



### 3-Motor Circle Diagram

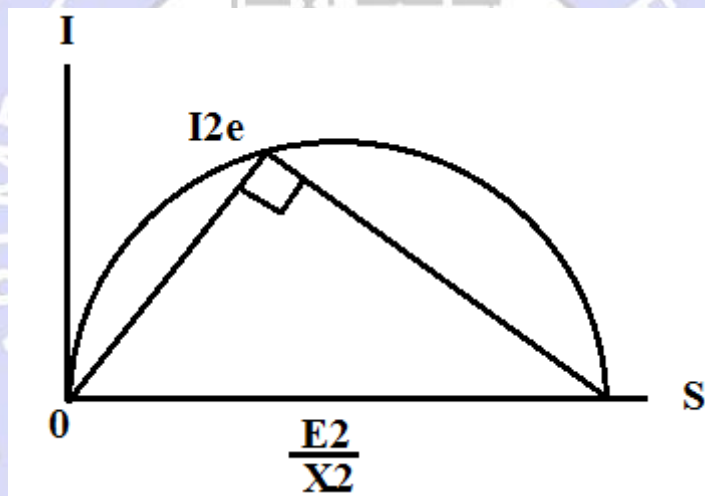
#### 3-1 Rotor Circle Diagram: -

$$I_{2e} = \frac{E_{2e}}{\sqrt{\left(\frac{R_{2e}}{s}\right)^2 + X_{2e}^2}} \dots\dots\dots eq(1)$$

$I_{2e}, E_{2e}, \cos \phi_2 = \text{function to } S \text{ } F(s)$

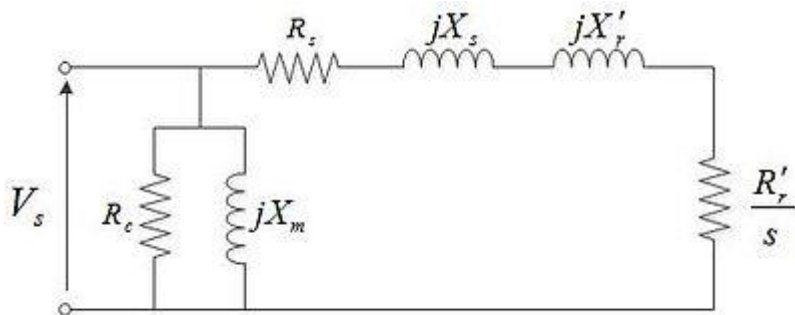
$$OD = \frac{OA}{\sin \phi_2} = \frac{I_{2e}}{\frac{X_{2e}}{Z_{2s}}} = \frac{E_{2s}}{Z_{2s}} \times \frac{Z_{2s}}{X_{2s}} = \frac{E_{2s}}{X_{2s}} = \frac{SE_2}{SX_2} = \frac{E_2}{X_2} = \text{CONSTANT}$$

$S=0, I_{2s}=0$



#### **Circle Diagram for the approximate equivalent cct.**

It is clear that the circuit to the right of points ab is similar to a series circuit ,having a constant voltage and reactance  $(X_1+X_{2e})$ but variable resistance (corresponding to different values of slip  $s$ )

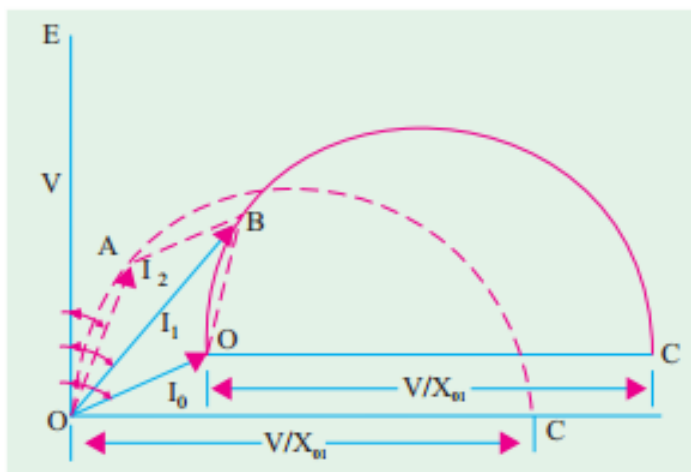


Hence, the end of current vector for  $I_{2e}$  will lie on a circle with a diameter of  $V/(X_1+X_{2e})$ . In fig.(2)  $I_{2e}$  is the rotor current referred to stator,  $I_e=I_0$  no load current or exciting current and  $I_1$  is the total stator current and is the

When  $I_{2e}$  is lagging and  $\phi_2=90^\circ$ , then the position of vector for  $I_{2e}$  will be along OC, right angles to the voltage vector OE, for any other value of  $\phi_2$ , point A will be move along the circle shown dotted.

