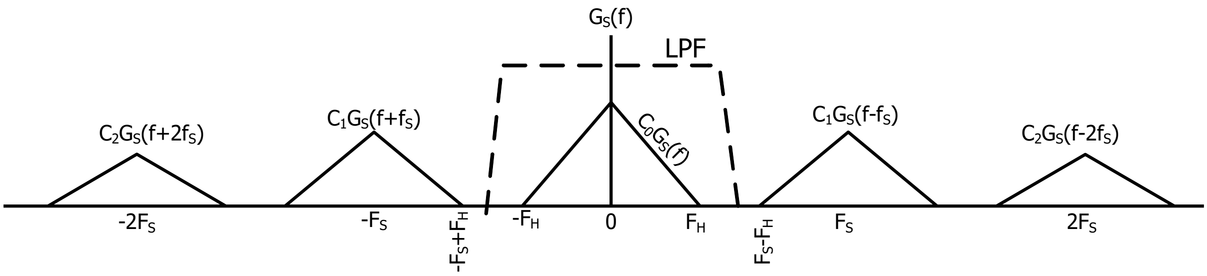
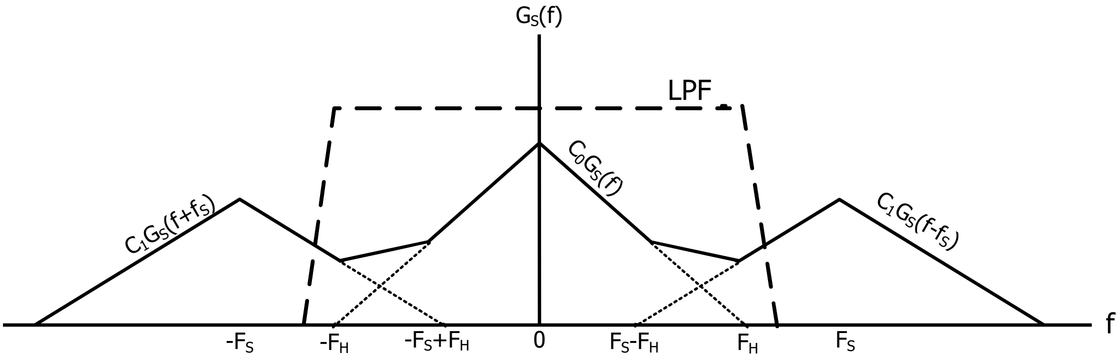
1.2.3 Reconstruction

To perform successful sampling, we keep $f_s - f_H \geq f_H$, i.e. $f_s \geq 2f_H$. This ensures the signal $G(f)$ always contained perfectly in $G_s(f)$. As a result, $G(f)$ can be recovered exactly by simply passing $G_s(f)$ through a LPF that passes only $C_0 G(f)$. Finally, we introduce a gain of $1/C_0$ in the LPF to restore the original signal.



1.2.4 Aliasing

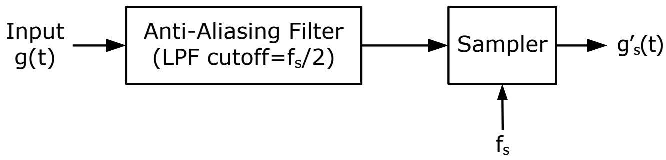
When $f_s < 2f_H$, the copies overlap, and hence, the perfect reconstruction becomes impossible. If this Nyquist criterion is not considered, the folded back portion overlaps the original spectrum. This results a new shape of the reconstructed spectrum filtered by LPF. And this, undoubtedly, gives a signal different from $g(t)$. This problem is called "aliasing".



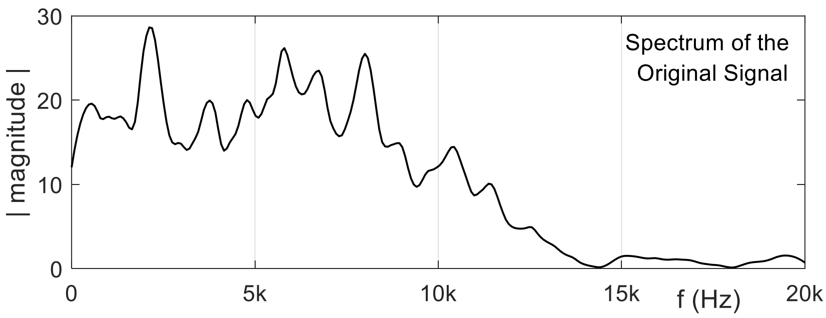
Two conditions are necessary to avoid the aliasing:

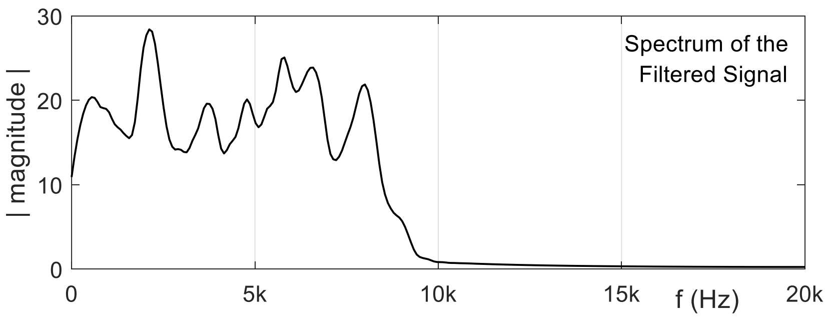
(1) The input signal must be limited according the Nyquist condition, i.e. $f_H \le \frac{f_s}{2}$.

(2) The sampling frequency must sufficiently greater than the maximum frequency component of the signal, i.e. $f_s \ge 2f_H$. This condition is called "Nyquist Rate".



The illustration below shows a baseband signal whose spectrum extends up to 20kHz. If we want to perform Nyquist sampling at the rate 20kHz, we must use an anti-aliasing filter whose cutoff frequency equals $\frac{f_s}{2} \le 10$kHz.





Another example from practical voice transmission systems. Although the average voice spectrum extends well up to 20kHz, most of the energy is concentrated in the range 100Hz→1kHz. For intelligibility, the bandwidth 3kHz is sufficient. A voice wave is pre-filtered by a 3.3kHz LPF and then sampled at 8kHz. These are the standard values for telephone sampling.