Part 2 SIGNALS & SPECTRA

2.1 BASIC DEFINITIONS

2.1.1 Classification of Signals

- Deterministic Signal vs. Random Signal.
- Periodic Signal vs. Non-periodic Signal.
- Analog Signal vs. Discrete Signal.
- Energy Signal vs. Power Signal: where the mean power and the total energy are:

$$P = x_{rms}^2 = \frac{1}{T}\int_0^T x^2(t)\cdot dt \quad , \quad E = \int_{-\infty}^{\infty} x^2(t)\cdot dt$$

- Power signal: If $x(t)$ is periodic, it has finite mean power but infinite energy.
- Energy signal: If $x(t)$ is non-periodic, it has finite energy and zero mean power.

2.1.2 Correlation

The correlation function between $x_1(t)$ and $x_2(t)$ is:

For finite energy signals:

$$R(\alpha) = \int_{-\infty}^{\infty} x_1(t)x_2(t+\alpha)dt$$

For finite power signals:

$$R(\alpha) = \frac{1}{T}\int_{-T/2}^{T/2} x_1(t)x_2(t+\alpha)dt$$

2.1.3 Convolution

The convolution between $x_1(t)$ and $x_2(t)$ is:

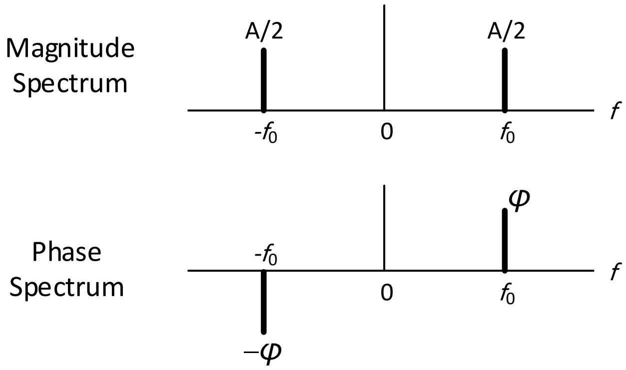
$$c_{12}(\alpha) = x_1 \star x_2 = \frac{1}{T}\int_{-\infty}^{\infty} x_1(t)\cdot x_2(-t+\alpha)\cdot dt$$

2.2 FOURIER TRANSFORM

Although an electrical signal physically exists in the time domain, we can also represent it in the *frequency domain*. We view the signal as it consists of sinusoidal components at various frequencies. This frequency-domain description is called: *spectrum*.

To understand the frequency spectrum, let's consider the signal $x(t) = A\cos(\omega_0 t + \varphi)$ where: A: the peak value or the amplitude, ω_0: The radian frequency and φ: The phase angle.

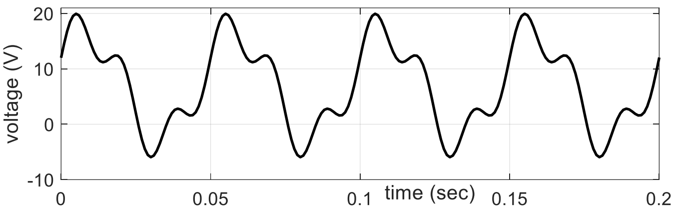
The spectrum of $x(t)$ will be look like:



Next, consider the periodic function:

$$x(t) = 7 + 10\cos(40\pi t - 60°) + 4\sin(120\pi t)$$

which is plotted in the time domain as:



