5. TEMPERATURE DISTRIBUTION IN MASS CONCRETE

One major difficulty is how to compute the temperature distribution in complex geometries, and how to incorporate the incremental construction into the analysis.

One of the challenges in mass concrete design is to maximize the thickness of the concrete layers without causing thermally induced cracks and to minimize the time between the placements of successive layers. The designer is under pressure from the contractor, who wants large layers to be placed in rapid succession in order to speed up construction. For large projects, major economic savings can be achieved when the size and placement of the lifts are perfectly orchestrated; the penalty for not coordinating construction of the layers is to incur large labour costs while the construction crew waits for the next placement or repairing, or even demolishing an overly thick layer that cracked as a result of thermal stresses.

This section introduces the finite element method, the most powerful tool available to compute temperature distributions in solid materials.

5.1 Thermal Properties of Concrete

***Coefficient of thermal expansion* (α)** is defined as the change in unit length per degree of temperature change. Selecting an *aggregate with a low coefficient of thermal expansion* when it is economically feasible and technologically acceptable, may, under certain conditions, become a critical factor for crack prevention in mass concrete. This is because the thermal shrinkage strain is determined both by the magnitude of temperature drop and the coefficient of linear thermal expansion of concrete; the latter, in turn, is controlled primarily by the coefficient of linear thermal expansion of the aggregate which is the primary constituent of concrete.

The reported values of the coefficient of linear thermal expansion for saturated portland cement pastes of varying water-cement ratios, for mortars containing 1:6 cement/natural silica sand, and for concrete mixtures with different aggregate types are approximately 18, 12, and 6 to 12 × 10^{-6} per °C, respectively. The coefficient of thermal expansion of commonly used rocks and minerals varies from about 5 × 10^{-6} per °C for limestones and gabbros to 11 to 12 × 10^{-6} per °C for sandstones, natural gravels, and quartzite. The coefficient of thermal expansion can be estimated from the weighted average of the components, assuming 70 to 80 percent aggregate in the concrete mixture.

Specific heat is defined as the quantity of heat needed to raise the temperature of a unit mass of a material by one degree. The specific heat of normal weight concrete is not very much affected by the type of aggregate, temperature, and other parameters. Typically the values of specific heat are in the range of 0.9 to 1.0 kJ/kg.°C.

Thermal conductivity gives the heat flux transmitted through a unit area of a material under a unit temperature gradient. The thermal conductivity of concrete is influenced by the mineralogical characteristics of aggregate, and by the moisture content, density, and temperature of concrete.

Thermal diffusivity is defined as:

$$\kappa = \frac{k}{c\rho}$$

where κ = diffusivity
k = conductivity
c = specific heat
ρ = density of concrete.

Heat will move more readily through a concrete with higher thermal diffusivity. For normal-weight concrete, the conductivity usually controls the thermal diffusivity because the density and specific heat do not vary much. Table 5.1 shows typical values of thermal conductivity and diffusivity for concretes made with different types of coarse aggregate.

Table 5.1a: Thermal conductivity

	Thermal conductivity	
Aggregate type	Btu in./h·ft^2·F	W/m·K
Quartzite	24	3.5
Dolomite	22	3.2
Limestone	18–23	2.6–3.3
Granite	18–19	2.6–2.7
Rhyolite	15	2.2
Basalt	13–15	1.9–2.2

Table 5.1b: Thermal diffusivity

Coarse aggregate	ft^2/h	m^2/h
Quartzite	0.058	0.0054
Dolomite	0.051	0.0047
Limestone	0.050	0.0046
Granite	0.043	0.0040
Rhyolite	0.035	0.0033
Basalt	0.032	0.0030

SOURCE: ACI Committee 207–Cooling and Insulating Systems for mass concrete, 1998.

5.2 Heat Transfer Analysis

Heat transfer is the exchange of thermal energy between physical systems. The rate of *heat transfer* is dependent on the temperatures of the systems and the properties of the intervening medium through which the *heat* is *transferred*. The three fundamental modes of *heat transfer* are **conduction**, **convection** and **radiation**.

The fundamental equation governing the distribution of temperature in a solid subjected to internal heat generation was developed by Fourier. Consider a parallelepiped representing a volumetric element of a material, with conductivity coefficient *k* (Fig. 5.1). The change in heat flux in the *x*-direction is given by the equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) dx\, dy\, dz \tag{5.1}$$

where *T* is the temperature.

