



3-2 Analysis Of A Power System In Per Unit: Steps :

- 1) Convert all three phase bus load MW and MVAR, generator MW and MVAR, to three phase per unit quantities using the 3phase system base.
- 2) Convert all actual resistance, inductive and capacitive reactance values to per unit using the Zbase for each particular part of the power system where each r, x etc. is located.
- 3) Carry out all calculations in per unit, solving for the per unit bus voltage magnitudes and bus phase angles, line currents, and transmission losses.
- 4) Convert from per unit back to actual MW, MVAR, MVA, and KV.

Example 1: Prove that the pu equivalent impedance of a two winding transformer is the same whether referred to the H.T side or L.T side.

Solution:

$$Z_{L\Omega} = Z_{H\Omega} \left(\frac{KV_L}{KV_H} \right)^2$$

$$Z_{L(pu)} = Z_{L\Omega} \frac{MVA}{(KV_L)^2}$$

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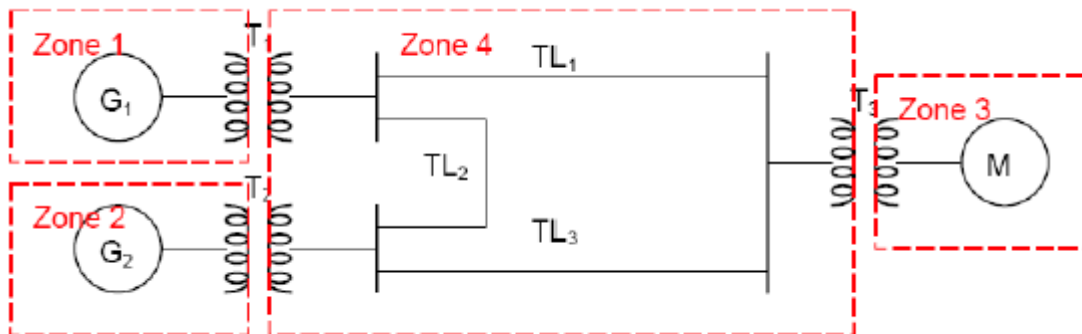
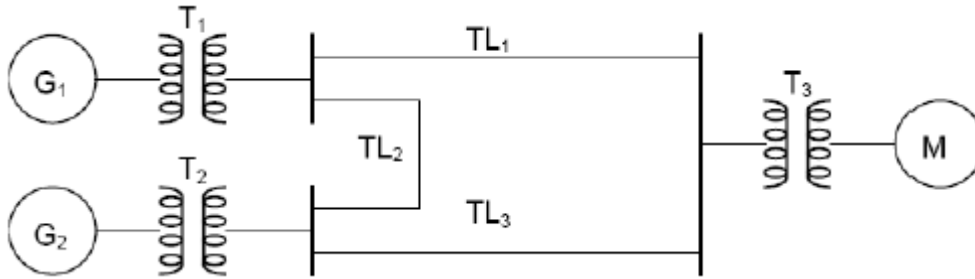
$$\therefore Z_{L(pu)} = Z_{H(pu)}$$

Example 2:

A one-line diagram of a three-phase power system is shown. Draw the impedance diagram of the power system, and mark all impedances in per unit. Use a base of 100 MVA and 138 kV for the transmission lines. All transformers are connected to step up the voltage of the generators to the transmission line voltages. Calculate the terminal voltage of G₂ (in pu) if G₁ is out of service and the motor draws 50 MW of power with 1 pu voltage at its terminals.

Equipment Ratings:

item	MVA	kV	X _{pu}	item	MVA	kV	
G ₁	45	13.2	0.15	T ₃	70	138 / 11.6	0.10
G ₂	55	18	0.12	Line 1	Z _{TL} = j40 Ω		
Motor	75	11.6	0.23	Line 2	Z _{TL} = j20 Ω		
T ₁	50	13.8 / 138	0.10	Line 3	Z _{TL} = j15 Ω		
T ₂	60	19.05 / 138	0.10				



Voltage zones:

Zone 1: $V_b = 13.8 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 2: $V_b = 19.05 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 3: $V_b = 11.6 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 4: $V_b = 138 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Impedances:

$$Z_{G1} = j0.15 \left(\frac{100 \text{ MVA}}{45 \text{ MVA}} \right) \left(\frac{13.2 \text{ KV}}{13.8 \text{ KV}} \right)^2 = j0.305 \text{ pu}$$

$$Z_{G2} = j0.12 \left(\frac{100 \text{ MVA}}{55 \text{ MVA}} \right) \left(\frac{18 \text{ KV}}{19.05 \text{ KV}} \right)^2 = j0.195 \text{ pu}$$

$$Z_M = j0.23 \left(\frac{100 \text{ MVA}}{75 \text{ MVA}} \right) \left(\frac{11.6 \text{ KV}}{11.6 \text{ KV}} \right)^2 = j0.307 \text{ pu}$$

$$Z_{T1} = j0.1 \left(\frac{100 \text{ MVA}}{50 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.2 \text{ pu}$$

$$Z_{T2} = j0.1 \left(\frac{100 \text{ MVA}}{60 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.167 \text{ pu}$$



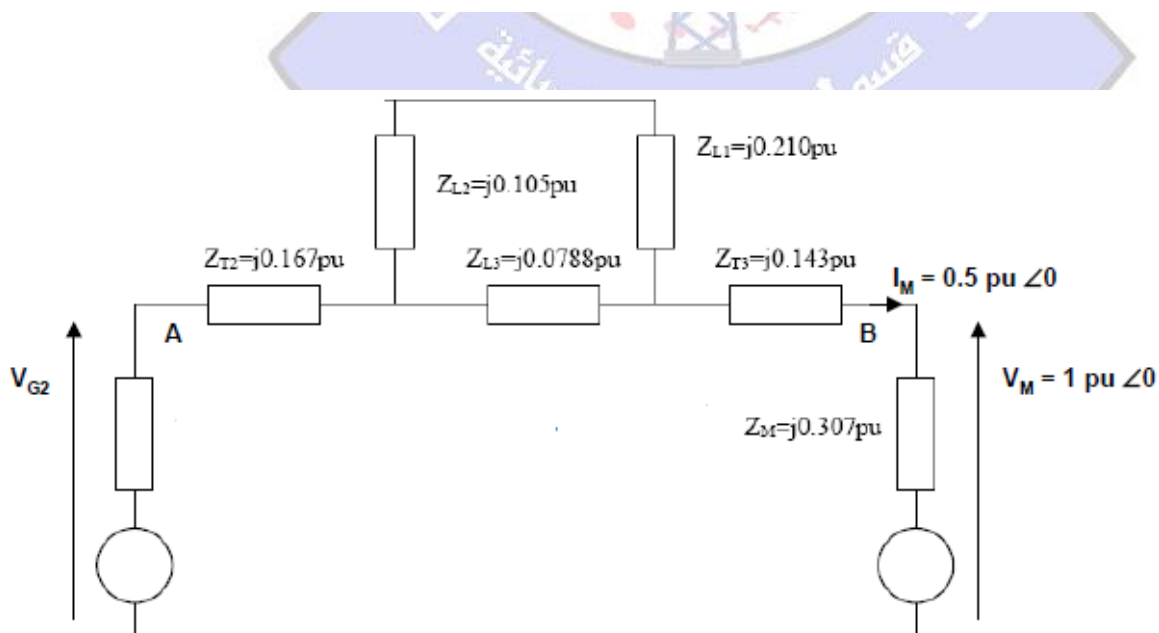
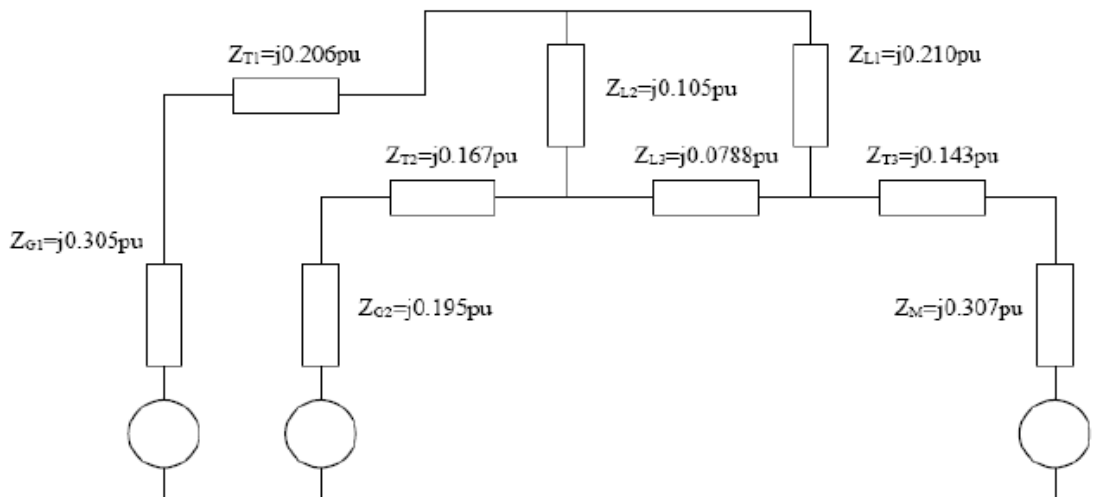
$$Z_{T3} = j0.1 \left(\frac{100 \text{ MVA}}{70 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.143 \text{ pu}$$

