Simplification of The Approximate Equivalent Circuit Of A Transformer:

We can replace the resistance R_2 of the secondary of fig(11) by additional resistance $\grave{R}_2$ in the primary circuit such the power absorbed in $\grave{R}_2$ when carrying the primary current is equal to that in R_2 due to the secondary current.

$$I_1^2\ \grave{R}_2 = I_2^2\ R_2 \ \rightarrow\ \grave{R}_2 = (\frac{I_2}{I_1})^2 \approx\ R_2 (\frac{V_1}{V_2})^2$$

Hence if R_e be a single resistance in the primary circuit equivalent. The primary and secondary resistances of the actual transformer.

$$R_e = R_1 + \grave{R}_2 = R_1 + R_2\,(\frac{V_1}{V_2})^2$$

Similarly, since the inductance of a coil is proportional to the square of the number of turns , the secondary leakage reactance X_2 can be replace by an equivalent reactance $\grave{X}_2$ in the primary circuit , such that:

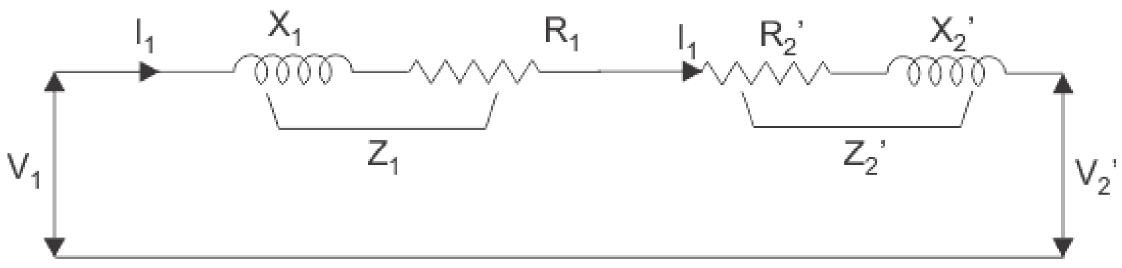
$$\grave{X}_2 = X_1 \left(\frac{N_1}{N_2}\right)^2 \approx X_1 (\frac{V_1}{V_2})^2$$

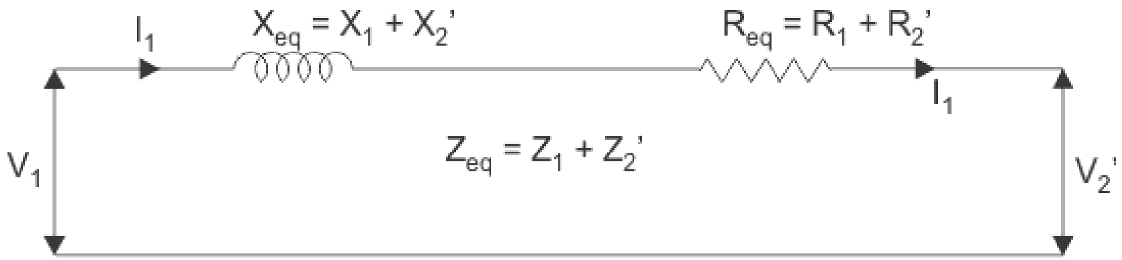
X_e be the single reactance in the primary cct . equivalent X_1 and X_2 of the actual transformer.

$$X_e = X_1 + \grave{X}_2 = X_1 + X_2\ \left(\frac{V_1}{V_2}\right)^2$$

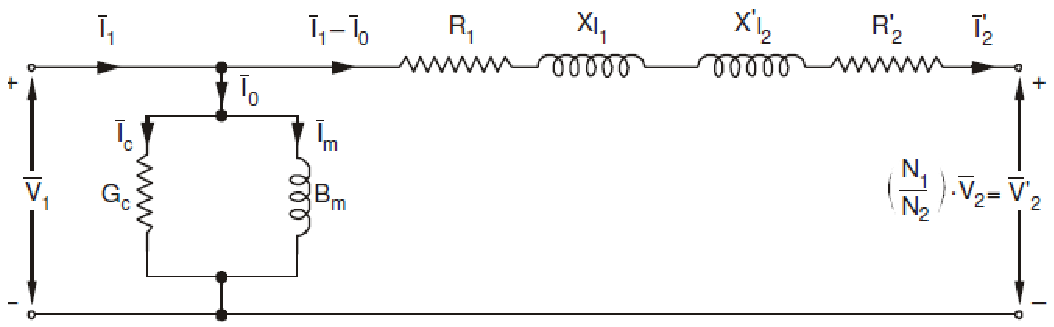
Z_e be the equivalent impedance of the primary and secondary winding referred to primary cct.

$$Z_e = \sqrt{R_e^2 + X_e^2} \quad , R_e = Z_e \cos\varphi\ , \quad X_e = Z_e \sin\varphi$$