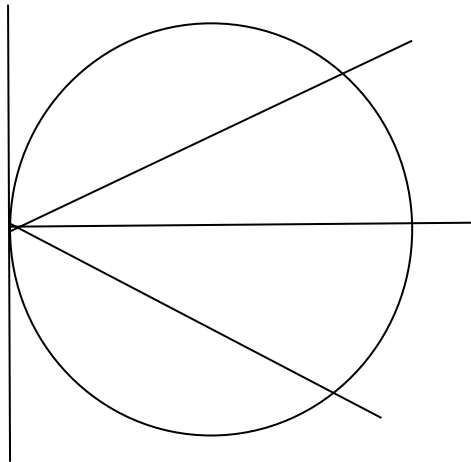
The exciting current  $I_e$  is drawn lagging  $V$  by an angle  $\phi_e$  if conductance  $G_i$  and susceptance  $B_m$  of the exciting circuit are assumed constant, then  $I_e$  and  $\phi_e$  are also constant.

The end of current vector for  $I_1$  is also seen to lie on another circle which is displaced from the dotted circle by an amount  $I_e$ . its diameter is still  $V/X_1+X_{2e}$  and is parallel to the horizontal axis OC. hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.



Fig(2)

**Complete Circle :-**



Motoring  $s=0 \rightarrow +1$

Braking  $s=1 \rightarrow +\infty$

Generating  $s=0 \rightarrow -\infty$

### 3-2 Motor Mechanical Characteristics

**-Torque from electromagnetic forces:-**

$$P_{em} = m_1 E_{2e} I_{2e} \cos \varphi_2 \dots \dots \dots (1)$$

$E_{2e}$  = induce emf in rotor coils

$$E_{2e} = 4.44 F_1 T_1 K \omega_1 \Phi \dots \dots \dots (2)$$

$\cos \varphi$  = Power Factor

$$\cos \varphi = \frac{R_{2e}}{Z_{2e}} = \frac{R_{2e}}{\sqrt{R_{2e}^2 + (S X_{2e})^2}} \dots \dots \dots (3)$$

We will have torque equ.

$$T_{em} = \frac{P_{em}}{\omega_1} = \frac{p}{2\pi f_1} \cdot P_{em} \dots \dots \dots (4)$$

$$= \frac{m_1 p}{2\pi f_1} (4.44 f_1 T_1 K \omega_1) I_{2e} \Phi \cos \varphi_2$$