$$Z_b = \frac{(138 \text{ KV})^2}{100 \text{ MVA}} = 190.4 \Omega$$

$$Z_{TL1} = \frac{j40 \Omega}{190.4 \Omega} = j0.21 \text{ pu}$$

$$Z_{TL2} = \frac{j20 \Omega}{190.4 \Omega} = j0.105 \text{ pu}$$

$$Z_{TL3} = \frac{j15 \Omega}{190.4 \Omega} = j0.0788 \text{ pu}$$





$$Z_L = \frac{j0.315 \cdot j0.0788}{j0.315 + j0.0788} = j0.063$$

$$Z_{AB} = j0.063 + j0.167 + j0.143 = j0.373$$

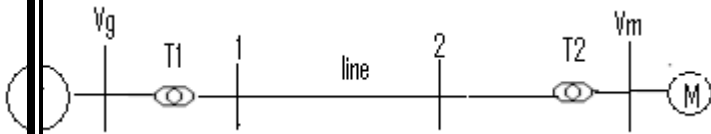
$$V_{AB} = I_M \cdot Z_{AB} = 0.5 \cdot j0.373 = 0.187 \angle 90$$

$$V_{G2} = V_{AB} + V_m = 1 + 0.187 \angle 90 = 1.017 \angle 10.6$$

Example 3: The three-phase power and line-line ratings of the electric power system shown in Figure are given below.

G:	60 MVA	20 kV	$X = 9\%$
T1:	50 MVA	20/200 kV	$X = 10\%$
T2:	50 MVA	200/20 kV	$X = 10\%$
M:	43.2 MVA	18 kV	$X = 8\%$
Line:	200 kV	$Z = 120 + j200 \Omega$	

Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.



Solution:

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is:

$$V_{B1} = 20 \left(\frac{200}{20} \right) = 200 \text{ KV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{Bm} = 200 \left(\frac{20}{200} \right) = 20 \text{ KV}$$

The generator and transformer reactances in per unit on a 100 MVA base,

$$G: \quad X = 0.09 \left(\frac{100}{60} \right) = 0.15 \text{ pu}$$

$$T_1: \quad X = 0.1 \left(\frac{100}{50} \right) = 0.2 \text{ pu}$$

$$T_2: \quad X = 0.1 \left(\frac{100}{50} \right) = 0.2 \text{ pu}$$

$$M: \quad X = 0.08 \left(\frac{100}{43.2} \right) \left(\frac{18}{20} \right)^2 = 0.15 \text{ pu}$$