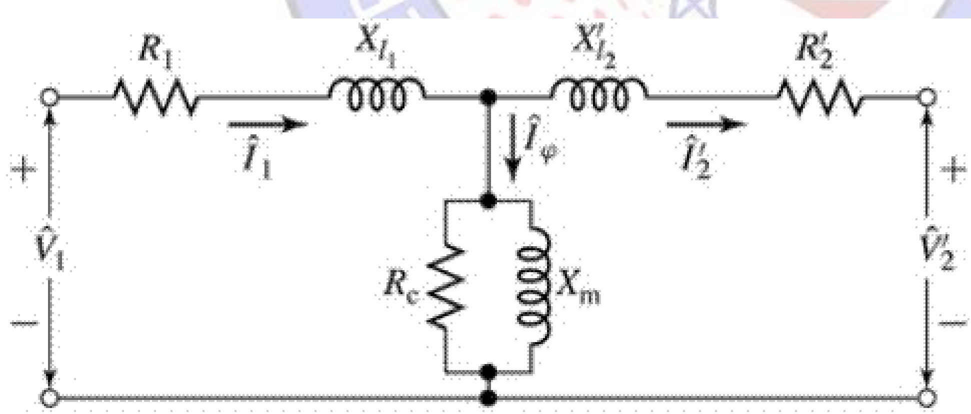




Approximate Equivalent Circuit of Transformer referred to Primary



(a) Equivalent Circuit Referred to Primary Side



<u>4-12 Efficiency of A Transformer:</u>

The losses which occur in a transformer on load can be divided into two groups:

1-Copper losses in primary and secondary windings namely $I_1^2\, R_1 + I_2^2\, R_2$.

2-Iron losses in the core due to hysteresis and eddy currents. The factor determining these losses have already been discussed in A.C machines.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 per cent between no load and full load its usual to assume the iron losses constant at all loads.

$P_c = total\ iron\ loss\ in\ core$

$$\sum_{loss} = P_c + P_{cu1} + P_{cu2} = P_c + I_1^2 R_1 + I_2^2\ R_2$$

ŋ=$\frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out}+losses} = \frac{I_2 V_2 \times Pf}{I_2 V_2 \times Pf + P_c + I_1^2 R_1 + I_2^2 R_2}$

OR $\frac{P_{in}-losses}{P_{in}} = 1 - \frac{losses}{P_{in}}$

$P_c = V_1 I_o \cos\emptyset o$

$P_{cu} = P_{cu1} + P_{cu2} = I_1^2 R_1 + I_2^2 R_2$

4-13 Condition for Maximum Efficiency of Transformer:

R2e the equivalent resistance of the primary and secondary windings referred to the secondary circuit.

$R_{2e} = R_1 (\frac{N_2}{N_1})^2 + R_2$

=a constant for gainer transformer

For any load current I_2,

Total copper loss=$I_2^2 R_{2e}$

And ŋ%=$\frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_c + P_{cu2}} = \frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_C + I_2^2 R_{2e}}$

For a normal transformer, V_2 is approximately constant, hence for a load of given power factor. The efficiency is maximum when the denominator of ŋ is a minimum.

$$\frac{d}{dI_2}(V_2 I_2 \times P.f + P_c + I_2^2 R_{2e}) = 0$$

$$\frac{d}{dI_2}\left(V_2 \times P.f + \frac{P_c}{I_2} + I_2 R_{2e}\right) = 0$$

$$\frac{-P_c}{I_2^2} + R_{2e} = 0 \rightarrow I_2^2 R_{2e} = P_c$$

Example1:

A 200 KVA, 6600V /400V, 50 HZ single phase transformer has 80 turns on the secondary. Calculate:

a- The approximate values of the primary and secondary current.
b- The approximate number of primary turns.

c- The maximum value of the flux.

Solution:

a- full-load primary current $I_1 = \frac{S}{V_1} = \frac{200\times 10^3}{6600} = 30.3\ A$

- full-load secondary current $I_2 = \frac{S}{V_2} = \frac{20\times 10^3}{400} = 500\ A$

b- $N_1 = N_2 \times \frac{V_1}{V_2} = \frac{80\times 6600}{400} = 1320$

c- $E_2 = 4.44 \times N_2 \times f \times \emptyset_m$

$400 = 4.44 \times 80 \times 50 \times \emptyset_m$

$\emptyset_m = 0.0225\ wb$.

Example 2: A 2200/200 V transformer draws a no-load primary current of 0.6 A and absorb 400 watts, find the magnetic and iron loss current.

Solution: iron loss current $=I_c = \frac{P}{V} = \frac{400}{2200} = 0.182\ A$

$I_o = \sqrt{I_o^2 + I_{mag}^2} \rightarrow I_o^2 = I_c^2 + I_{mag}^2$

$0.6^2 = 0.182^2 + \mathrm{I}_{\mathrm{mag}}^2$

$I_{\mathrm{mag}} = 0.572\ \mathrm{A}$

Example 3: A single phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current 3A at a power factor 0.2 lagging. Calculate the primary current and power factor when the secondary current is 280 A, power factor of 0.8 lagging. (voltage drop-in winding be negligible).