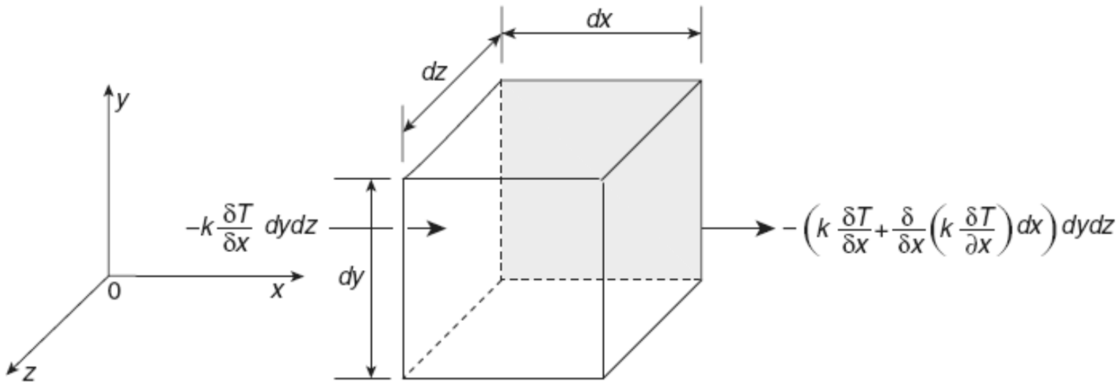

Figure (5.1): Heat flux in the x-direction

Similarly the *y* and *z* directions:

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) dy\, dx\, dz \tag{5.2}$$

and

$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) dz\, dy\, dx \tag{5.3}$$

Addition of the flux variation in the three directions, Eqs. (5.1) to (5.3) determines the amount of heat introduced in the interior of the element per unit time:

$$\left\{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right\} dx\, dy\, dz \tag{5.4}$$

In the above derivation the material was considered isotropic. Considering it also homogeneous, Eq. (5.4) becomes

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)dx\,dy\,dz \tag{5.5}$$

For a material with mass density ρ and specific heat c, the increase of internal energy in the element is given by

$$\rho c\,dx\,dy\,dz\,\frac{\partial T}{\partial t} \tag{5.6}$$

where t is the time.

When the material does not generate any heat, we equate Eqs. (5.5) and (5.6), obtaining

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c\frac{\partial T}{\partial t} \tag{5.7}$$

And then Eq. (5.7) is rewritten as

$$\kappa\nabla^2 T=\dot{T} \tag{5.8}$$

where

$$\dot{T}=\frac{\partial T}{\partial t}$$

$$\nabla^2 T=\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

$\kappa=\frac{k}{c\rho}$ = thermal diffusivity

Consider the case when heat generation occurs inside the material. Equation (5.5) when added to the quantity of heat generated in the interior of the element per unit of time, $wd_xd_yd_z$, can be equated with the increase of internal energy in the element. Therefore, the Fourier equation is obtained

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+w=\rho c\frac{\partial T}{\partial t} \tag{5.9}$$

or

$$k\nabla^2 T+w=\rho c\dot{T} \tag{5.10}$$

In the steady-state, T and w are not function of time, therefore Eq. (5.10) becomes:

$$k\nabla^2 T+w=0 \tag{5.11}$$

Note, Eq. (5.8) is applicable to any isotropic homogeneous material. We will concentrate on the problem of determining temperature distribution in mass concrete. In this case, the heat generation rate w is associated with the adiabatic temperature rise. For a concrete with a density ρ and a cement content β (kg/m^3), the relationship between the adiabatic temperature rise T_a and the heat of hydration Q_h is given by

$$T_a = \frac{\beta}{c\rho} Q_h \quad (5.12)$$

The heat of hydration Q_h is obtained per unit mass of cement, therefore the factor β/ρ must be used to calculate the heat of hydration per unit mass of concrete. The heat generation rate w is related to the heat of hydration by the following equation:

$$w = \beta \frac{dQ_h}{dt} \quad (5.13)$$

Using Eq. (5.12) we obtain

$$w = \rho c \frac{dT_a}{dt} \quad (5.14)$$

In order to determine a unique solution to the Fourier Eq. (5.10), adequate initial and boundary conditions must be given. They should be compatible with the physical conditions of the particular problem.

5.2.1 Initial condition

The initial condition must be defined by prescribing the temperature distribution throughout the body at time zero as a known function of x, y, and z.

$$T(x, y, z, t = 0) = f(x, y, z) \quad (5.15)$$

5.2.2 Boundary conditions

I. Prescribed temperature boundary. The temperature existing on a portion of the boundary of the body Γt is given as

$$T(x, y, z, t) = f(x, y, z, t) \qquad x, y, z \text{ on } \Gamma_t \quad (5.16)$$

This condition is also known as Dirichlet or essential boundary condition. In mass concrete, this condition may exist in the concrete-water contact, where the convection is small, making the temperature of the concrete that is in contact with water the same as that of the water.

II. Prescribed heat flow boundary. A prescribed heat flow boundary condition can be expressed as

$$k \frac{\partial T}{\partial n}(x, y, z, t) = q_n(x, y, z, t); \qquad x, y, z \text{ on } \Gamma_q \quad (5.17)$$

where q_n is the given amount of heat flow at point (x, y, z), and n is the outward normal to the surface.