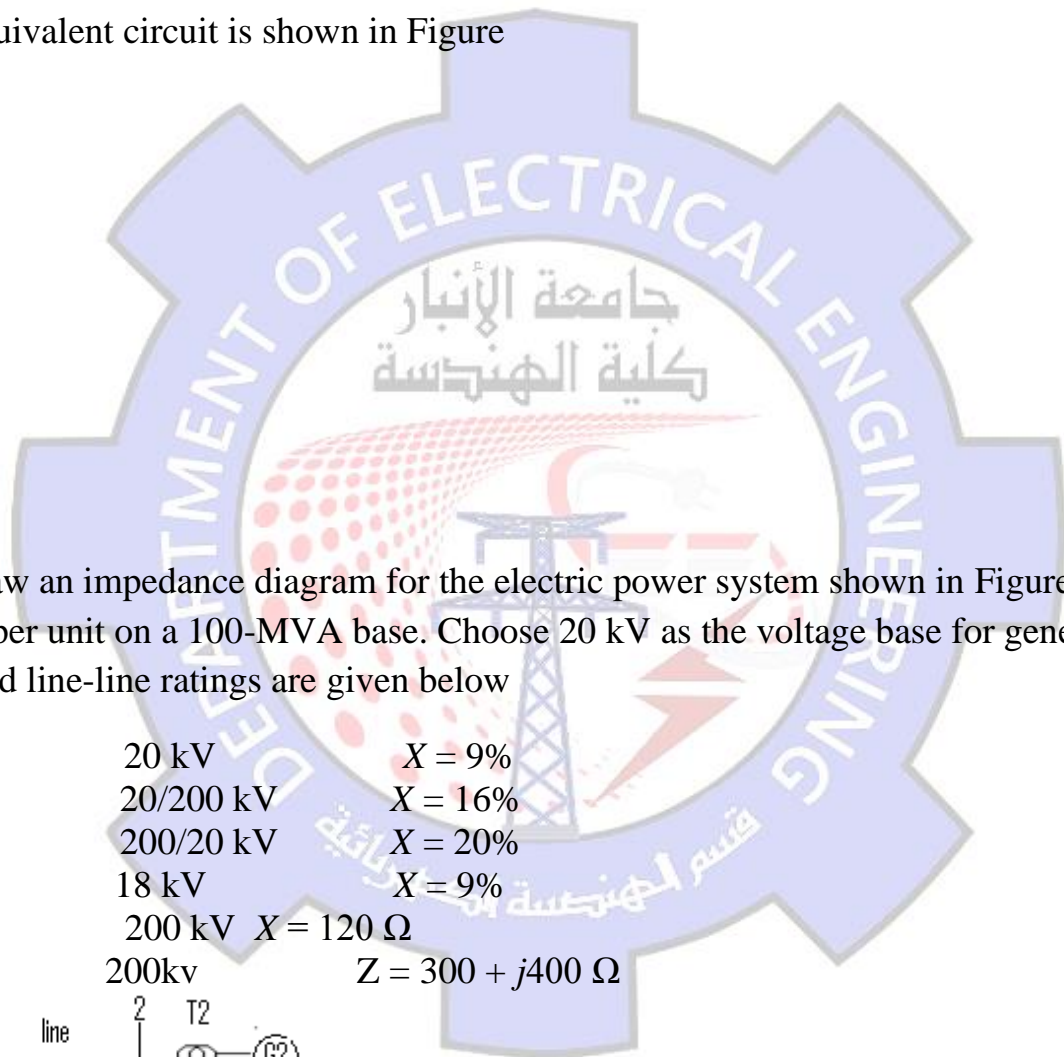
The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \Omega$$

The per unit line impedance is

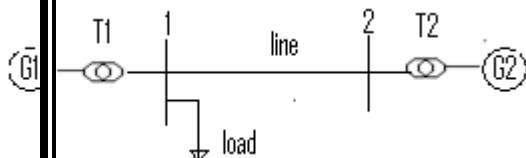
$$\text{line: } Z_{line} = \left(\frac{120 + j200}{400} \right) = 0.3 + j0.5 \text{ pu}$$

The per unit equivalent circuit is shown in Figure



Example 4: Draw an impedance diagram for the electric power system shown in Figure showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below

G1	90 MVA	20 kV	$X = 9\%$
T1	80 MVA	20/200 kV	$X = 16\%$
T2	80 MVA	200/20 kV	$X = 20\%$
G2	90 MVA	18 kV	$X = 9\%$
Line:	200 kV	$X = 120 \Omega$	
Load:	200kv	$Z = 300 + j400 \Omega$	



Solution:

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is:

$$V_{B1} = 20 \left(\frac{200}{20} \right) = 200 \text{ KV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200 \text{ kV}$, and on its LV side at



$$V_{BG2} = 200 \left(\frac{20}{200} \right) = 20 \text{ KV}$$

The generator and transformer reactance in per unit on a 100 MVA base,

$$G_1: \quad X = 0.09 \left(\frac{100}{90} \right) = 0.1 \text{ pu}$$

$$T_1: \quad X = 0.16 \left(\frac{100}{80} \right) = 0.2 \text{ pu}$$

$$T_2: \quad X = 0.2 \left(\frac{100}{80} \right) = 0.25 \text{ pu}$$

$$G_2: \quad X = 0.09 \left(\frac{100}{90} \right) \left(\frac{18}{20} \right)^2 = 0.081 \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \Omega$$

