Mechanical Engineering
Design

Lecturer

Mazin Yaseen Abbood (Teacher) – Ph.D. Applied Mechanics

University of Anbar
Mechanical Department

Main reference

Shigley's Mechanical Engineering Design, Eighth Edition

<u>Syllabus</u>

1. Load and Stress Analysis
2. Failures Resulting from Static Loading
3. Fatigue Failure Resulting from Variable Loading
4. Shafts and Shaft Components
5. Screws, Fasteners, and the Design of Nonpermanent Joints
6. Welding, Bonding, and the Design of Permanent Joints
7. Mechanical Springs
8. Rolling−Contact Bearings
9. Lubrication and Journal Bearings
10. Gears

Introduction

Mechanical design is a complex undertaking, requiring many skills. Extensive relationships need to be subdivided into a series of simple tasks. The complexity of the subject requires a sequence in which ideas are introduced and iterated. To design is either to formulate a plan for the satisfaction of a specified need or to solve a problem. If the plan results in the creation of something having a physical reality, then the product must be functional, safe, reliable, competitive, usable, manufacturable, and marketable.

The complete design process, from start to finish, is often outlined as in Fig.(1).The process begins with an identification of a need and a decision to do something about it. After many iterations, the process ends with the presentation of the plans for satisfying the need. Depending on the nature of the design task, several design phases may be repeated throughout the life of the product, from inception to termination.

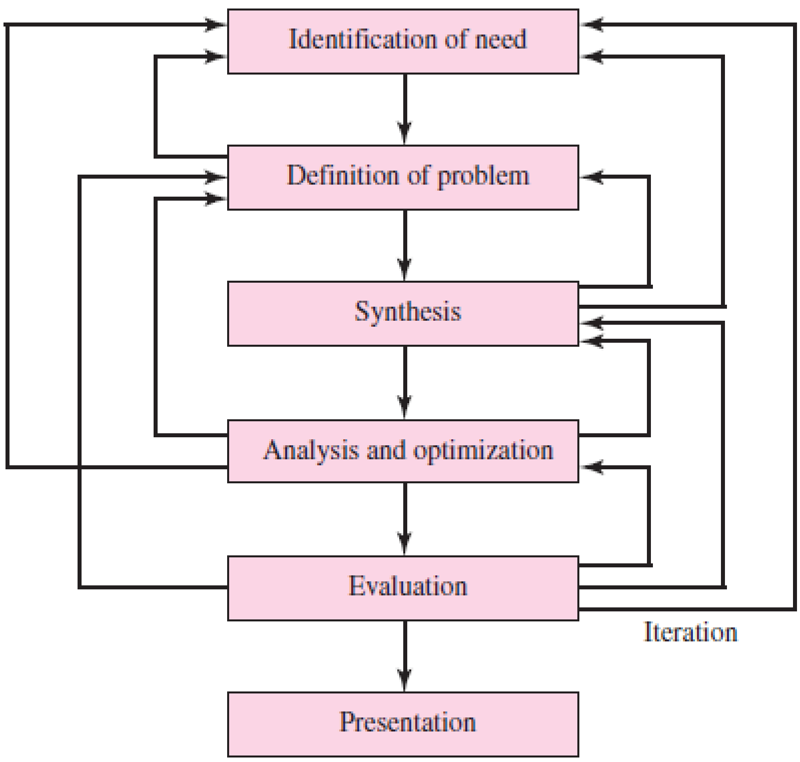


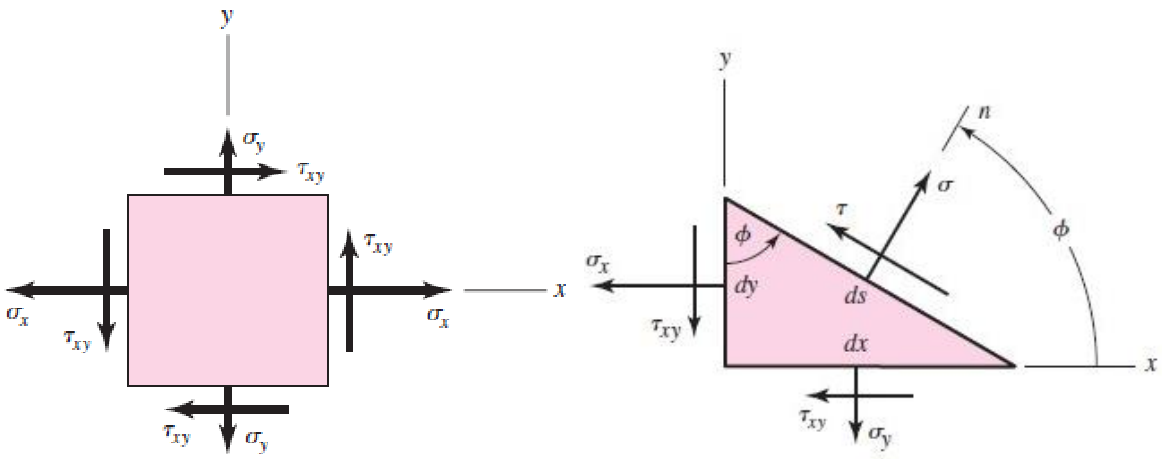
Figure (1)

The phases in design, acknowledging the many feedbacks and iterations

1. Load and Stress Analysis

The ability to quantify the stress condition at a critical location in a machine element is an important skill of the engineer. Why? Whether the member fails or not is assessed by comparing the (damaging) stress at a critical location with the corresponding material strength at this location.