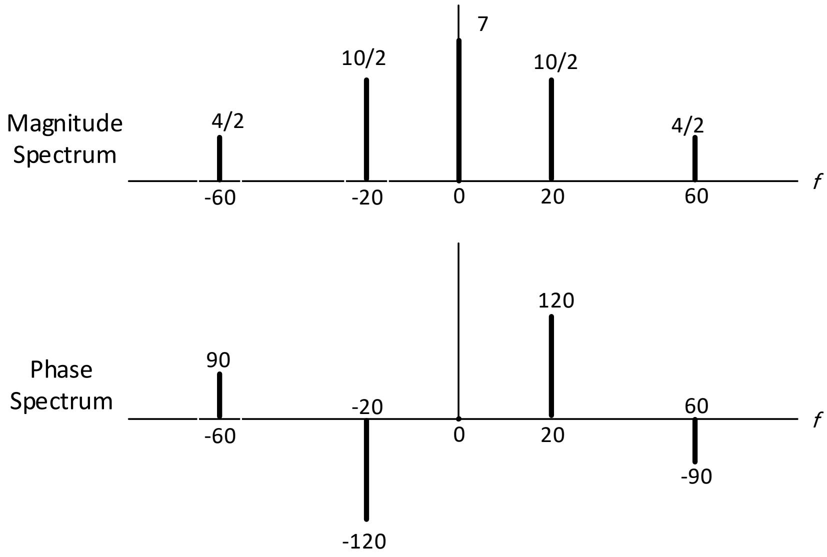
To get the frequency spectrum of $y(t)$, first, we convert $y(t)$ to cosines using:

$\sin\omega t = \cos(\omega t - 90°)$ and $-\cos\omega t = \cos(\omega t \pm 180°)$.

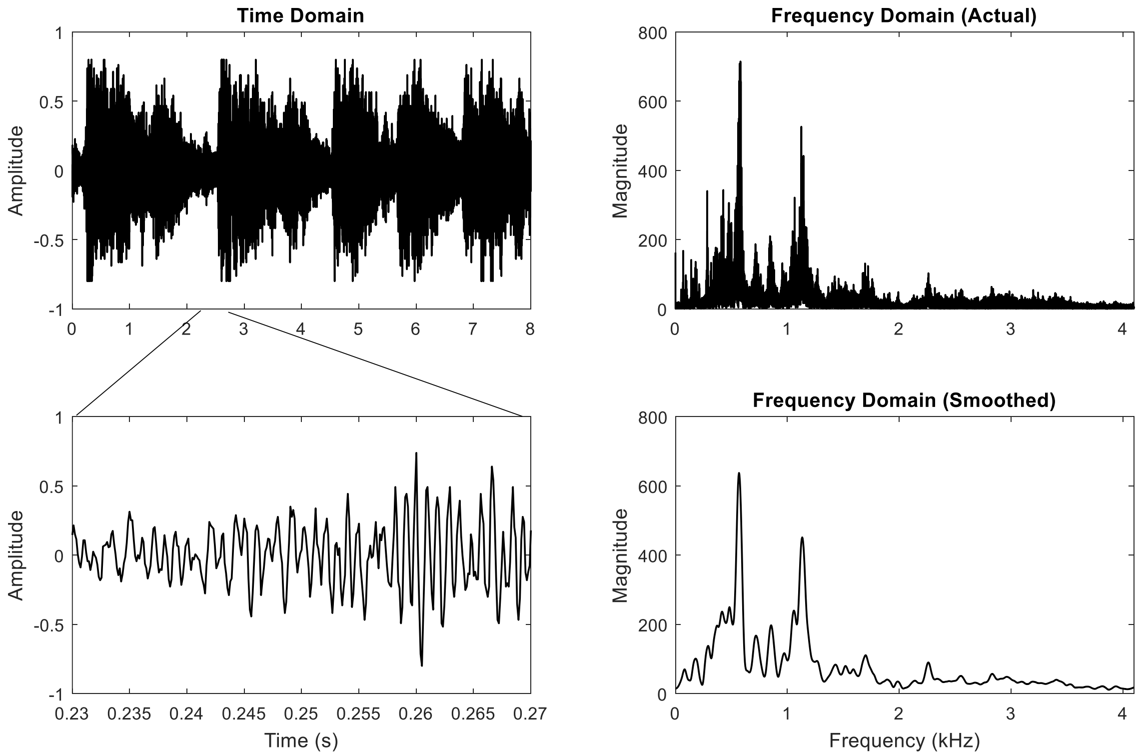
This yields to:

$$y(t) = 7\cos(2\pi 0 t) + 10\cos(2\pi 20 t + 120°) + 4\cos(2\pi 60 t - 90°)$$

Whose spectrum will be:



Practical signals, such as speech, consist of large number of frequency components, and they may look like:



The basic equations relating the time domain version $x(t)$ and the frequency domain version $X(f)$ are known as the Fourier Transform, which are:

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

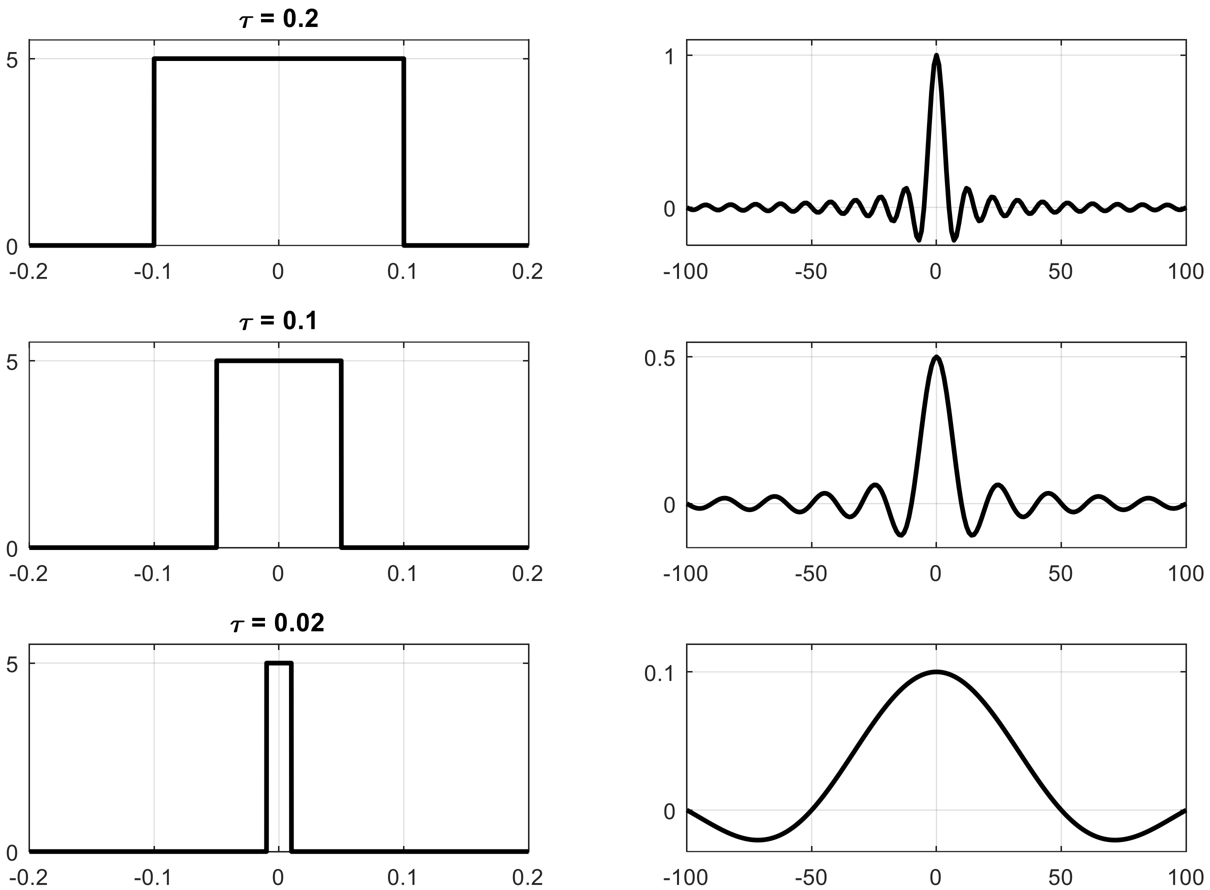
$$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{j\omega t}df$$

These general equations are valid for non-periodic signals whose frequency domain are continuous. The spectrum of periodic functions is discrete, and the Fourier Transform becomes:

$$X_n = \mathcal{F}\{x(t)\} = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

$$x(t) = \mathcal{F}^{-1}\{X_n\} = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

For example, when $A = 5\text{V}$, and $\tau = (0.2, 0.1, 0.02)$ seconds, we get the first nulls respectively located at $(5, 10, 50)\text{Hz}$.