III. Convection boundary condition. The rate of heat transfer across a boundary layer is given by

$$k\frac{\partial T}{\partial n}(x,y,z,t) = h(T_e - T_s)^N \qquad x,y,z \; on \; \Gamma_h \tag{5.18}$$

where h = heat transfer coefficient
T_e = known temperature of the external environment
T_s = surface temperature of the solid
Γ_h = portion of the boundary surface undergoing convective heat transfer

For a linear convection boundary condition, N = 1, and Eq. (5.18) becomes

$$k\frac{\partial T}{\partial n}(x,y,z,t) = h(T_e - T_s) = g(x,y,z,t) - hT_s \tag{5.19}$$

where $g\,(x,y,z,t) = hT_e$

IV. Radiation boundary condition. Heat transfer by radiation between boundary condition surface Γ, and its surroundings can be expressed by

$$q_r(x,y,z,t) = V\sigma\left(\frac{1}{\frac{1}{\varepsilon_r}+\frac{1}{\varepsilon_s}-1}\right)\left[T_r^4 - T_s^4\right]; \qquad x,y,z \; on \; \Gamma_r \tag{5.20}$$

where V = radiation view factor
σ = Stefan-Boltzmann constant
ε_r = emissivity of the external radiation source
ε_s = emissivity of the surface
T_r and T_s = absolute temperature of the radiation source and the surface, respectively.

5.3 Finite Element Formulation

The finite element method is a powerful tool to solve thermal problems. The method is completely general with respect to geometry, material properties, and arbitrary boundary conditions. Complex bodies of arbitrary shape, including several different anisotropic materials, can be easily represented. For mass concrete structures, the significant boundary conditions that apply are Cases I and III, that is, the prescribed temperature and the convection boundary conditions. Many approaches exist that present a finite element formulation for temperature distribution in mass concrete. Below, we will follow the approach suggested by Souza Lima et al. (1976). First, we will start with the *steady-state* case, and then move to the *transient-state* case. The objective is to solve the Fourier equation given the necessary initial and boundary conditions.

Consider a body with the two different boundary conditions: Γ_t where the temperature is prescribed and Γ_h where there is a convection boundary condition (see Fig. 5.2).

For a point P in Γ_t (steady-state case)

$$T = f(P) \tag{5.21}$$

and for a point P in Γ_h (steady-state case)

$$k\frac{\partial T}{\partial n} = g(P) - hT \tag{5.22}$$

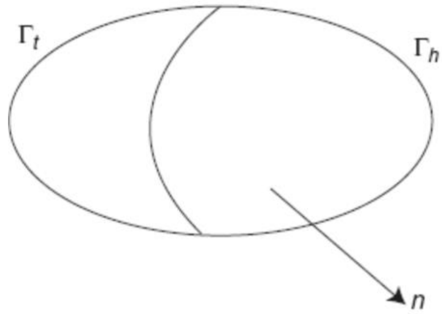

Figure (5.2): BC's in which the temperature is prescribed at Γ_t and convection is prescribed at Γ_h.

Consider a continuous and differentiable function Φ in the domain shown in Figure (5.2) with the condition $\Phi = 0$ along Γ_t. No condition on Φ is imposed along Γ_h. Φ is often referred to as "weighting function" and is relevant to note that it is, and will remain, arbitrary.

Multiplying both sides of Eq. (5.11) by Φ, we obtain

$$k\Phi\nabla^2 T = -\Phi w \tag{5.23}$$

Integrating the above equation in domain V,

$$k\int_V \Phi\nabla^2 T\, dV = \int_V -\Phi w\, dV \tag{5.24}$$

Using the divergence theorem in the left side of Eq. (5.24)

$$k\int_V \Phi\nabla^2 T\, dV = -k\int_V \left(\frac{\partial\Phi}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial\Phi}{\partial z}\frac{\partial T}{\partial z}\right) dV + k\int_V \Phi\frac{\partial T}{\partial n}\, dS \tag{5.25}$$

Since $\Phi = 0$ along Γ_t and using the boundary conditions defined above, Eq. (5.19) becomes

$$k\int_\Gamma \Phi\frac{\partial T}{\partial n}\, dS = k\int_{\Gamma_h} \Phi\frac{\partial T}{\partial n}\, dS = k\int_{\Gamma_h} \Phi g\, dS - h\int_{\Gamma_h} \Phi T\, dS \tag{5.26}$$

Introducing Eqs. (5.25) and (5.26) into Eq. (5.23)

$$k \int_V \left(\frac{\partial\Phi}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial\Phi}{\partial z}\frac{\partial T}{\partial z}\right) dV + h \int_{\Gamma_h} \Phi T \, dS = \int_V \Phi\omega \, dV + \int_{\Gamma_h} \Phi g dS \quad (5.27)$$

Equation (5.27) can be used to solve the steady-state Fourier heat equation. Consider a set of n functions Φ_i with $\Phi_i = 0$ on Γ_t. Thus any temperature field satisfying the boundary condition on Γ_h also satisfies Eq. (5.27) for each of the functions Φ_i.

