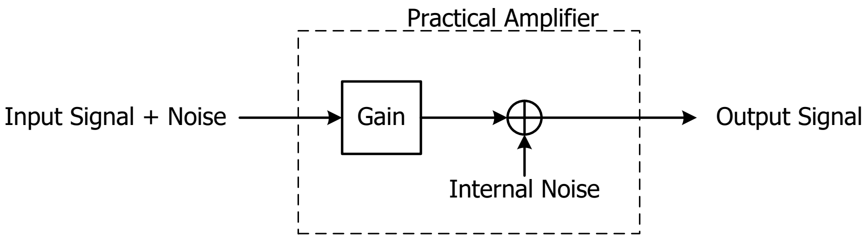
4.5 AMPLIFIER NOISE

Since the received signal is very weak, it is crucial to have an amplifier at an early stage of the reception. Because the received signal has already been corrupted by the noise from the external noise, it is important to make the internal noise within the receiver itself as small as possible. Since we cannot practically implement a *noiseless* amplifier, we may consider *theoretically* an amplifier like:



Let the received signal S_r comprise S_i as the message without noise, and N_i as the additive noise. So, if the input to the amplifier of the receiver is $S_r = S_i + N_i$, the output will be:

$$S_o = S_r G + N_{int} = GS_i + (GN_i + N_{int}) = GS_i + N_o$$

Where: G is the gain of the ideal amplifier, N_{int} is the internally generated noise. It is obvious that the final noise component represents the addition of the internal and the amplified value of the external noise. To discuss the noise within the receiver, we need to use describe the following measures and terms.

4.6 NOISE COMPUTATIONS

4.6.1 SNR

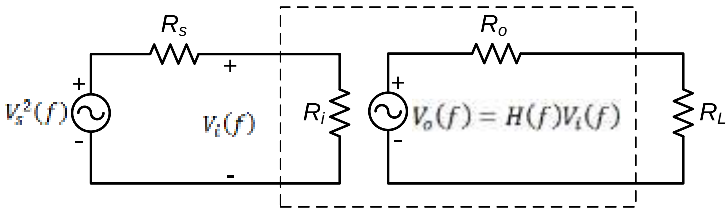
The Signal-to-Noise Ratio (SNR) is an important measure for the performance of a system. It reflects the amount of noise that has been induced in the system in proportion to the desired signal.

$$\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\overline{s^2(t)}}{\overline{n^2(t)}} \quad \text{or} \quad \left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{S}{N}\right)$$

4.6.2 Available Power and T_E

The figure below depicts the circuit model of a noiseless amplifier inserted between a source V_s and a load R_L. This model has an input resistance R_i, an output resistance R_o, and a voltage transfer function $H(f)$. The source generates a mean square voltage density $V_s^2(f)$ representing noise or an information signal or both. The available power density from the source is $\eta_s(f) = V_s^2(f)/4R$. The available power density at the output of the amplifier is:

$$\eta_o(f) = \frac{V_o^2(f)}{4R_o} = \frac{|H(f)|^2 V_i^2(f)}{4R_o} = \frac{|H(f)|^2}{4R_o}\left(\frac{R_i}{R_s + R_i}\right)^2 V_s^2(f)$$

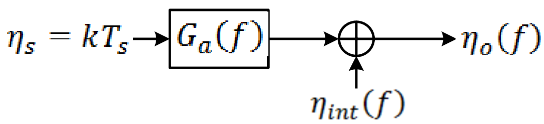


We define $G_a(f)$ as the gain of the amplifier's available power:

$$G_a(f) \approx \frac{\eta_o(f)}{\eta_s(f)} = \frac{V_o^2(f)R_s}{V_s^2(f)R_o} = \left(\frac{|H(f)|R_i}{R_s + R_i}\right)^2 \frac{R_s}{R_o}$$

Assuming that the source generates thermal white noise at the temperature T_s, then $\eta_s = kT_s$. The available noise power at the output of a noiseless amplifier is:

$$\eta_o(f) = G_a(f)\eta_s(f) = G_a(f)kT_s$$



Since the internal noise is independent of the source noise, we write:

$$\eta_o(f) = G_a(f)kT_s + \eta_{int}(f)$$

Integration then yields the total available output noise power:

$$N_o = \int_0^\infty \eta_o(f)df = kT_s\int_0^\infty G_a(f)df + \int_0^\infty \eta_{int}(f)df$$

Most amplifiers in a communication system have a frequency-selective response, with maximum power gain G and noise equivalent bandwidth B_N, so:

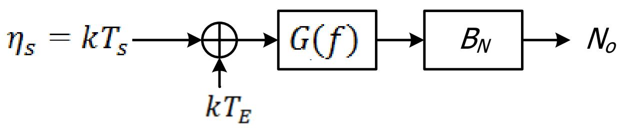
$$GB_N = \int_0^\infty G_a(f)df$$

We define the effective noise temperature of the amplifier to be:

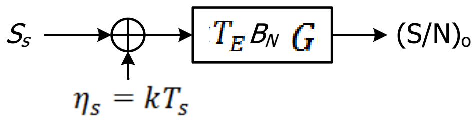
$$T_E = \frac{1}{GkB_N}\int_0^\infty \eta_{int}(f)df$$

Hence, the total output noise power becomes:

$$N_o = Gk(T_s + T_E)B_N$$



Now let the figure below represent a noisy amplifier with signal plus white noise at the input.



So, the available signal power at the output will be $S_o = GS_s$. Thus

$$\left(\frac{S}{N}\right)_o = \frac{GS_s}{N_o} = \frac{S_s}{k(T_s + T_E)B_N}$$

Since $\left(\frac{S}{N}\right)_s = \frac{S_s}{kT_sB_N}$, then:

$$\left(\frac{S}{N}\right)_o = \frac{1}{1 + T_E/T_s}\left(\frac{S}{N}\right)_s$$

we see that in general $\text{SNR}_o < \text{SNR}_s$, however we may get $\text{SNR}_o \approx \text{SNR}_s$ when $T_E \ll T_s$. At carrier frequencies below ~30MHz, the internal noise has little effect, and the amplifier appears to be noiseless. At higher frequencies, T_E becomes significant which often affect the design of the receivers and the repeaters. Some very low-noise amplifiers have $T_E \approx$ 10K → 30K while it is $T_E \approx$ 1000K for some systems. Knowing T_E is not necessarily the ambient temperature of the amplifier, sometimes cryogenic cooling is employed to lower the noise temperature. In this class, we set $T_E = 290$K.