Main properties of the Fourier Transform: (Str. Fig: 3.3, Tables: 3.1 and 3.2)

Signal (in TD)	Spectrum (in FD)
$x(t)$	$X(f)$
$a \cdot x(t)$	$a \cdot X(f)$
$x_1(t) + x_2(t)$	$X_1(f) + X_2(f)$
$x_1(t) \times x_2(t)$	$X_1(f) \star X_2(f)$
$x_1(t) \star x_2(t)$	$X_1(f) \times X_2(f)$

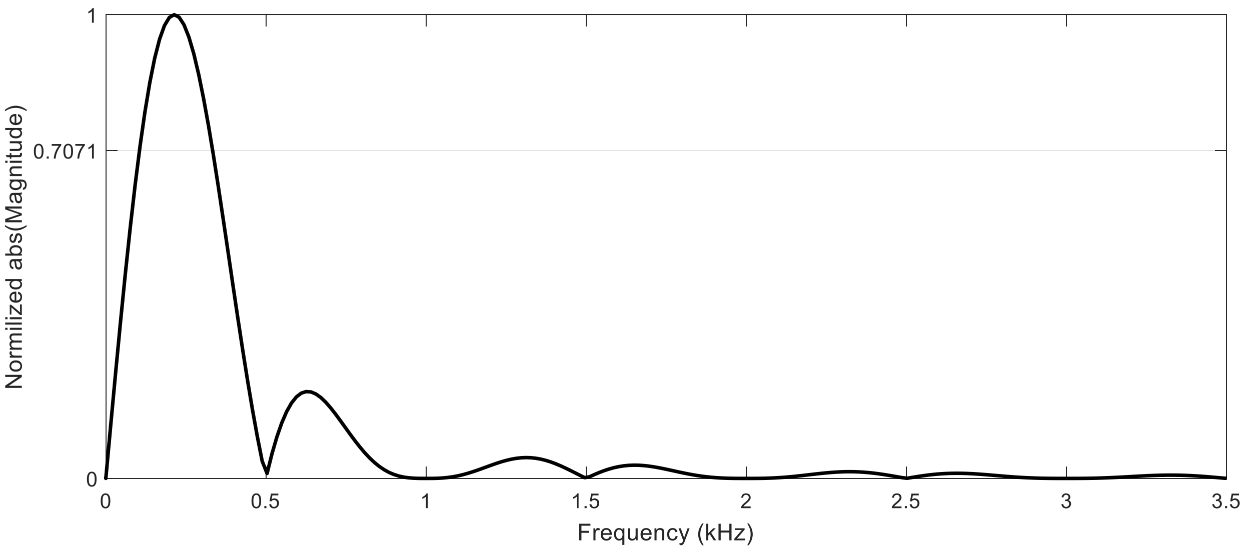
2.3 BANDWIDTH OF A SYSTEM

The set of all frequencies that are present in a signal is the frequency *content*, and if the frequency content consists of all frequencies below some given B, then the signal is said to be *bandlimited to* B. Some bandlimited signals are:

- Telephone quality speech: frequency extends up to∼4kHz.
- Audible music: the frequency extends up to ∼20kHz.
- An ideal square pulse: the frequency extends up to infinity!

Several definitions of the bandwidth are commonly used.

- *Absolute bandwidth*: it contains all the frequency componets of the signal, i.e. the spectrum is zero outside it.
- *3dB bandwidth (or the half-power bandwidth)*: it contains the frequency components whose values are at least $1/\sqrt{2}$ times the maximum component.
- *Null-to-null bandwidth (or zero-crossing bandwidth)*.
- *Power banwidth*: it contains the frequency components that sum ∼99% of the total power.

