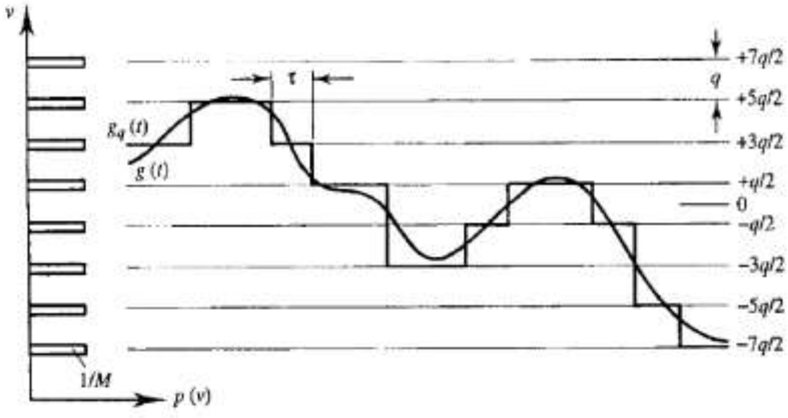
1.6.5 Signal to Quantization Noise Ratio

It is important to consider the quantization noise in the overall system quality. To calculate SN_qR of a uniformly quantized signal, it is suitable to make the following assumptions:

(1) Linear quantization (i.e. equal increments between quantization levels).

(2) Zero mean signal (i.e. symmetrical PDF around the 0 Volt).

(3) Uniform signal PDF (i.e. all signal levels equally likely).



(a) Analogue and quantised signals ($g(t)$ and $g_q(t)$ respectively)

(b) Quantised minus analogue signal, $\varepsilon_q(t) = g_q(t) - g(t)$

Let: L be the number of the levels of the quantizer, l be the number of bits per a PCM word ($L = 2^l$), and V_p be the peak design level of the quantizer. The quantization interval q becomes:

$$q = \frac{2V_p}{L-1}$$

The PDF of the allowed levels is given by:

$$p(v) = \sum_{\substack{k=-L \\ k=\text{odd}}}^{L} \frac{1}{L}\delta\left(v - \frac{qk}{2}\right)$$

The mean square signal after quantization is:

$$\overline{v^2} = \int_{-\infty}^{\infty} v^2 p(v)dv = \frac{2}{L}\left[\int_0^{\infty} v^2\delta\left(v-\frac{q}{2}\right)dv + \int_0^{\infty} v^2\delta\left(v-\frac{3q}{2}\right)dv + \cdots\right]$$

$$= \frac{2}{L}\left(\frac{q}{2}\right)^2\left[1^2+3^2+5^2+\cdots+(L-1)^2\right] = \frac{2}{L}\left(\frac{q}{2}\right)^2\left[\frac{L(L-1)(L+1)}{6}\right]$$

$$\therefore \overline{v^2} = \frac{q^2}{12}(L^2-1)$$

Denoting the quantization error (i.e. the difference between the unquantized and quantized signals) as ε_q, then the PDF of ε_q is uniform:

$$p(\varepsilon_q) = \begin{cases} \frac{1}{q} & -\frac{q}{2} \leq \varepsilon_q < \frac{q}{2} \\ 0 & \text{elsewhere} \end{cases}$$

The mean square quantization error (noise) is:

$$\overline{\varepsilon_q^2} = \int_{-q/2}^{q/2} \varepsilon_q^2 \, p(\varepsilon_q) \, d\varepsilon_q = \frac{q^2}{12}$$

Therefore, the average $\text{SN}_\text{q}\text{R}$ will be:

$$\text{SN}_\text{q}\text{R} = \overline{v^2}/\overline{\varepsilon_q^2} = L^2 - 1$$

Since the peak signal level is $\frac{qL}{2}$ Volts then the peak $\text{SN}_\text{q}\text{R}$ will be:

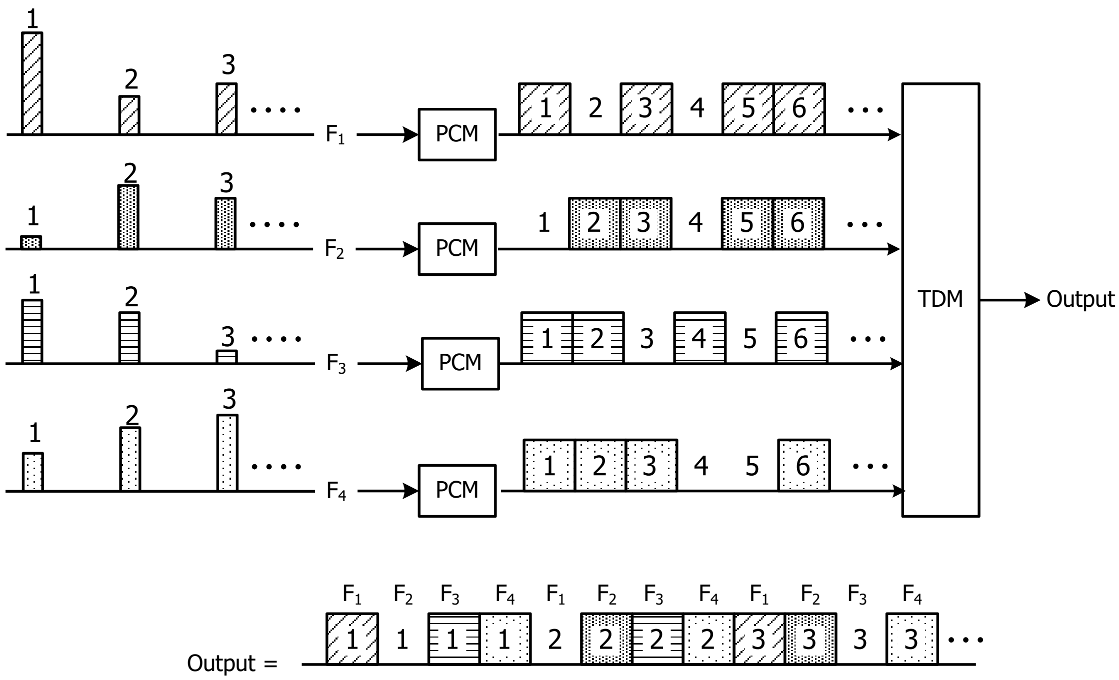
$$\text{SN}_\text{q}\text{R} = \frac{(Lq/2)^2}{\overline{\varepsilon_q^2}} = 3L^2$$

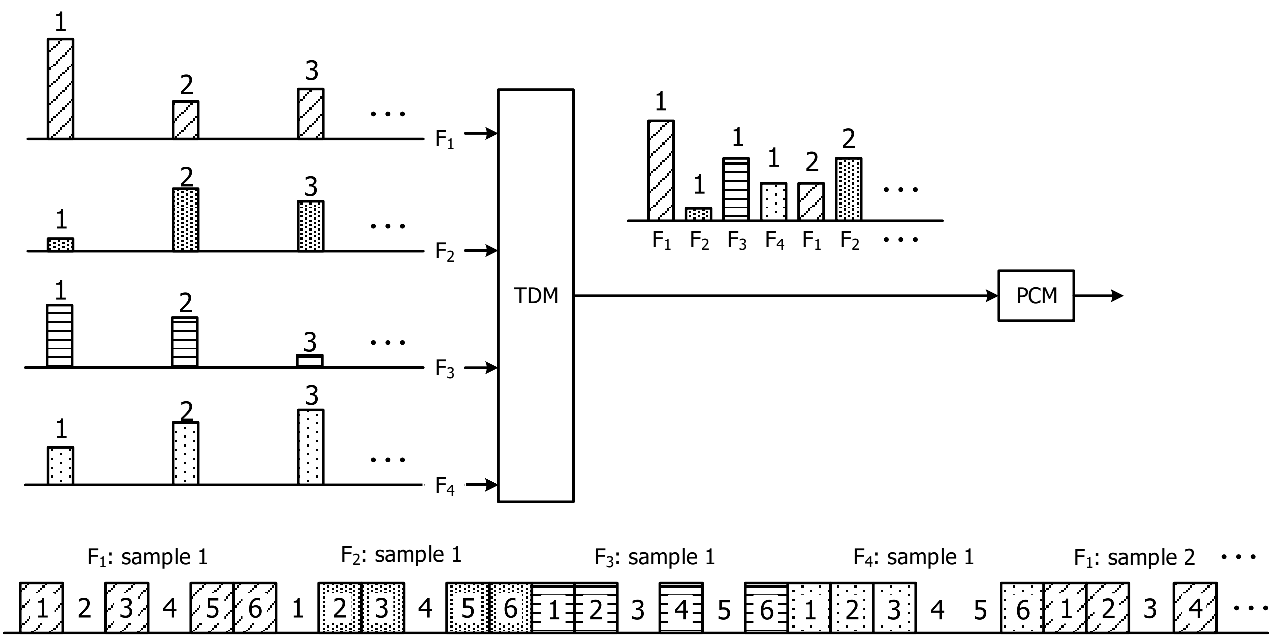
We may express it in decibels as $\text{SN}_\text{q}\text{R} = 6.02l + \alpha$

Where $\alpha = 4.77$ for the peak $\text{SN}_\text{q}\text{R}$, and $\alpha = 0$ for the average $\text{SN}_\text{q}\text{R}$. This equation is called the *6dB rule*, and it points out that: an additional 6dB improvement in the $\text{SN}_\text{q}\text{R}$ is obtained for each bit added to the PCM word.

1.6.6 PCM Multiplexing

The output PCM signal rate $R_{TDM} = NR_b = Nlf_s$ (in bps) where: N = number of multiplexed signals, l = number of bits per sample and f_s = sampling frequency.





What is the difference between *Information Rate* and *Baud Rate*?