4- Nodal Analysis

In per-phase analysis the components of the power transmission system are modeled and represented by passive impedances or equivalent admittances accompanied, where appropriate, by active voltage or current sources. In the steady state, for example, a generator can be represented by the circuit of either Fig.l (a) or Fig.1 (b). The circuit having the constant emf E_s ' series impedance Z_a ' and terminal voltage V has the voltage equation

$$E_s = IZ_a + V$$

Dividing across by Za gives the current equation for Fig. l (b)

$$I_s = \frac{E_s}{Z_a} = I + VY_a$$

where $Y_a = 1/Z_a$ ' Thus, the emf E_s and its series impedance Z_a can be interchanged with the current source I_s and its shunt admittance Y_a , provided

$$I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a}$$

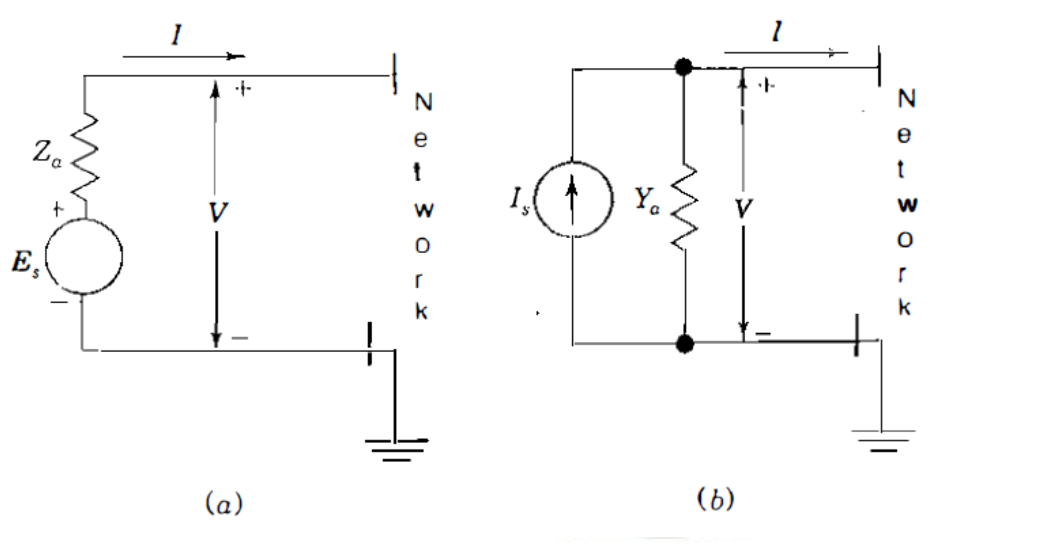
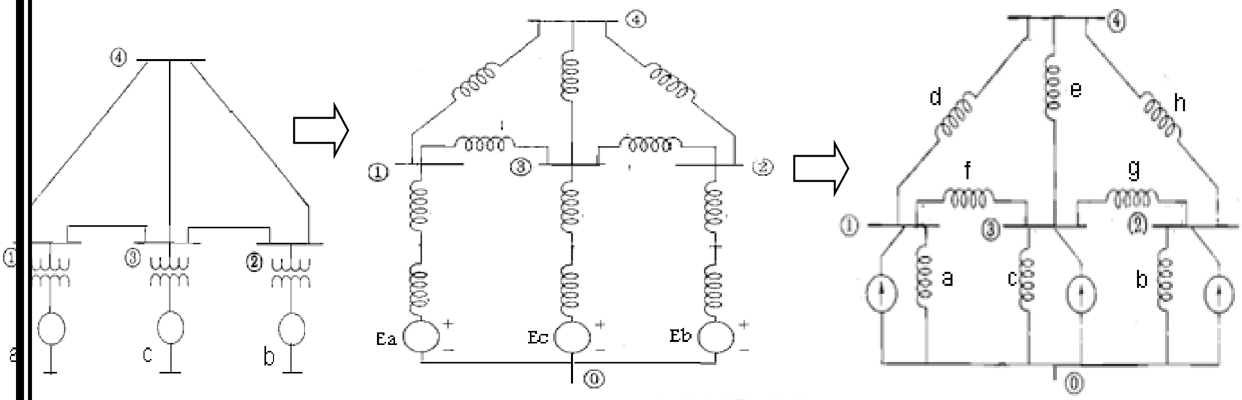


Figure 1Circuits illustrating the equivalenc of sources when $I_s = E_s/Z_a$ and $Y_a = 1/Z_a$

Systematic formulation of equations determined at nodes of a circuit by applying Kirchoff current law. In electrical power networks, the reference node is the ground (major node), then replace the e.m.f,s and series impedance by equivalent current source and shunt admittances.