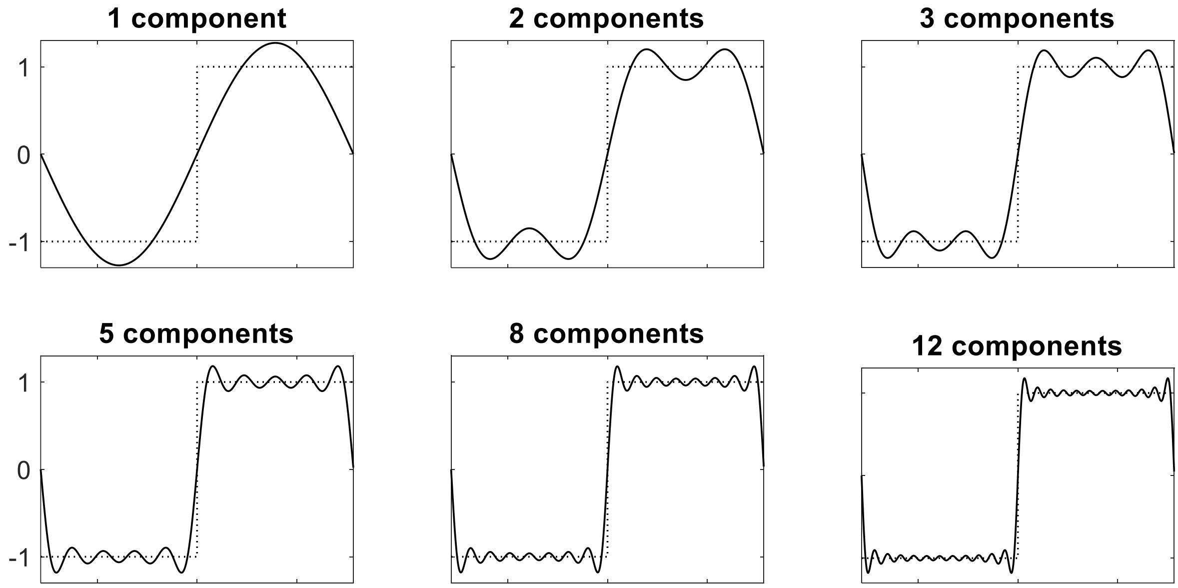


Limiting the frequency response of a system and channel removes some of the frequency components of these signals and causes the time-domain representation to be distorted.

Although real communication systems have a solid theoretical base, they also involve many practical considerations. There is always a trade-off between fidelity to the original signal (that is, the absence of any distortion of its waveform) and such factors as bandwidth and cost.

Increasing the bandwidth often increases the cost of a communication system. Not only is the hardware likely to be more expensive, but bandwidth itself may be in short supply.

In communication over cables, the total bandwidth of a given cable is fixed by the technology employed. The more bandwidth used by each signal, the fewer signals can be carried by the cable.



$$x(t) = \frac{4}{\pi}\left\{\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots + \frac{1}{2k-1}\sin(2k-1)t\right\}$$

Generally, the bandwidth of a system (B) is defined as: the interval of positive frequencies over which the magnitude $|V_O(f)/V_I(f)|$ remains within -3dB (that is $\frac{1}{\sqrt{2}}$ in voltage or ½ in power).

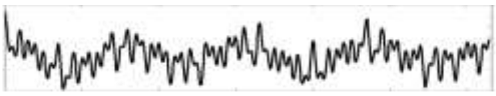
Note: We consider the required B for transmitting a baseband signal = Symbol Rate ÷ 2

2.4 FILTERS

A filter is a device that passes electric signals at certain frequency ranges while preventing the passage of others. For illustration, let the input signal is:

$$x(t) = 2 + 3\cos(2\pi 800t) + 4\cos(2\pi 1200t + 45°) + 5\cos(2\pi 6000t) + 6\cos(2\pi 9000t) + 7\cos(2\pi 11500t) + 8\cos(2\pi 13000t) + 9\cos(2\pi 20000t)$$

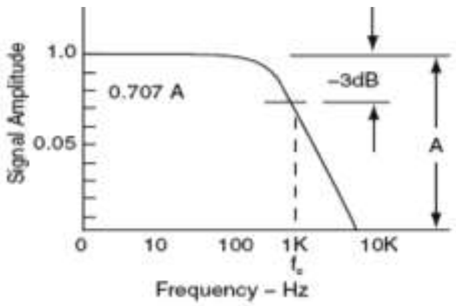
This is plotted in time domain (T.D.) as:

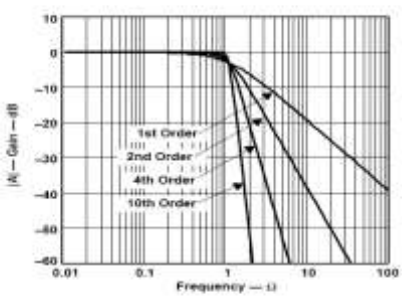


Plot it in frequency domain (F.D.)

