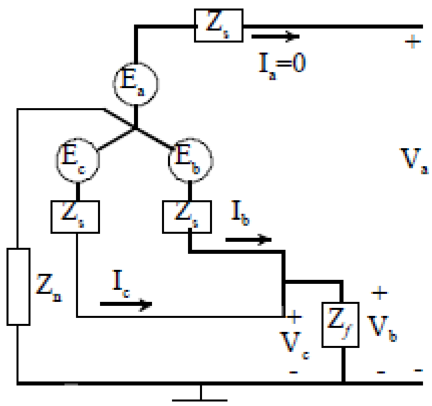
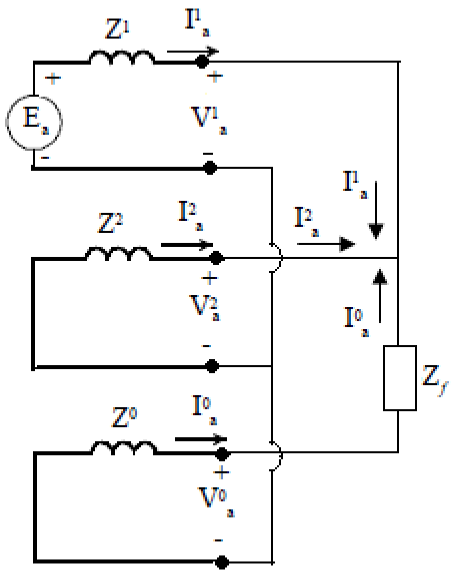




1- Double Line-To-Ground Fault (L-L-G fault)

The boundary conditions at the fault point are:

$V_b = V_c = Z_f(I_b + I_c)$ ………………..1

$I_a = 0$ …………2

From equation 2

$$\boxed{I_a^0 + I_a^1 + I_a^2 = 0} \quad \text{……….…………3}$$

$V_b \& V_c$ in terms of symmetrical components are

$$\begin{Bmatrix} V_b = V_a^0 + a^2 V_a^1 + a\, V_a^2 \\ V_c = V_a^0 + a\, V_a^1 + a^2 V_a^2 \end{Bmatrix} \quad \text{……………..4}$$

$$\boxed{\therefore V_a^1 = V_a^2} \quad \text{……………….5}$$

Eqn. 1 in terms of symmetrical components

$V_b = Z_f(I_a^0 + a^2 I_a^1 + a\, I_b^2 + I_a^0 + a\, I_a^1 + a^2 I_a^2)$

$\quad = Z_f(2I_a^0 - I_a^1 - I_a^2)$

$V_b = 3Z_f\, I_a^0$ ……………………………………6

Substitute 6&5 in 4

$3Z_f\, I_a^0 = V_a^0 + (a^2 + a)V_a^1$

$$\boxed{3Z_f\, I_a^0 = V_a^0 - V_a^1} \quad \text{……………………………7}$$

$$\text{We have} \begin{Bmatrix} V_a^0 = 0 - Z^0 I_a^0 \\ V_a^1 = E_a - Z^1 I_a^1 \\ V_a^2 = 0 - Z^2 I_a^2 \end{Bmatrix} \quad \text{………………8}$$

Substitute 8 in 7

$I_a^0 = -\frac{E_a - Z^1 I_a^1}{Z^0 + 3Z_f}$ ……………………………………9

Using 8 in 5

$I_a^2 = -\frac{E_a - Z^1 I_a^1}{Z^2}$ …………………………………10

Substitute 9&10 in 2&3

$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}}$ ……………………………11

The terminal conditions of L-L-G fault are in the equations 3, 5 and 7. Hence the impedance seen by E_a is: $Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}$ and from this we note that the impendence Z^1 in series with the parallel combination of Z^2 and $Z^0 + 3Z_f$.

Thus the sequence networks can be connected as shown in the figure above.

Finally the fault current is found from:

$$I_f = I_b + I_c = 3I_a^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 12$$

Example:

The one-line diagram of a simple power system is shown in the figure. The neutral of each generator is grounded through a current-limiting reactor of 0.25 per unit on a 100-MVA base. The system data is tabulated below. The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase. Determine the fault current for the following faults:

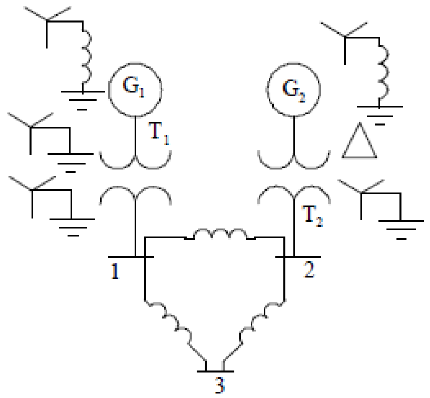
(a) A balanced three-phase fault at bus3 through fault impedance $Z_f = j0.1$ per unit.

