THEORY OF PLASTICITY

INROUDUCTION

The theory of plasticity is the branch of mechanics that deals with the calculation of stresses and strains in a body, made of ductile material, permanently deformed by a set of applied forces. The theory is based on certain experimental observations on the macroscopic behavior of metals in uniform states of combined stresses. The observed results are then idealized into a mathematical formulation to describe the behavior of metals under complex stresses. Unlike elastic solids, in which the state of strain depends only on the final state of stress, the deformation that occurs in a plastic solid is determined by the complete history of the loading. The plasticity problem is, therefore, essentially incremental in nature, the final distortion of the solid being obtained as the sum total of the incremental distortions following the strain path.

Up to now we have concentrated on the elastic analysis of structures. In these analyses we used superposition often, knowing that for a linearly elastic structure it was valid. However, an elastic analysis does not give information about the loads that will actually collapse a structure. An indeterminate structure may sustain loads greater than the load that first causes a yield to occur at any point in the structure. In fact, a structure will stand as long as it is able to find redundancies to yield. It is only when a structure has exhausted all of its redundancies will extra load causes it to fail. Plastic analysis is the method through which the actual failure load of a structure is calculated, and as will be seen, this failure load can be significantly greater than the elastic load capacity.

In ductile metals, under favorable conditions, plastic deformation can continue to a very large extent without failure by fracture. Large plastic strains do occur in many metal-working processes, which constitute an important area of application of the theory of plasticity. While elastic strains may be neglected in such problems, the continued change in geometry of the workpiece must be allowed for in the theoretical treatment. Severe plastic strains are produced locally in certain mechanical tests such as the hardness test and the notch tensile test. The significance of these tests cannot be fully appreciated without a knowledge of the extent of the plastic zone and the associated state of stress. Situations in which elastic and plastic strains are comparable in magnitude arise in a number of important structural problems when the loading is continued beyond the elastic limit. Structural designs based on the estimation of collapse loads are more economical than elastic designs, since the plastic method takes full advantage of the available ductility of the material.

THE STRESS–STRAIN BEHAVIOR

A uniaxial tensile stress on a ductile material such as mild steel typically provides the following graph of stress versus strain:

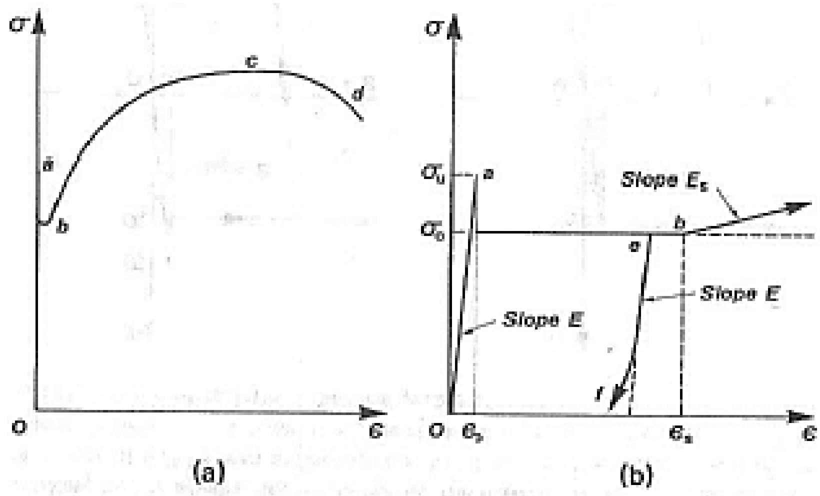
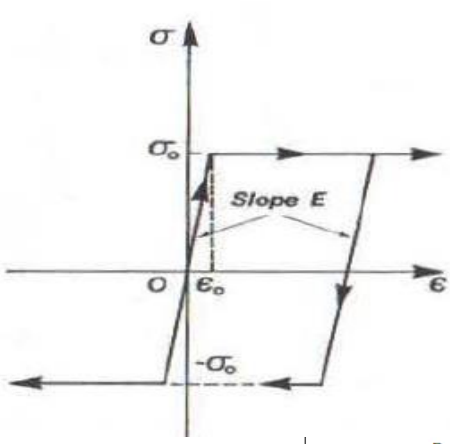


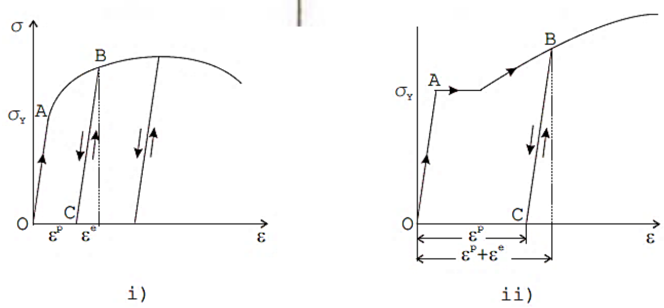
Fig. a represent behavior up to rupture while Fig. b represent yield range. As can be seen, the material can sustain strains far in excess of the strain at which yield occurs before failure. This property of the material is called its ductility. ***Ductility*** *is a measure of a material's ability to undergo significant plastic deformation before rupture*

Though complex models do exist to accurately reflect the above real behavior of the material, the most common, and simplest, model is the idealized stress-strain curve. This is the curve for an ideal elastic-plastic material (which doesn't exist), and the graph is:



As can be seen, once the yield has been reached it is taken that an indefinite amount of strain can occur. It must be sufficiently ductile for the idealized stress-strain curve to be valid.

Let us consider the uniaxial tension test with the subsequent unloading for two materials: i) pure cooper, and ii) soft-annealed carbon steel (Steels with higher carbon content, and most high-alloy steels, which are allowed to air cool after hot working, such as forging or hot rolling, are usually hard to machine. Soft annealing reduces the hardness and makes the material easier to machine) as shown in Figure, where the strain and stress are defined as follows:



$$\varepsilon = \frac{\Delta l}{l_0}, \quad \sigma = \frac{F}{A_0}, \quad \dot{\varepsilon} < 10^{-3}/s.$$

Since the deformed cross-section at tension shrinks, the true stress should actually be defined as F/A, where A is the current cross-section area. However, at small strains of the order $\varepsilon < 1\%$ the error is not so grave.

Looking at the stress-strain curve one can recognize two different types of material response in the elastic and elasto-plastic regions. In the purely elastic region (within the line OA) no residual strain is observed: the specimen assume its original length after the load is removed. For most of metals the stress is proportional to the strain so that the Hooke law is valid. The purely elastic region ends at point A corresponding to the yield stress σ_Y. Beyond this purely elastic region we observe for cooper

i) a "mild" transition to the elasto-plastic region, while for steel

ii) a sharp yield stress marked by a nearly horizontal segment. If the specimen is loaded beyond this yield stress, it begins to deform plastically. The specimen shows a residual strain after unloading. The total strain is additively decomposed into the elastic and plastic parts

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + \varepsilon^p$$

Where τ_f = shear stress on the failure plane
c = apparent cohesion
σ_f =normal stress on the failure plane
f = angle of internal friction

If the stress condition for any other soil sample is represented by a Mohr circle that lies below the failure envelope, every plane within the sample experiences a shear stress which is smaller than the shear strength of the sample. Thus, the point of tangency of the envelope to the Mohr circle at failure gives a clue to the determination of the inclination of the failure plane.