$$= \frac{1}{\sqrt{2}} (m_1 K \omega_1 T_1 I_{2e} \cos \varphi_2) P \Phi$$

$$T_{em} = K_T \Phi m I_{2e} \cos \varphi_2 \dots \dots \dots (*)$$

$$* kT = \frac{1}{\sqrt{2}} p m_1 k \omega_1 T_1$$

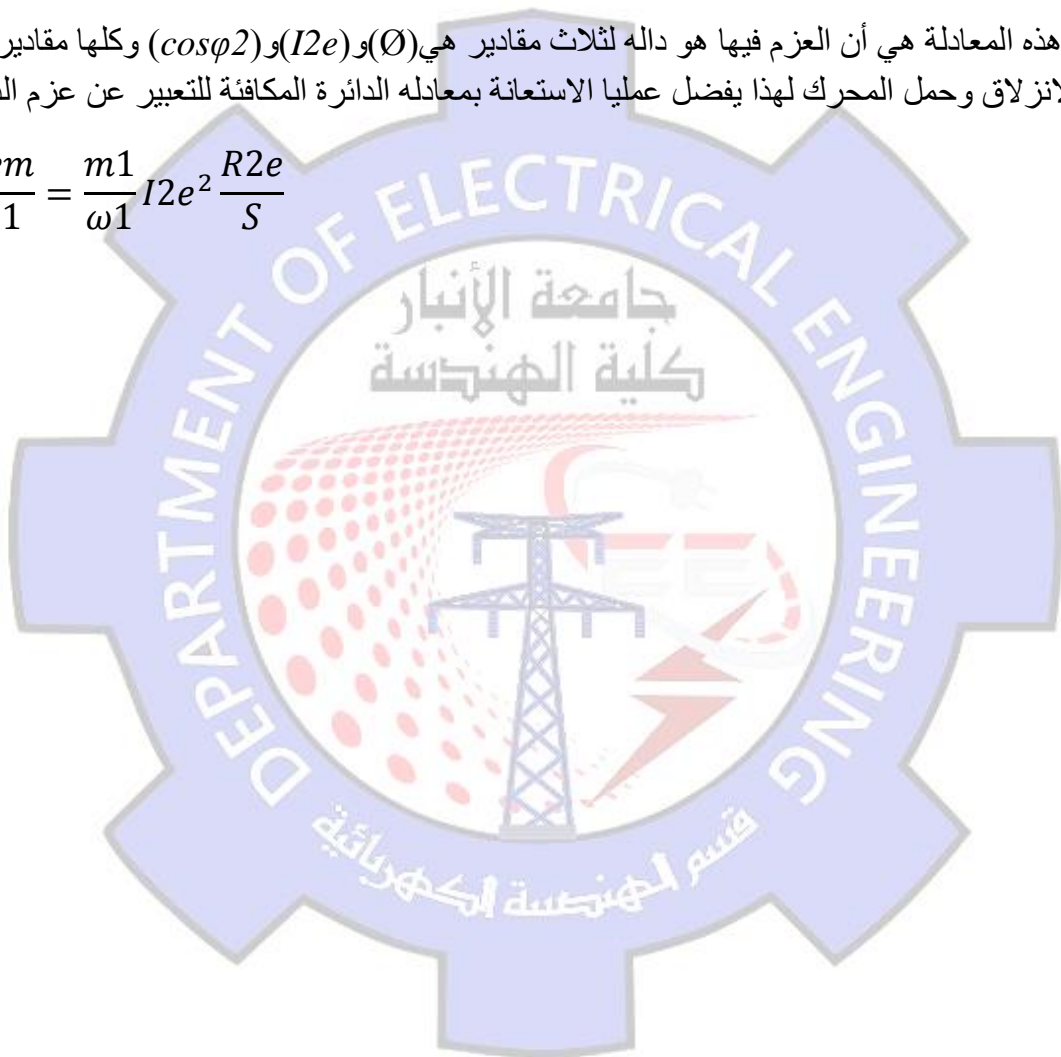
المعادلة(\*) تعبر عن عزم الدوران في جميع المكائن الكهربائية وهي تعني بأن مقدار العزم يتناسب طردياً مع الفيض المغناطيسي ( $\Phi$ ) ومع المكونة الفعالة لتيار الدوار.

وان مكونه التيار تعتمد على مقدار وزاوية الطور لهذا التيار، وتعتمد زاوية الطور على طبيعة ممانعة لفيده الدوار

$$T_{em} \propto \Phi, I_2 e, \cos \varphi$$

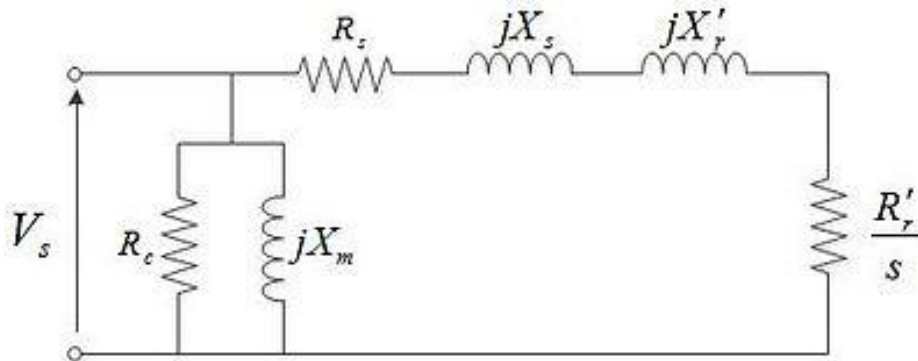
إن المشكلة في هذه المعادلة هي أن العزم فيها هو داله لثلاث مقادير هي ( $\Phi$ ) و ( $I_2 e$ ) و ( $\cos \varphi$ ) وكلها مقادير متغيره لها علاقة بمقدار الانزلاق وحمل المحرك لهذا يفضل عملياً الاستعانة بمعادله الدائرة المكافئة للتعبير عن عزم الدوران.

$$T_{em} = \frac{P_{em}}{\omega_1} = \frac{m_1}{\omega_1} I_2 e^2 \frac{R_2}{s}$$



### 3-3 Torque From Electromagnetic Power

-For Approximate Equivalent cct.



$$Z_{2t} = \left( R_1 + \frac{R_2 e}{s} \right) + j(X_1 + X_2 e), \quad V_1 = E_1$$

$$I_2 e = \frac{V_1}{Z_{2t}} = \frac{V_1}{\sqrt{\left( R_1 + \frac{R_2 e}{s} \right)^2 + (X_1 + X_2 e)^2}}$$

$$P_{em} = \frac{m I_2 e^2 R_2 e}{s}$$

$$T_{em} = \frac{m_1 P_{em}}{\omega_1} = \frac{m_1}{\omega_1} \left[ \frac{v_1^2 \cdot \frac{R_2 e}{s}}{\left( R_1 + \frac{R_2 e}{s} \right)^2 + (X_1 + X_2 e)^2} \right]$$