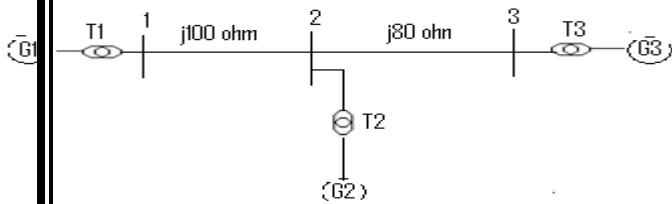
The per unit line impedance is

$$\text{line:} \quad Z_{line} = \left(\frac{300 + j400}{400} \right) = 0.75 + j1 \text{ pu}$$

The per unit equivalent circuit is shown in Figure

Example 5: Draw an impedance diagram for the electric power system shown in Figure showing all impedances in per unit on a 30MVA base. Choose 6.9 kV as the voltage base for generator. The three-phase power and line-line ratings are given below

G1	20 MVA	6.9 kV	$X = 15\%$
G2	10 MVA	6.9 kV	$X = 15\%$
G3	30 MVA	13.8 kV	$X = 15\%$
T1	25 MVA	6.9/115 kV	$X = 10\%$
T2	12.5 MVA	6.9/115 kV	$X = 10\%$
T3	10 MVA	7.5/75 kV	$X = 10\%$
Line 1-2:	$X = j100 \Omega$		Line 1-3: $X = j80 \Omega$



Solution:

$$G_1: X = 0.15 \left(\frac{6.9}{6.9} \right)^2 \left(\frac{30}{20} \right) = 0.225 pu$$

$$\text{Base voltage in } L_{1-2} = 6.9 \left(\frac{115}{6.9} \right) = 115 KV$$

$$\text{Base impedance} = \frac{(115)^2}{30} = 440 \Omega$$

$$L_{1-2} K = \frac{100}{440} = 0.2268 pu$$

$$L_{2-3} K = \frac{80}{440} = 0.1814 pu$$

$$T_1: X = 0.1 \left(\frac{6.9}{6.9} \right)^2 \left(\frac{30}{25} \right) = 0.12 pu$$

$$\text{Base KV in } G_2 = 115 \left(\frac{6.9}{115} \right) = 6.9 KV$$

$$G_2: X = 0.15 \left(\frac{30}{10} \right) \left(\frac{6.9}{6.9} \right)^2 = 0.45 pu$$

$$T_2: X = 0.1 \left(\frac{30}{12.5} \right) \left(\frac{115}{115} \right)^2 = 0.25 pu$$

$$\text{Base voltage} = 115 \left(\frac{7.5}{75} \right) = 11.5 KV$$

$$G_3: X = 0.15 \left(\frac{13.8}{11.5} \right)^2 \left(\frac{30}{30} \right) = 0.216 pu$$

$$T_3: X = 0.1 \left(\frac{30}{10} \right) \left(\frac{75}{115} \right)^2 = 0.127 pu$$

HW.

Q/A 100 MVA, 13.8 KV, 3-phase generator has a reactance of 20%. The generator is connected to a 3-phase transformer T I rated 100 MVA 12.5 KV 1110 KV with 10% reactance. The h.v. side of the transformer is connected to a transmission line of reactance 100 ohm. The far end of the line is connected to a step down transformer T 2' made of three single-phase transformers each



rated 30 MVA, 60 KV / 10 KV with 10% reactance the generator supplies two motors connected on the l.v. side T2 as shown in Fig. E.6.2. The motors are rated at 25 MVA and 50 MVA both at 10 KV with 15% reactance. Draw the reactance diagram showing all the values in per unit. Take generator rating as base.

Q2/ Figure below shows single-line diagram of a power system. The ratings of the generators and transformers are given below:

G1: 25 MVA, 6.6 kV, $x_{g1}=0.20$ pu

G2: 15 MVA, 6.6 kV, $x_{g2} = 0.15$ pu

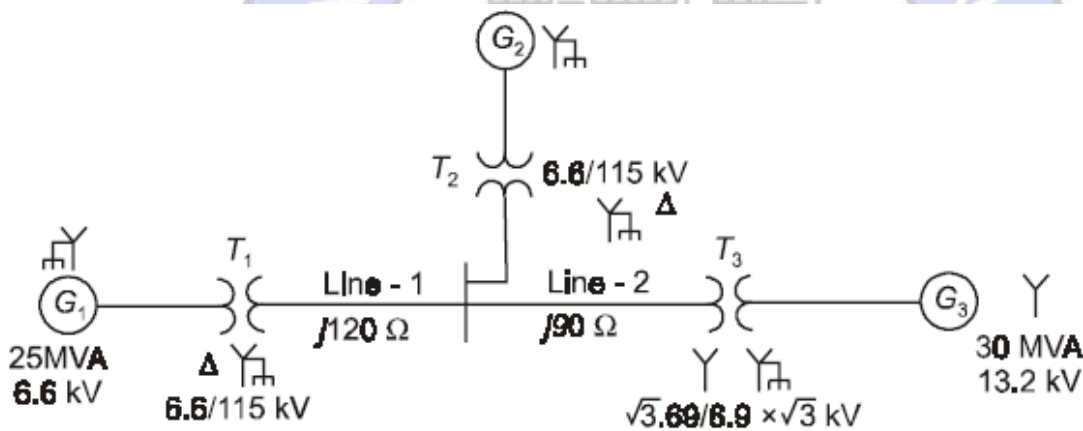
G3: 30 MVA, 13.2 kV, $x_{g3} = 0.15$ pu

T₁ : 30 MVA, 6.6 fl - 115 Y kV, $x_{T1} = 0.10$ pu

T₂ : 15 MVA, 6.6 fl - 115 Y kV, $x_{T2} = 0.10$ pu

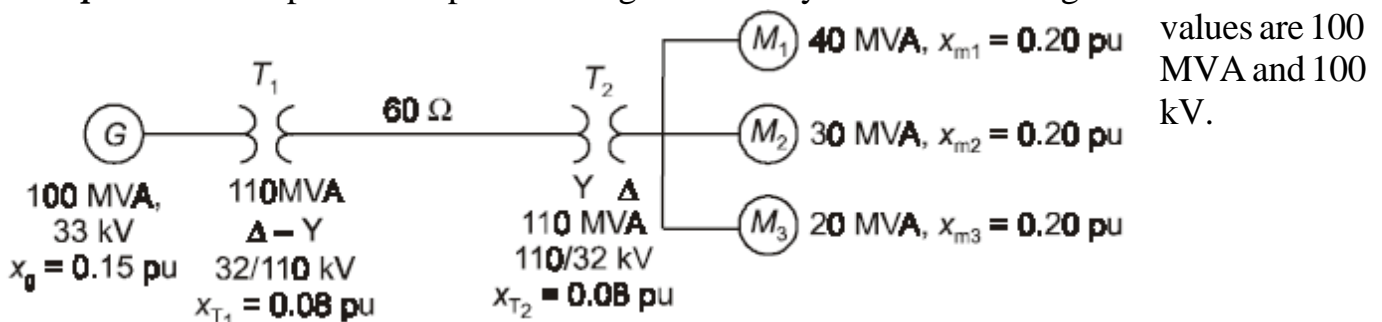
T₃ : Single-phase unit each rated 10 MVA, 6.9/69 kV, $x_{T3} = 0.10$ pu.

Draw per-unit circuit diagram using base values of 30 MVA and 6.6 kV in the circuit of generator-1.

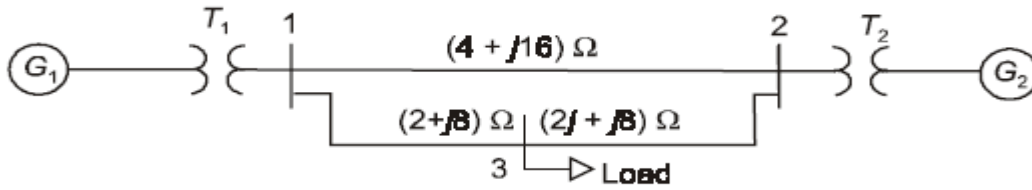


Q3: A 100 MVA, 33 kV, three phase generator has a reactance of 15%. The generator is connected to the motors through a transmission line and transformers as shown in Fig. below. Motors have rated inputs of 40 MVA, 30 MVA and 20 MVA at 30 kV with 20% reactance-each. Draw the per-unit circuit diagram.

Example: Draw the per-unit impedance diagram of the system shown in Fig. Assumed base



values are 100 MVA and 100 kV.



- G1 50 MVA, 12.2 kV, $x_{g1} = 0.10$ pu
G2 20 MVA, 13.8 kV, $x_{g2} = 0.10$ pu
T1 80 MVA, 12.2/132 kV, $x_{T1} = 0.10$ pu
T2 40 MVA, 13.8/132 kV, $x_{T2} = 0.10$ pu
Load 50 MVA, 0.80 pf lagging operating at 124 kV.

