



Symmetrical Components

The Solution of unsymmetrical fault problems can be obtained by either (a) Kirchhoff's laws or (b) Symmetrical components method. The latter method is preferred because of the following reasons:

- (i) it is a simple method and gives more generality to be given to fault performance studies.
- (ii) It provides a useful tool for protection engineers, particularly in connection with tracing out of fault currents

In 1918, Dr. C.L. Fortescue, an American scientist, showed that any unbalanced system of 3-phase currents (or voltages) may be regarded as being composed of three separate sets of balanced vectors

- (1) A balanced * system of 3-phase currents having a positive (or normal) phase sequence. These are called positive phase sequence components.
- (2) A balanced system of 3-phase currents having the opposite or negative phase sequence. These are called negative phase sequence components.
- (3) A system of three currents equal in magnitude and having zero phase displacement. These are called zero phase sequence components.

The positive, negative and zero phase sequence components are called the symmetrical components of the original unbalanced system. the term symmetrical is appropriate because the unbalanced 3-phase system has been resolved into three sets of balanced (or symmetrical) components.

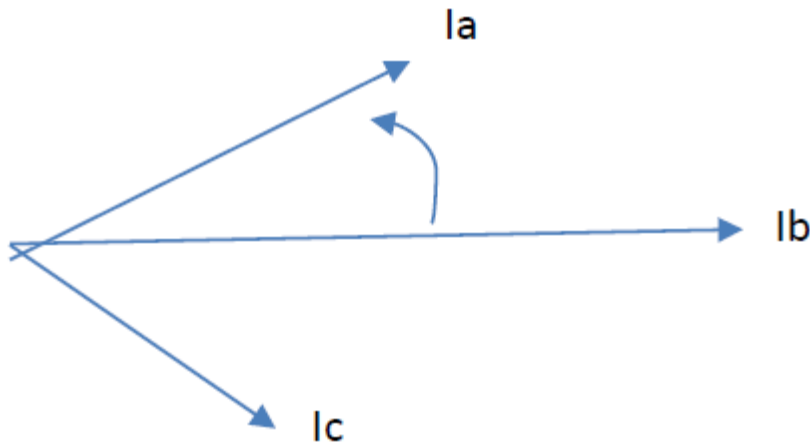
The subscripts 1,2 and 0 are generally used to indicate positive, negative and zero phase sequence components respectively.

For instance, I_{a0} indicates the zero phase sequence components of the current in the first phase similarly I_{b1} implies the positive phase sequence components of current in the second phase.

Let us now apply the symmetrical components theory to an unbalanced 3-phase system.

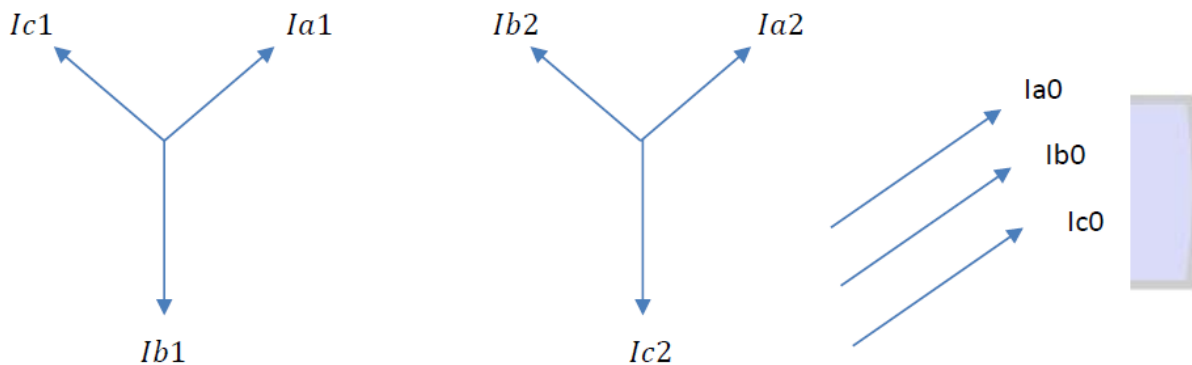
Suppose an unsymmetrical fault occurs on a 3-phasesystems having phase sequence ABC.

According to symmetrical components theory, the resulting unbalanced currents I_a, I_b and I_c (see Fig.1) can be resolved into:



Fig(1)

1. A balanced system of 3-phase currents I_a , I_b and I_c having positive phase sequence (i.e. ABC) as shown in Fig.(2i) these are the positive phase sequence components .