Equation (5.27) may be used instead of Eqs. (5.11), (5.21), and (5.22) to solve approximately for T in the following manner:

$$T = \Phi_0 + \sum c_i \Phi_i \quad (5.28)$$

where c_i are unknown constants and Φ_0 is any smooth function satisfying the boundary condition on Φ_t. Of course the above may not satisfy Eq. (5.11) at every point in the body. However, substituting T given by Eq. (5.28) into Eq. (5.27) a system of linear equations is obtained that allows the determination of the coefficients c_i. By increasing the number of coefficients in Eq. (5.28) a better approximation to the solution is obtained. Φ_i are referred to as interpolation functions and they are almost invariably polynomials.

Finite element analysis idealizes the continuum by an assemblage of discrete elements or sub-regions. These elements may be of variable size and shape and are interconnected by a finite number of nodal points P_i. The interpolation functions Φ_i, should be chosen so that coefficients c_i are numerically equal to temperature T, at n nodal points P_i previously chosen in the domain. In order for the equality $c_i = T(P_i)$ to be true at the nodal points P_i the following conditions should be obeyed: $\Phi_i(P_i) = 1$, $\Phi_j(P_i) = 0 (j \neq i)$, and $\Phi_0(P_i) = 0$.

It is convenient to introduce a matrix formulation: $\{T\}$ is a vector of n elements with values $T(P_i)$ and $\{w\}$ is a vector of n elements with the values

$$\int_V \Phi_i w \, dV = \int_{\Gamma_h} \Phi_i g \, dS - w_{oi} \quad (5.29)$$

where w_{oi} is the value of the first term of Eq. (5.24), for $\Phi = \Phi_i$ and T is replaced by the function Φ_0. For the steady-state case, this notation leads to

$$[K]\{T\} = \{w\} \quad (5.30)$$

where $[K]$ is the conductivity matrix ($n \times n$) with values

$$[K] = K_{ij} = \int_V \nabla^T \Phi_i k \nabla^T \Phi_j \, dV \quad (5.31)$$

Determining the heat transfer in mass concrete is additionally complicated, because it involves the solution of the transient case and the continuous change of boundaries as construction progresses. To solve this problem, an incremental calculation of the linear transient problem is introduced. In the transient case, the Fourier equation is given by Eq. (5.10), which differs from the steady-state case by the term $\rho c\dot{T}$ and because w is a function of time. Using the divergence theorem:

$$k \int_V \left(\frac{\partial\Phi}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial\Phi}{\partial z}\frac{\partial T}{\partial z}\right) dV + h \int_{\Gamma_h} \Phi T \, dS$$

$$= \int_V \Phi\omega \, dV + \int_{\Gamma_h} \Phi g \, dS - \rho c \int_V \Phi \dot{T} \, dv \qquad (5.32)$$

Or, if the matrix notation is used:

$$[k]\{T\} = \{w\} - [c]\{\dot{T}\} \qquad (5.33)$$

where [c] is the capacity matrix ($n \times n$) with values

$$c_{ij} = \rho c \int_V \Phi_i \Phi_j \, dV \qquad (5.34)$$

To integrate Eq. (5.33), an incremental method is usually employed. Taking small interval Δt

$$\{\dot{T}\} = \frac{1}{\Delta T}[\{T(t)\} - \{T(t - \Delta T)\}] \qquad (5.35)$$

and incorporating Eq. (5.35) into Eq. (5.33)

$$\left([k] + \frac{1}{\Delta T}[c]\right)\{T(t)\} = \{w\} + \frac{1}{\Delta T}[c]\{T(t - \Delta T)\} \qquad (5.36)$$

Starting from a known initial temperature distribution, we proceed stepwise. Equation (5.36) allows the determination of $\{T(\Delta t)\}$ for the first step. Once the new temperature is known, we proceed to the next step, giving a new increment Δt and continuing the process until the distribution of temperatures over the period of time of interest is known.

5.4 Examples of Application

Typical problems that a concrete technologist faces when studying thermal stresses in mass concrete include the type of aggregate, amount of pozzolan, size of the concrete lift, and temperature of fresh concrete that might affect the maximum temperature rise in concrete. To study these parameters a finite element model of a concrete block placed on a foundation rock can be developed, as shown in Figure (5.3). The finite element mesh is made of 385 nodal points and 344 elements. Note that the size of elements in the concrete block is much smaller than in the foundation; because we are mainly interested in temperature distribution inside the concrete block. The material properties for different types of concrete and for the foundation rock are shown in Table 5.2. An important parameter in thermal analysis is the adiabatic temperature rise. Figure (5.4) shows the assumed values for different levels of pozzolan replacements.