Solution:

If $\grave{I}_1$ represents the component of the primary current to neutralize the demagnetizing effect of the secondary current. The power –turns due to $\grave{I}_1$ must be equal and opposite to those due to I_2.

$\frac{I_2}{\grave{I}_1} = \frac{N_1}{N_2} = \frac{280}{\grave{I}_1} = \frac{1000}{200}$

$\grave{I}_1 \times 1000 = 280 \times 200 \rightarrow \grave{I}_1 = 56\ A$

$\cos \emptyset_2 = 0.8 \rightarrow \sin \emptyset_2 = 0.6$

$\cos \emptyset_o = 0.2 \rightarrow \sin \emptyset_o = 0.98$

$I_1 \cos \emptyset_1 = \grave{I}_1 \cos \emptyset_2 + I_o \cos \emptyset_o$

$=56 \times 0.8 + 3 \times 0.2 = 45.4\ A$

$I_1 \sin \emptyset_1 = \grave{I}_1 \sin \emptyset_2 + I_o \sin \emptyset_o$

$=56 \times 0.6 + 3 \times 0.98 = 36.54\ A$

$I_1 = \sqrt{45.4^2 + 36.54^2} = 58.3\ A$

$\tan^{-1}\frac{I_1 \sin \emptyset_2}{I_1 \cos \emptyset_1} = \tan^{-1}\frac{36.54}{45.4} = 38°$

Primary power factor= $\cos \emptyset_1 = \cos 38° = 0.78\ lagging$

Example 4:

A 50 KVA, 4400/220 V transformer has R_1=3.45Ω, R_2=0.009 have values of reactance are X_1=5.2Ω and X_2=0.015Ω. Calculate for the transformer:1-equivalent resistance as referred to primary. 2-equivalent resistance as referred to secondary. 3-equivalent reactance as referred to both primary and secondary. 4-equivalent impedance as referred to both primary and secondary. 5-total P_{cu} loss. first used individual resistance two windings and secondary using equivalent resistance as referred to each side.

Solution: $I_1 = \frac{S}{V_1} = \frac{50\times10^3}{4400} = 11.36\ A$

$I_2 = \frac{S}{V_1} = \frac{50\times10^3}{220} = 227\ A$

$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{220}{4400} = 0.05$

$R_{e1} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{0.05^2} = 7.05\Omega$

$R_{e2} = K^2 R_{e1} = 0.05^2 \times 7.05 = 0.0176\Omega$

Or $R_{e2} = R_2 + \frac{R_1}{K^2} = 0.009 + \frac{3.45}{0.05^2} = 0.0176\ A$

$X_{e1} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{0.05^2} = 11.2\Omega$

$X_{e2} = X_2 + \frac{X_1}{K^2} = 0.015 + \frac{5.2}{0.05^2}$

$X_{e1} = k^2 X_{e1} = 0.05^2 \times 11.2 = 0.028\Omega$

4- $Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{7.05^2 + 11.2^2} = 13.23\Omega$

$Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{0.0176^2 + 0.028^2} = 0.03311\Omega$

5-$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = 910\text{W}$

$\mathrm{P_{cu}} = \mathrm{I_1^2 R_{e1}} = 11.36^2 \times 7.05 = 910\ W$

$= I_2^2 R_{e2} = 227^2 \times 0.017 = 910\ w$

Example 5: The primary and secondary of a 500 KVA transformer have resistance of 0.42Ω and 0.0011Ω respectively. The primary and secondary voltages are 6600 V and 400 V, and the iron losses is 2.9 kW. Calculate the efficiency at:

a-full-load, b-half-load. Assuming the p.f of the load 0.8.

solution: $I_2 = \frac{S}{V_2} = \frac{500\times10^3}{400} = 1250\ A$

$I_1 = \frac{S}{V_1} = \frac{500\times10^3}{6600} = 75.8\ A$

$P_{cu2} = \mathrm{I_2^2 R_2} = 1250^2 \times 0.0011 = 1720\ W$

$P_{cu1} = I_1^2 R_1 = 75.8^2 \times 0.42 = 2415\ W$

$\mathrm{P_{cu} = P_{cu1} + P_{cu2}} = 4135 + 2.9 \times 10^3 = 7035\ \mathrm{W}$

$\mathrm{P_2 = P_{out} = S_x} \cos\varphi = 500 \times 10^3 \times 0.8 = 400\ \mathrm{KW}$

$\mathrm{P_1 = P_{in} = P_2} + \sum P_{loss} = 400 + 7.035 = 407.035\ \mathrm{KW}$

$\frac{400}{407..035} \times 100 = 98.27\%$η=

b-half load $P_{cu} = 4135 \times 0.5^2 = 1034\ W$

$\sum P_{loss} = 1034 + 2.9 = 3.934\ KW$

$\frac{200}{203.9} \times 100 = 98.08\%$η=