2. A balanced system of 3-phase currents I_{a2} , I_{b2} and I_{c2} having negative phase sequence (i. e. ACB) as shown in Fig. (2ii). These are the negative phase sequence components.

3. A system of three currents I_{a0} , I_{b0} and I_{c0} equal in magnitude with zero phase displacement from each other as shown in Fig. (2iii). These are the zero phase sequence components.

The current in any phase is equal to the vector sum of positive, negative and zero phase sequence currents in that phase as shown in Fig. 3

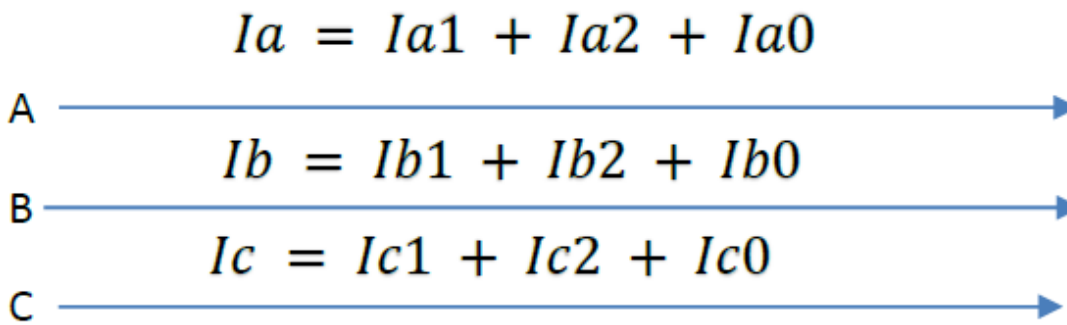


Fig. 3

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_b = I_{b1} + I_{b2} + I_{b0}$$

$$I_c = I_{c1} + I_{c2} + I_{c0}$$



Operator (a): It is the vector which is a unit vector at an angle of 120 degrees.

$$a = 1\angle 120^\circ, a^2 = 1\angle 240^\circ = 1\angle -120^\circ \text{ and } a^3 = 1.$$

apply operator (a) on the symmetrical components:

positive sequence equation:

$$I_{a^1} = I_{a^1}\angle 0^\circ = I_{a^1}$$

$$I_{b^1} = I_{a^1}\angle 240^\circ = a^2 I_{a^1}$$

$$I_{c^1} = I_{a^1}\angle 120^\circ = a I_{a^1}$$

negative sequence equations

$$I_{a^2} = I_{a^2}\angle 0^\circ = I_{a^2}$$

$$I_{b^2} = I_{a^2}\angle 120^\circ = a I_{a^2}$$

$$I_{c^2} = I_{a^2}\angle 240^\circ = a^2 I_{a^2}$$

zero sequence equations

$$I_{a^0} = I_{b^0} = I_{c^0}$$



“abc” currents can be expressed in terms of operator (a)

$$I_a = I_a^0 + I_a^1 + I_a^2$$

$$I_b = I_a^0 + a^2 I_a^1 + a I_a^2$$

$$I_c = I_a^0 + a I_a^1 + a^2 I_a^2$$

In matrix
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} \quad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}, \quad A^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_a^0 = \frac{1}{3} \{I_a + I_b + I_c\}$$

$$I_a^1 = \frac{1}{3} \{I_a + a I_b + a^2 I_c\}$$

$$I_a^2 = \frac{1}{3} \{I_a + a^2 I_b + a I_c\}$$

Also for voltages :

$$V_a = V_a^0 + V_a^1 + V_a^2$$

$$V_b = V_a^0 + a^2 V_a^1 + a V_a^2$$

$$V_c = V_a^0 + a V_a^1 + a^2 V_a^2$$





$$V_a^0 = \frac{1}{3} [V_a + V_b + V_c]$$

$$V_a^1 = \frac{1}{3 \{V_a + aV_b + a^2V_c\}}$$

$$V_a^2 = \frac{1}{3} \{V_a + a^2V_b + aV_c\}$$

Often it understand that the quantities with superscript “0”, “1”, or “2” will have subscript “a”. It is noted that I_{a0} is the sum of the three “abc” components. Thus if the abc components are balanced their sum is zero hence the zero-sequence component is also zero. And we know that in 3-ph sys. $I_a + I_b + I_c = I_n$. Comparing with above equations **$I_n = 3I_{a0}$**