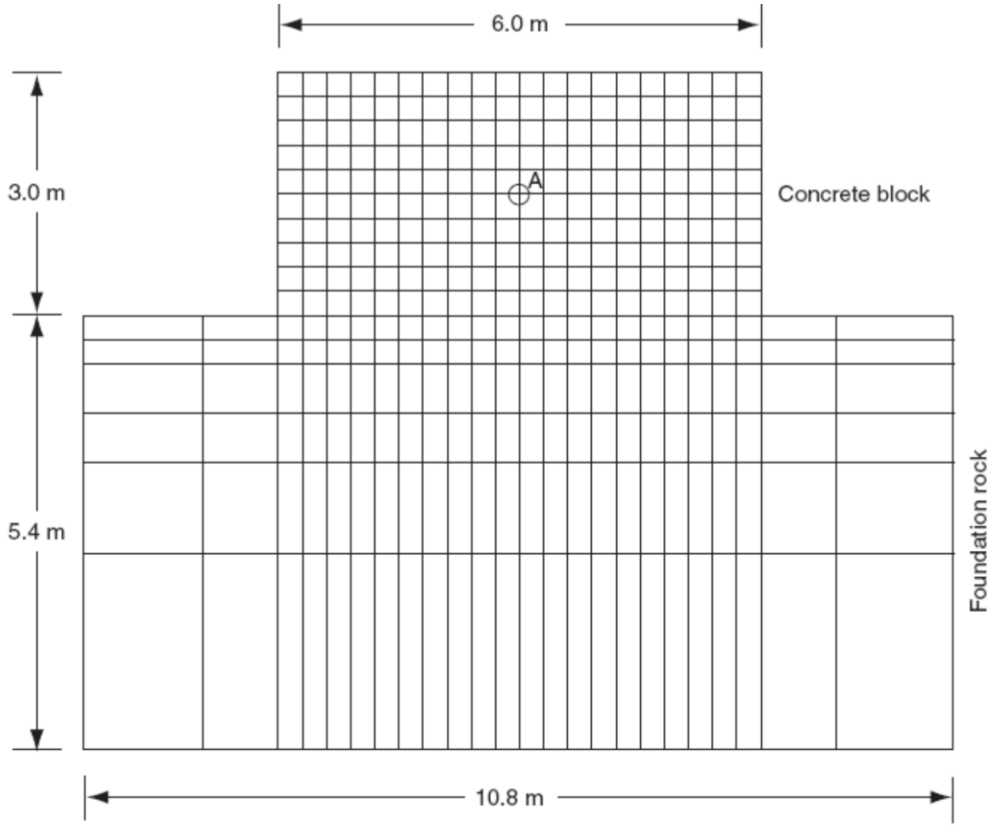

Figure (5.3): Finite element mesh of a concrete block and foundation.

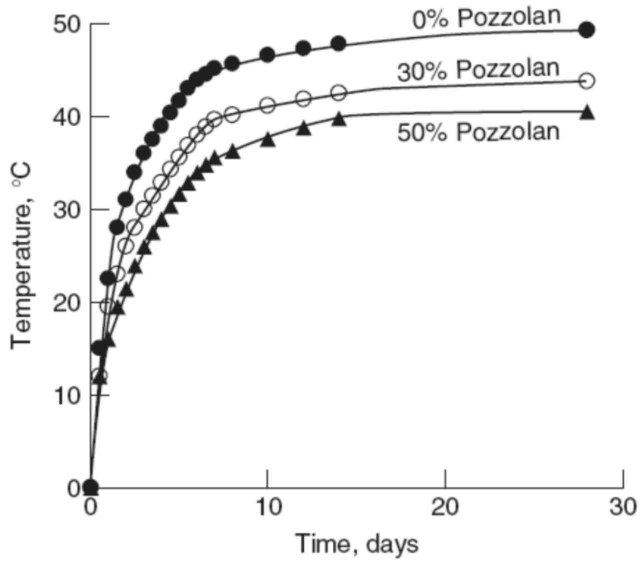

Figure (5.4): Effect of percentage of pozzolan on the adiabatic temperature rise.

Table 5.2: Properties of concrete and foundation rock

	Properties of concrete made with different aggregates			
	Basalt	Granite	Gravel	Foundation rock
Thermal conductivity (kcal/m.h.°C)	1.740	2.367	3.690	2.800
Specific heat (kcal.kg. °C)	0.24	0.23	0.22	0.20
Thermal diffusivity (m^2/h)	0.0029	0.0042	0.0075	0.0050
Density (kg/m^3)	2500	2450	2400	2800
Cement consumption (kg/m^3)	315	315	315	—

To illustrate the importance of lift thickness, consider the mesh shown in Figure (5.3). Assume that the concrete was placed either: (**a**) in two lifts of 1.50 m placed 3 days apart, or (**b**) in one lift of 3.00 m. Given that the temperature distribution changes with time, however, the designer is usually concerned with the maximum temperature distribution that occurs within the concrete block. Figure (5.5) shows the maximum temperature distribution in the concrete block for both cases. The maximum temperature with two 1.50-m lifts was 46°C, which is likely to cause fewer problems than the 56°C reached using the one 3.00-m lift.

