiency is usually not of prime concern. Rather, the use of QPSK is motivated by the fact that it is less sensitive to some types of interference (jamming).

In an FH/MFSK system, the PN sequence makes the carrier hop over a number of frequencies in a pseudo-random manner, with the result that the spectrum of the transmitted signal is spread in a sequential manner.

Naturally, the direct-sequence and frequency-hop spectrum-spreading techniques may be employed in a single system. The resulting system is referred to as *hybrid* DS/FH *spread-spectrum system*. The reason for seeking a hybrid approach is that advantages of both the direct-sequence and frequency-hop spectrum-spreading techniques are realized in the same system.

A discussion of spread-spectrum communications would be incomplete without some reference to jammer waveforms. The jammers encountered in practice include the following types:

1. *The barrage noise jammer*, which consists of band-limited white Gaussian noise of high average power. The barrage noise jammer is a brute-force jammer that does not exploit any knowledge of the antijam communication system except for its spread bandwidth.
2. *The partial-band noise jammer*, which consists of noise whose total power is evenly spread over some frequency band that is a subset of the total spread bandwidth. Owing to the smaller bandwidth, the partial-band noise jammer is easier to generate than the barrage noise jammer.
3. *The pulsed noise jammer*, which involves transmitting wideband noise of power

$$J_{\text{peak}} = \frac{J}{p}$$

   for a fraction p of the time, and nothing for the remaining fraction $1 - p$ of the time. The average noise power equals J.
4. *The single-tone jammer*, which consists of a sinusoidal wave whose frequency lies inside the spread bandwidth; as such, it is the easiest of all jamming signals to generate.
5. *The multitone jammer*, which is the tone equivalent of the partial-band noise jammer.

In addition to these five, many other kinds of jamming waveforms occur in practice. In any event, there is no single jamming waveform that is worst for all spread-spectrum systems, and there is no single spread-spectrum system that is best against all possible jamming waveforms.

Notes and References

1. The definition of spread-spectrum modulation presented in the Introduction is adapted from Pickholtz, Schilling, and Milstein (1982). This paper presents a tutorial review of the theory of spread-spectrum communications.

   For introductory papers on the subject, see Viterbi (1979), and Cook and Marsh (1983). For books on the subject, see Dixon (1984), Holmes (1982), Ziemer and Peterson (1985, pp. 327–649), Cooper and McGillem (1986, pp. 269–411), and Simon, Omura, Scholtz, and Levitt (1985, Volumes I, II, and III). The three-volume book by Simon et al. is the most exhaustive treatment of spread-spectrum communications available in the open literature. The development of spread-spectrum communications dates back to about the mid-1950s. For a historical account of these techniques, see Scholtz (1982). This latter paper traces the origins of spread-spectrum communications back to the 1920s. Much of the historical material presented in this paper is reproduced in Chapter 2, Volume I, of the book by Simon et al.

   The book edited by Tantaratana and Ahmed (1998) includes introductory and advanced papers on wireless applications of spread-spectrum modulation. The papers are grouped into the following categories: spread-spectrum technology, cellular mobile systems, satellite communications, wireless local area networks, and global positioning systems (GPS).

2. For further details on maximal-length sequences, see Golomb (1964, pp. 1–32), Simon, Omura, Scholtz, and Levitt (1985, pp. 283–295), and Peterson and Weldon (1972). The last reference includes an extensive list of polynomials for generating maximal-length sequences; see also Dixon (1984). For a tutorial paper on pseudo-noise sequences, see Sarwate and Pursley (1980).

3. Table 7.1 is extracted from the book by Dixon (1984, pp. 81–83), where feedback connections of maximal-length sequences are tabulated for shift-register length m extending up to 89.

4. For detailed discussion of the synchronization problem in spread-spectrum communications, see Ziemer and Peterson (1985, Chapters 9 and 10) and Simon et al. (1985, Volume III).

5. The original papers on Gold sequences are Gold (1967, 1968). A detailed discussion of Gold sequences is presented in Holmes (1982).

Problems

Pseudo-Noise Sequences

7.1 A pseudo-noise (PN) sequence is generated using a feedback shift register of length $m = 4$. The chip rate is 10^7 chips per second. Find the following parameters:

(a) PN sequence length.

(b) Chip duration of the PN sequence.

(c) PN sequence period.