2.4.1 Low Pass Filter (LPF)

$B = 0 \rightarrow f_k$



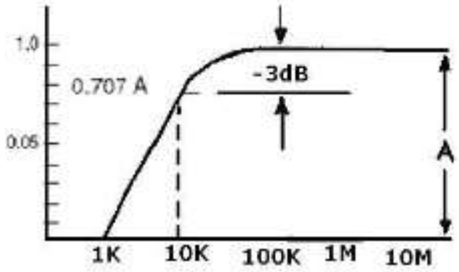


$y(t) = 2 + 3\cos(2\pi 800t)$, This is plotted in T.D. as:

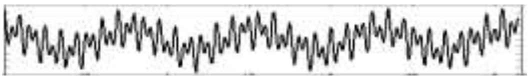
Plot it in frequency domain (F.D.)!

2.4.2 High Pass Filter (HPF)

$B = f_k \to \infty$



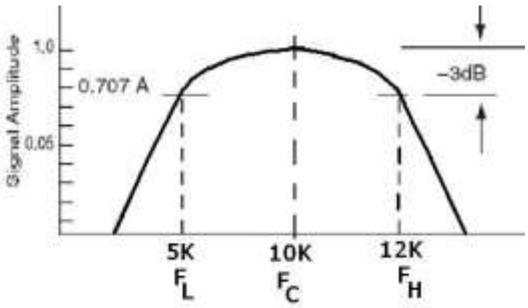
$y(t) = 7\cos(2\pi 11500t) + 8\cos(2\pi 13000t) + 9\cos(2\pi 20000t)$, plotted in T.D. as:

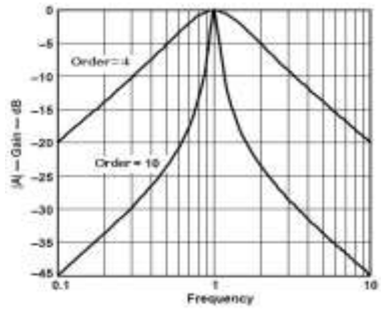


Plot it in frequency domain (F.D.)!

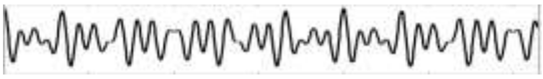
2.4.3 Band Pass Filter (BPF)

$B = f_L \to f_H$, centered at f_C





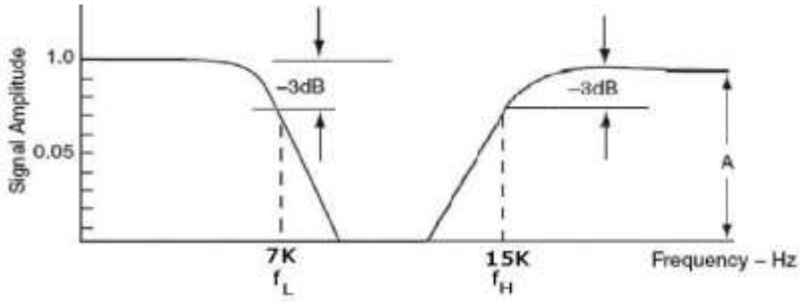
$y(t) = 5\cos(2\pi 6000t) + 6\cos(2\pi 9000t) + 7\cos(2\pi 11500t)$



Plot it in frequency domain (F.D.)!

2.4.4 Band Reject Filter (BRF)

$B = (0 \rightarrow f_L) + (f_H \rightarrow \infty)$



$y(t) = 2 + 3\cos(2\pi 800t) + 4\cos(2\pi 1200t + 45°) + 5\cos(2\pi 6000t) + 9\cos(2\pi 20000t)$

2.4.5 All Pass Filter (APF)

H.W. Find the output signal of the mentioned filters if the input signal is:

$$x(t) = -2.3 + 7\cos(2\pi 50t) + \cos(2\pi 950t) + 3\cos(2\pi 2340t + 45°) + 8\cos(2\pi 6720t) + 4.4\cos(2\pi 8800t) + 5.7\cos(2\pi 11000t) + 6\cos(2\pi 1400t) + 9\cos(2\pi 10^6 t)$$