The thermal diffusivity of concrete is controlled mainly by the aggregate. To analyze the effect of thermal diffusivity on the temperature distribution, consider three types of aggregates: basalt, gravel, and granite. The temperature evolution for the point A, (indicated in Fig. 5.3) is shown in Figure (5.6). Concrete made with gravel has the highest thermal diffusivity, therefore, it dissipates heat faster and, consequently, shows the smallest temperature rise.

The use of pozzolans is an efficient method of controlling the temperature rise in concrete. The performance of three concrete mixtures is compared, and the adiabatic temperature rise for each type of concrete is shown in Figure (5.4). The advantage of including pozzolans is illustrated in Figure (5.7), where the maximum temperature rise is significantly reduced when such replacements are used.

Refrigeration is another powerful method of controlling temperature rise in mass concrete. Pre-cooling can be achieved by replacing the mixing water by ice or by cooling the coarse aggregate. Post-cooling can be achieved by circulation of cold water through pipes embedded in concrete. Usually pre-cooling is preferred because it is more economical and does not involve extra labour, such as the embedding of pipes, pumping cold water, and eventually re-grouting the pipes. The importance of temperature of fresh concrete is shown in Figure (5.8). When the concrete is placed at 25°C, the maximum temperature is 52°C, compared to 42°C when the concrete is placed at 10°C.

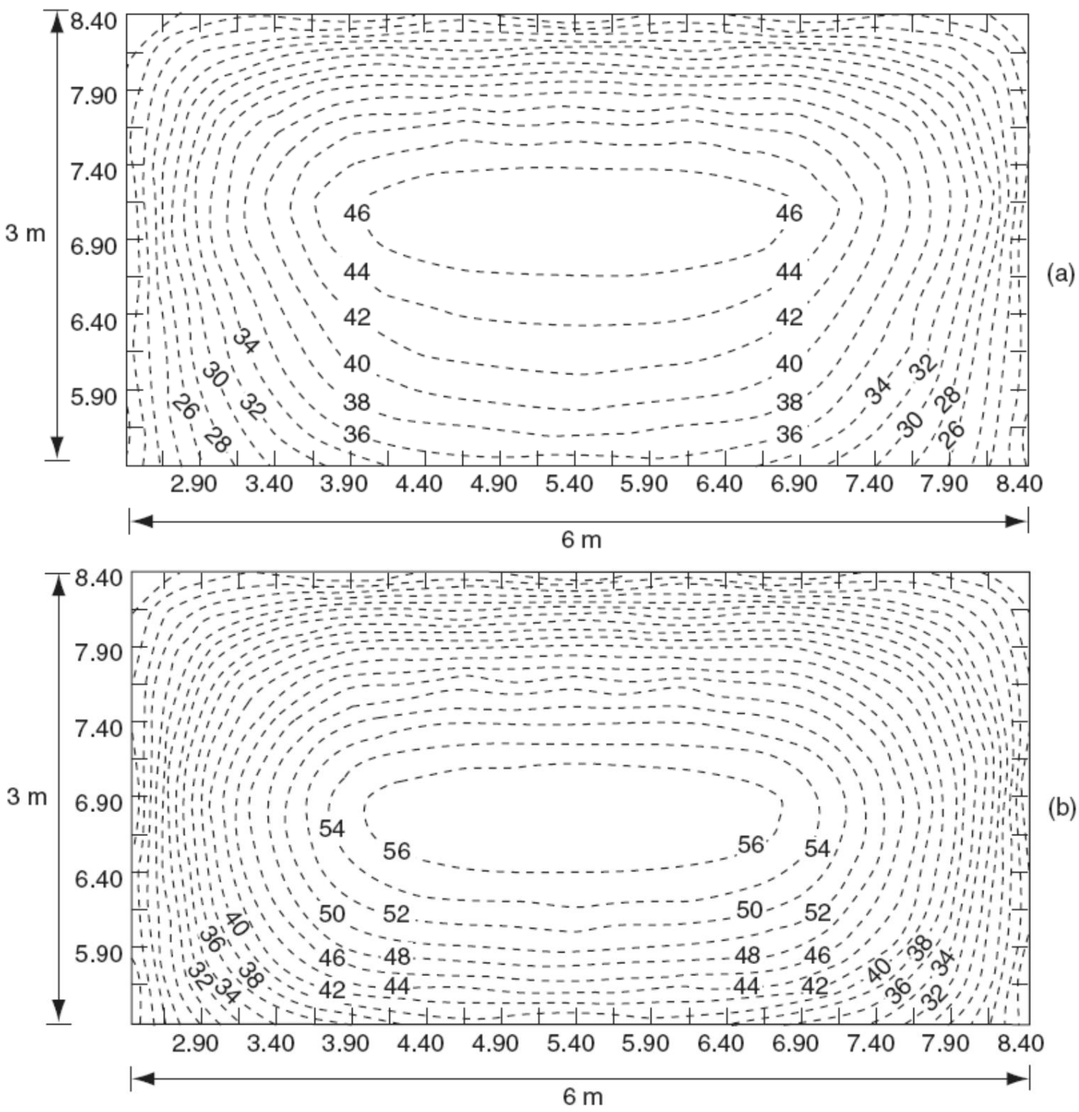

Figure (5.5): Maximum temperature distribution in the concrete block.