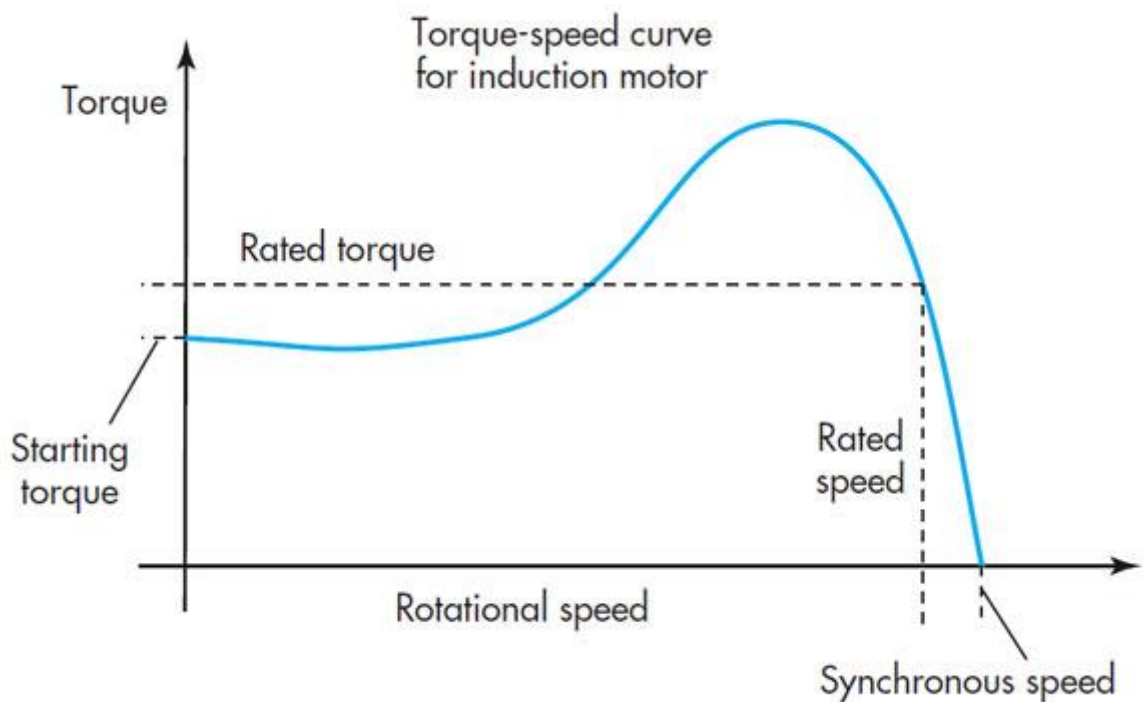
$T_{em}$  will variable with  $(V_1, R_2 e, S)$

At  $S=1$  and  $V_1 = \text{const.} \rightarrow T \propto R_2 e$

عند ثبوت مقدار الجهد المسلط فإن العزم يتناسب طردياً مع مقدار مقاومه لفيض الدوار عند بدء الحركة ( $s=1$ ). ولزيادة العزم ينبغي زيادة مقدار المقاومة.

في الحالات الاعتيادية يكون مقدار الجهد المسلط ومقدار المقاومة في الدوار ثابتين فإن العزم يتغير بتغير الانزلاق ( $s$ ) ويعطي منحنى (Torque slip curve)





$$T_{em} = \frac{P_{em}}{\omega_1} = \frac{m_1}{\omega_1} \left( \frac{V_1^2 \frac{R_{2e}}{s}}{\left( R_1 + \frac{R_{2e}}{s} \right)^2 + \dots} \right)$$

Multiply by  $s^2/s^2$  for 3phase motor

$$T_{em} = \frac{3}{\omega_1} \left( \frac{V_1^2 R_{2e} S}{(SR_1 + R_{2e})^2 + S^2(X_1 + X_{2e})^2} \right)$$

If  $s=0$  then  $T_{em}=0$

For  $s \downarrow \downarrow (s < s_m)$  then  $SR_1$  and  $S^2(X_1 + X_{2e})^2$  are neglected then we will have

$$T_{em} = \left( \frac{3 V_1^2}{\omega_1 R_{2e}} \right) s$$

The torque is directly proportional to the slip and the torque/slip curve is a straight line.

For  $S \uparrow \uparrow (S > S_m)$  neglect  $SR_1=0$  then

$$T_{em} = \left( \frac{3 V_1^2 R_{2e}}{\omega_1 (X_1 + X_{2e})^2} \right) \cdot \frac{1}{s}$$



Then the torque is inversely proportional to the slip and the torque is continuously dropping starting from critical slip at  $S=\infty$  then the  $T_{em} = 0$

