4.6.3 Noise Figure

Another measure of amplifier noise is the noise figure F (also called 'figure-of-merit'). It is expressed as $F = \text{SNR}_I \div \text{SNR}_o$ (normally in dB). Now, for the case where $T_s = T_0 = 290\text{K}$,

$$F = 1 + \frac{T_E}{T_0} \qquad \text{or} \qquad F_{dB} = 10\log_{10}(F)$$

A very noisy amplifier has $T_E \gg T_0 \rightarrow F \gg 1$

At room temperature, $F = 2$

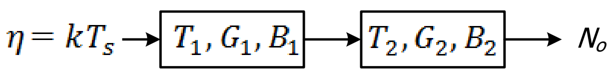
A low-noise amplifier has $T_E < T_0 \rightarrow 1 < F < 2$: it is the case receivers usually should work at.

HW: What is the value of F of a perfect system?

4.6.4 Noise in Multi-Stage Systems

Receivers comprise consecutive different stages to process the incoming signal. Since each stage generates internal noise differently, they must be carefully designed to perform the optimum reception.

To develop expressions for the overall performance of the system in terms of the parameters of the individual stages, let's consider the following cascade of two noisy two-ports sub-systems. In this figure, the subscripts identify: the effective noise temperature, the maximum power gain, and the noise bandwidth per stage.



The overall power gain then equals the product, i.e. $G = G_1G_2$

The total output noise power consists of three terms:

(1) Source noise amplified by both stages;
(2) Internal noise from the first stage, amplified by the second stage;
(3) Internal noise from the second stage.

Assuming $B_2 \leq B_1$ and $B_N \approx B_2$, thus:

$$N_o = GkT_sB_N + G_2(G_1kT_1B_N) + G_2kT_2B_N = Gk\left(T_s + T_1 + \frac{T_2}{G_1}\right)B_N$$

The overall effective noise temperature and noise figure are:

$$T_E = T_1 + \frac{T_2}{G_1} \qquad \text{and} \qquad F = 1 + \frac{T_1}{T_0} + \frac{T_2}{G_1T_0} = F_1 + \frac{F_2 - 1}{G_1}$$

The foregoing analysis readily generalizes to:

$$T_E = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots \qquad \text{and} \qquad F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

Both expressions bring out the fact that:

The first stage plays a critical role and it must be given careful attention in system design.

See Str. Example 4.7.5

4.6.5 Time Representation of Bandpass Noise

We can approximate the noise with a phasor representation as: the portion of the noise which is the in-phase component $n_c(t)$, and the quadrature component $n_s(t)$. So:

$$n(t) = n_c(t)\cos(\omega_0 t) - n_s(t)\sin(\omega_0 t)$$

Where f_0 = center frequency. Note that: $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$

4.7 Noise in AM Systems

4.7.1 Synchronous DSB-SC

We assume synchronous demodulation. So, if the input is $y(t) = m(t)\cos(\omega_c t)$, then:

$S_I = \overline{y^2(t)} = \frac{1}{2}\overline{m^2(t)}$, and as the output is $\frac{1}{2}m(t)$ then: $S_O = \overline{\left[\frac{1}{2}m(t)\right]^2} = \frac{1}{4}\overline{m^2(t)} = \frac{1}{2}S_I$.

If the input noise is $n_I(t)$ then $N_I = \overline{n_I^2(t)}$, The multiplier output

$$r(t) = n_I(t)\cos(\omega_c t) = [n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)]\cos(\omega_c t)$$

$$= \frac{1}{2}n_c(t) + \frac{1}{2}n_c(t)\cos(2\omega_c t) - \frac{1}{2}n_s(t)\sin(2\omega_c t)$$

The output from the LPF will be: $n_o(t) = \frac{1}{2}n_c(t)$, so:

$$N_O = \overline{n_o^2(t)} = \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}\overline{n_I^2(t)} = \frac{1}{4}N_I$$

As a result:

$$\therefore \frac{S_O}{N_O} = 2\frac{S_I}{N_I}$$

This means that the detector improves the SNR in DSB-SC system by factor of two. This improvement is achieved because the coherent (synchronous) detector rejects $n_s(t)$ and halves $n_c(t)$.

4.7.2 Synchronous DSB-LC

The use of synchronous detector for DSB-LC provides the same advantages mentioned above.

The same computations can be used here as well by using $m(t)$ as $\left(A + m(t)\right)$ instead. So:

$S_I = \frac{1}{2}A^2 + \frac{1}{2}\overline{m^2(t)}$ assuming $\overline{m^2(t)} = 0$.

Hence $\quad \dfrac{S_O}{N_O} = \dfrac{2\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \cdot \dfrac{S_I}{N_I}$

Note: for $m(t) = \cos(\omega_m t)$, then:

$$\frac{S_O}{N_O} = 2\rho \cdot \frac{S_I}{N_I}$$

4.7.3 Envelope Detector DSB-LC

The input signal to the envelope detector is

$\Phi(t) = y(t) + n_I(t) = [A + m(t)]\cos(\omega_c t) + n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$

For high input SNR, it is just like synchronous. i.e.:

$$\frac{S_O}{N_O} = \frac{2\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \cdot \frac{S_I}{N_I}$$

For input SNR$<$ 10dB

$$\frac{S_O}{N_O} = \frac{S_I}{\eta f_m}$$

In fact, for input SNR$<$ 1 (0dB), the noise takes over the control of the envelop.

4.7.4 SSB-SC

Since the SSB-SC signal is: $y(t) = m(t)\cos(\omega_c t) \pm \hat{m}(t)\sin(\omega_c t)$, we have:

$S_I = \overline{y^2(t)} = \dfrac{1}{2}\overline{m^2(t)} \pm \dfrac{1}{2}\overline{\hat{m}^2(t)}$

But $\hat{m}(t)$ is only 90° shifting of $m(t)$, so $S_I = \overline{m^2(t)}$. And: $S_O = \overline{\left[\frac{1}{2}m(t)\right]^2} = \frac{1}{4}\overline{m^2(t)} = \frac{1}{4}S_I$. This results in:

$$\therefore \frac{S_O}{N_O} = \frac{S_I}{N_I}$$