For node (1):

$$I_1 = V_1Y_a + (V_1 - V_3)Y_f + (V_1 - V_4)Y_d$$

$$I_1 = V_1(Y_a + Y_f + Y_d) - V_3Y_f - V_4Y_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$0 = (V_4 - V_1)Y_d + (V_4 - V_2)Y_h + (V_4 - V_3)Y_e$$

$$0 = -V_1Y_d - V_2Y_h - V_3Y_e + V_4(Y_d + Y_e + Y_h) \quad \ldots\ldots\ldots\ldots 2$$

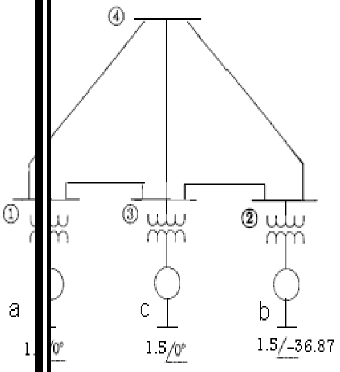
$$\begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{vmatrix}$$

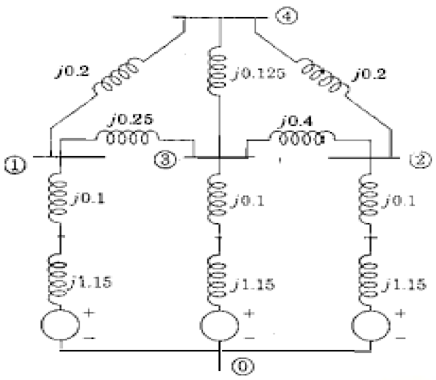
The (Y) matrix is designated (Y_{bus}) and called (the bus admittance matrix)

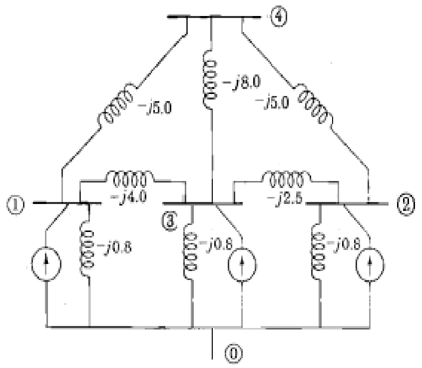
1- Each diagonal element in the nodal admittance matrix (Y_{11} Y_{22} Y_{33} Y_{44}) is called self admittance and it is the sum of the admittances of the branches terminating in each node.
2- Each off-diagonal element of the nodal admittance matrix (all the other admittances) called the mutual admittances and each equal the negative of the sum of all admittances connected between the nodes identified by the double subscript.
3- If no direct connection exists between any two nodes the corresponding off- diagonal elements will be zero.

Example:

Write in matrix form the node equations necessary to solve for the voltages of the numbered buses of the figure below.







Solution:

$I_1 = I_3 = \frac{1.5\angle 0}{j1.25} = 1.2\angle -90 = -j1.2 \quad \text{p.u}$

$I_2 = \frac{1.5\angle -36.87}{j1.25} = 1.2\angle -126.87 = -0.72 - j0.96 \quad \text{p.u}$

Self admittances

Y_{11} = -j5 – j4 – j0.8 = -j9.8

Y_{22} = -j5 – j2.5 – j0.8 = -j8.3

Y_{33} = -j4 – j2.5 – j8 –j0.8 =-j15.3

Y_{44} = -j5 – j5 – j8 = -j18

Mutual admittances

$Y_{12} = Y_{21}$ =0 , $Y_{23} = Y_{32}$ =j2.5, $Y_{13} = Y_{31}$ =j4

$Y_{14} = Y_{41}$ =j5, $Y_{24} = Y_{42}$ =j5, $Y_{34} = Y_{43}$ =j8

The node equations in matrix form

$$\begin{bmatrix} -j1.2 \\ -0.72-j0.96 \\ -j1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.8 & j0 & j4 & j5 \\ j0 & -j8.3 & j2.5 & j5 \\ j4 & j2.5 & -j15.3 & j8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

		j5 j5 j8 -j18		