*The size of the concrete lift is an important parameter in the temperature distribution in mass concrete. Thick lifts are attractive for fast construction, but high temperatures are usually generated in the concrete. Smaller lifts generate much lower temperatures, however, they may create problems with the construction scheduling. It is the responsibility of the engineer to establish the optimal thickness of the lifts. For this, a thermal analysis is usually performed using the finite element method. As an illustration, a thermal analysis was conducted for the finite element mesh shown in Fig (5.3), with two conditions: (**a**) two concrete lifts of 1.50 m placed 3 days apart, and (b) one lift of 3.00 m. The temperature distribution in the concrete is shown above. For case (a) the maximum temperature in the concrete is 46°C that is much lower than the 56°C for case (**b**). For this analysis, the temperature of the fresh concrete was assumed to be 17°C, the aggregate was assumed to be granite, and no pozzolan was used.*

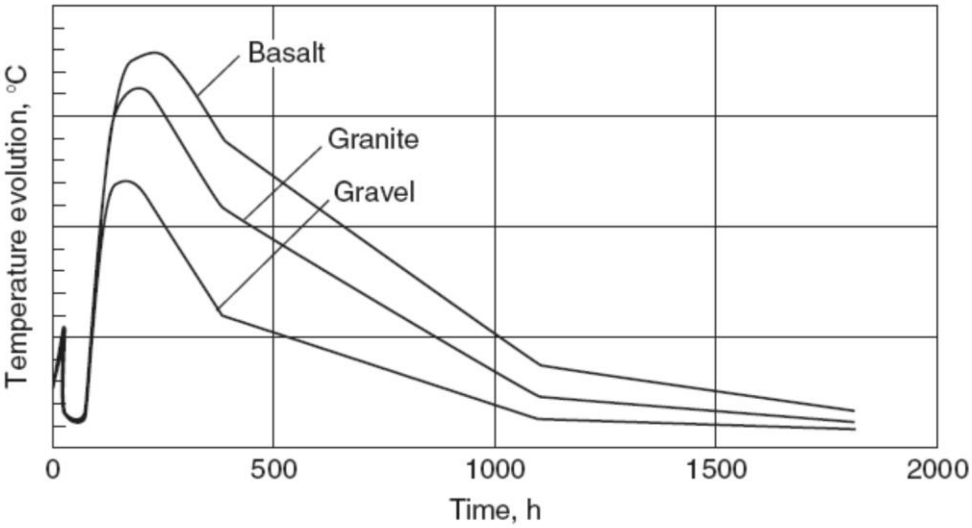

Figure (5.6): Temperature evolution for concrete with different thermal diffusivities.
Thermal diffusivity of concrete is a property which greatly influences the temperature distribution within the mass. Higher thermal diffusivity leads to faster heat loss which results in a lower maximum temperature. It may not always be desirable to have a rapid dissipation of heat, because the concrete may not have enough tensile strength at earlier ages. For this study two lifts of 1.50 m each was considered. The reason for the first temperature drop is given in the caption of Fig. (5.7).

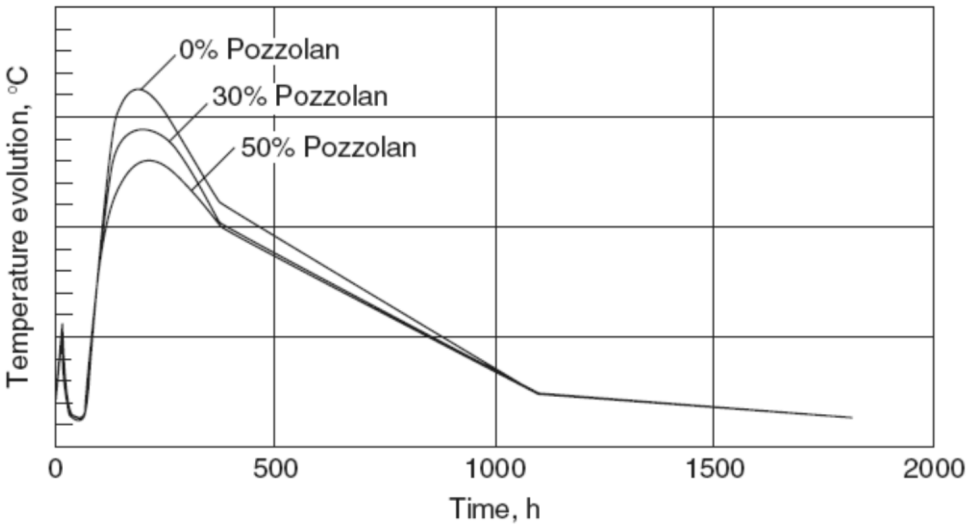

Figure (5.7): Influence of Pozzolan on the temperature of concrete.
Pozzolans can significantly reduce the temperature inside mass concrete. The plot above shows the temperature evolution for point A of Fig.(5.3). Placement consisted of two concrete lifts of 1.50 m 3 days apart. Point A is at the top of the first lift, so there is an initial temperature increase, followed by a quick heat loss to the ambient temperature (17°C) until the next lift is placed. The temperature then increases up to a maximum, after which the block starts to cool.