(b) A single line-to-ground fault at bus 3 through fault impedance $Z_f = j0.1$ per unit

(c) A line-to-line fault at bus 3 through fault impedance $Z_f = j0.1$ per unit

(d) A double line-to-ground fault at bus 3 through fault impedance $Z_f = j0.1$ per unit

Item	Base MVA	Voltage rating	X^1	X^2	X^0
G_1	100	20 KV	0.15	0.15	0.05
G_2	100	20 KV	0.15	0.15	0.05
T_1	100	20/220 KV	0.1	0.1	0.1
T_2	100	20/220 KV	0.1	0.1	0.1
L_{12}	100	220 KV	0.125	0.125	0.3
L_{13}	100	220 KV	0.15	0.15	0.35
L_{23}	100	220 KV	0.25	0.25	0.7125

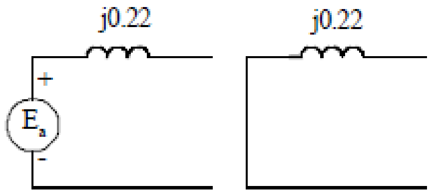


Solution:
Find the Thevenin impedance viewed from bus 3 (faulted bus). The delta transforms to a Y as in the figure below

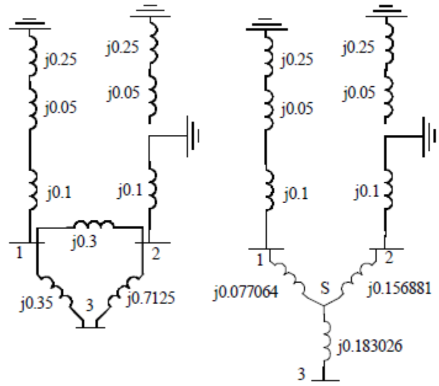
$$Z_{1S} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143$$

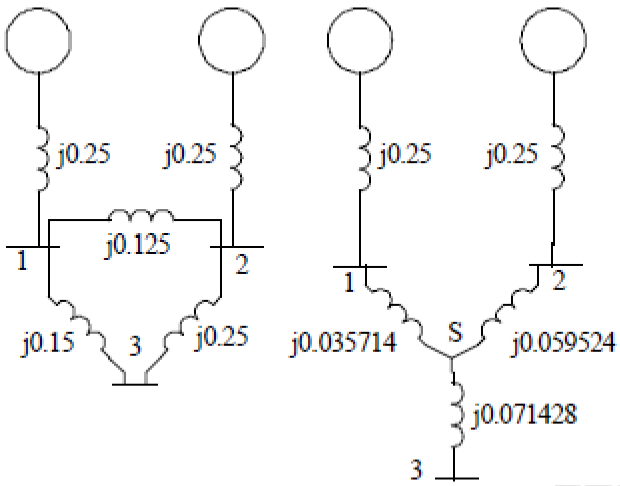
$$Z_{2S} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238$$

$$Z_{3S} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286$$



Positive-sequence Negative-sequence



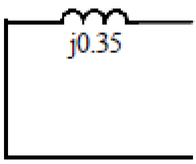


$$Z^1 = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286$$
$$= j0.22$$

The positive- and negative-sequence networks are shown to the right. The only difference between them in this case is that the source is missing in the negative-sequence network. Hence we have:

$$Z^2 = Z^1 = j0.22$$

Now, the zero-sequence network is constructed based on the transformer connections and is shown in the figures to the right. Find the equivalent zero sequence circuit. The result is the simple circuit shown below.



Zero-sequence

(a) Balanced three-phase fault at bus 3.

In this case only a positive network will be used. All generators replaced by on voltage have a voltage of 1.0 per unit, hence the fault current is:

$$I_f = I_{3a} = \frac{V_{3a}(0)}{Z^1 + Z_f} = \frac{1}{j0.22 + j0.1} = -j3.125$$

(b) Single line-to-ground fault at bus 3.

$$I_{3a}^{0} = I_{3a}^{1} = I_{3a}^{2} = \frac{V_{3a}(0)}{Z^1 + Z^2 + Z^0 + 3Z_f}$$
$$= \frac{1}{j0.22 + j0.22 + j0.35 + 3 \times j0.1}$$
$$= -j0.9174\ p.u$$
$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^{0} \\ I_{3a}^{1} \\ I_{3a}^{2} \end{bmatrix} = \begin{bmatrix} -j2.7523 \\ 0 \\ 0 \end{bmatrix} p.u$$

(c) Line-to-line fault at bus 3.

The zero-sequence component of current is zero, i.e. I_{3a}^{0}.

$$I_{3a}^{1} = -I_{3a}^{2} = \frac{V_{3a}(0)}{Z^1 + Z^2 + Z_f}$$
$$= \frac{1}{j0.22 + j0.22 + j0.1}$$
$$= -j1.8519\ p.u$$

The fault current is:

$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^{0} \\ I_{3a}^{1} \\ I_{3a}^{2} \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix} p.u$$

(d) Double line-to-ground fault at bus 3.

$$I_a^1 = \frac{V_{3a}(0)}{Z^1 + \frac{Z^2(Z^0 + 3\mathrm{Z_f})}{Z^2 + Z^0 + 3\mathrm{Z_f}}}$$
$$= \frac{1}{j0.22 + \frac{j0.22(j0.35 + 0.3)}{j0.22 + j0.35 + 0.3}}$$
$$= -j2.6017\ p.u$$

$$I_a^2 = -\frac{V_{3a}(0) - Z^1 I_a^1}{Z^2}$$
$$= \frac{1 - (j0.22)(-j2.6017)}{j0.22}$$
$$= j1.9438$$

$$I_a^0 = -\frac{V_{3a}(0) - Z^1 I_a^1}{Z^0 + 3Z_f} = \frac{1 - (j0.22)(-j2.6017)}{j0.35 + 0.3} = j0.6579$$

The fault current is:

$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^0 \\ I_{3a}^1 \\ I_{3a}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.058\angle 165.93 \\ 4.058\angle 14.07 \end{bmatrix} p.u$$

$$\mathrm{I}_f = I_b + I_c = 1.9732\angle 90$$