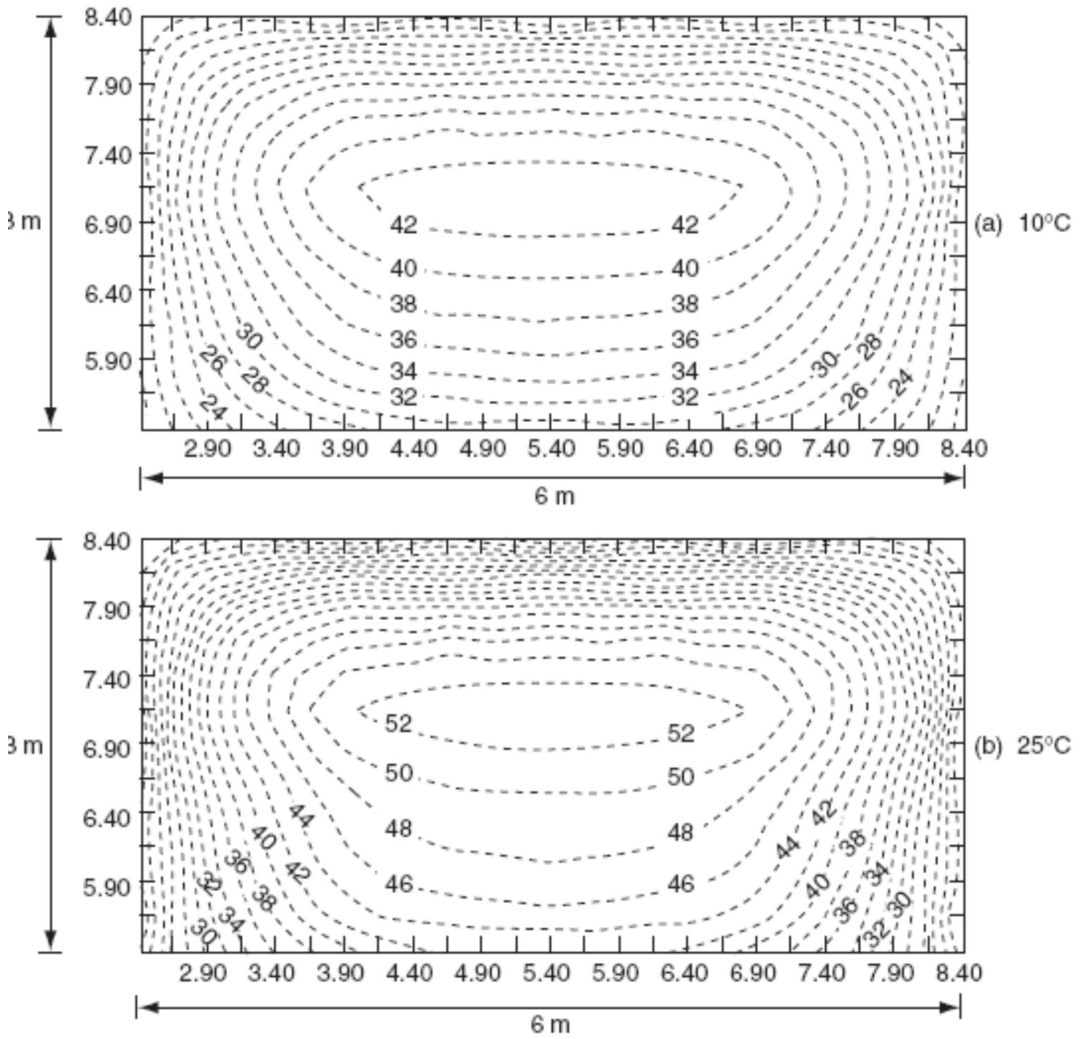

Figure (5.8): Effect of fresh concrete on the maximum temperature distribution.

One of the most effective methods of controlling the temperature rise in mass concrete is by lowering the temperature of fresh concrete. A simple method is to use ice instead of mixing water or pre-cooling the aggregate. Unlike change in the size of lift, modifications in the temperature of fresh concrete do not affect construction scheduling. This finite element analysis assumed three different temperatures of fresh concrete: 10, 17, and 25°C. The temperature distribution for fresh concrete placed at a temperature of 17°C is shown in Fig. (5.5a). The concrete was placed in two 1.50 m lifts each, with granite as the coarse aggregate and an ambient temperature of 17°C.