1-1. Mohr's Circle for Plane Stress



(a) (*b*)

Figure (1-1)

Suppose the *dx dy dz* element of Fig. (1–1*a*) is cut by an oblique plane with a normal *n* at an arbitrary angle ϕ counterclockwise from the *x* axis as shown in Fig. (1–1*b*). This section is concerned with the stresses σ and τ that act upon this oblique plane. By summing the forces caused by all the stress components to zero, the stresses σ and τ are found to be

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi \qquad \text{1-1}$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \qquad \text{1-2}$$

Equations (1–1) and (1–2) are called the *plane-stress transformation equations*. Differentiating Eq. (1–1) with respect to ϕ and setting the result equal to zero gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \text{1-3}$$

Equation (1–3) defines two particular values for the angle $2\phi_p$, one of which defines the maximum normal stress σ_1 and the other, the minimum normal stress σ_2. These two stresses are called the *principal stresses,* and their corresponding directions, the *principal directions.* The angle between the principal directions is 90°. It is important to note that Eq. (1–3) can be written in the form

$$\frac{\sigma_x - \sigma_y}{2}\sin 2\phi_p - \tau_{xy}\cos 2\phi_p = 0 \qquad \text{a}$$

Comparing this with Eq. (1–2), we see that $\tau = 0$, meaning that the *surfaces containing principal stresses have zero shear stresses*. In a similar manner, we differentiate Eq. (1–2), set the result equal to zero, and obtain

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \qquad \text{1-4}$$

Equation (1–4) defines the two values of $2\phi_s$ at which the shear stress τ reaches an extreme value. The angle between the surfaces containing the maximum shear stresses is 90°. Equation (1–4) can also be written as

$$\frac{\sigma_x - \sigma_y}{2}\cos 2\phi_p + \tau_{xy}\sin 2\phi_p = 0 \qquad \text{b}$$

Substituting this into Eq. (1–1) yields

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \qquad \text{1-5}$$

Equation (1–5) tells us that the two surfaces containing the maximum shear stresses also contain equal normal stresses of

$(\sigma_x + \sigma_y)/2$. Comparing Eqs. (1–3) and (1–4), we see that tan $2\phi_s$ is the negative reciprocal of tan $2\phi_p$. This means that $2\phi_s$ and $2\phi_p$ are angles 90° apart, and thus the angles between the surfaces containing the maximum shear stresses and the surfaces containing the principal stresses are ±45◦. Formulas for the two principal stresses can be obtained by substituting the angle $2\phi_p$ from Eq. (1–3) in Eq. (1–1). The result is

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{1-6}$$

In a similar manner the two extreme-value shear stresses are found to be

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{1-7}$$

Your particular attention is called to the fact that an extreme value of the shear stress *may not be the same as the actual maximum value.*

It is important to note that the equations given to this point are quite sufficient for performing any plane stress transformation. However, extreme care must be exercised when applying them. For example, say you are attempting to determine the principal state of stress for a problem where σ_x = 14 MPa, σ_y = −10 MPa, and τ_{xy} = −16 MPa. Equation (1–3) yields ϕ_p = −26.57° and 63.43° to locate the principal stress surfaces, whereas, Eq. (1–6) gives σ_1 = 22 MPa and σ_2 = −18 MPa for the principal stresses. If all we wanted was the principal stresses, we would be finished. However, what if we wanted to draw the element containing the principal stresses properly oriented relative to the *x*, *y* axes? Well, we have two values of ϕ_p and two values for the principal stresses. How do we know which value of ϕ_p corresponds to which value of the principal stress? To clear this up we would need to substitute one of the values of ϕ_p into Eq. (1–1) to determine the normal stress corresponding to that angle. A graphical method for expressing the relations developed in this section, called *Mohr's circle diagram,* is a very effective means of visualizing the stress state at a point and keeping track of the directions of the various components associated with plane stress. Equations (1–1) and (1–2) can be shown to be a set of parametric equations for σ and τ, where the parameter is 2ϕ. The relationship between σ and τ is that of a circle plotted in the σ, τ plane, where the center of the circle is located at $C = (\sigma, \tau) = [(\sigma_x + \sigma_y)/2, 0]$ and has a radius of

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}.$$

A problem arises in the sign of the shear stress. The transformation equations are based on a positive ϕ being counterclockwise, as shown in Fig. (1–2). If a positive τ were plotted above the σ axis, points would rotate clockwise on the circle 2ϕ in the opposite direction of rotation on the element. It would be convenient if the rotations were in the same direction. One could solve the problem easily by plotting positive τ below the axis. However, the classical approach to Mohr's circle uses a different convention for the shear stress.

1-2. Mohr's Circle Shear Convention

This convention is followed in drawing Mohr's circle:

• Shear stresses tending to rotate the element clockwise (cw) are plotted *above* the σ axis.

• Shear stresses tending to rotate the element counterclockwise (ccw) are plotted *below* the σ axis.

For example, consider the right face of the element in Fig. (1–1*a*). By Mohr's circle convention the shear stress shown is plotted *below* the σ axis because it tends to rotate the element counterclockwise. The shear stress on the top face of the element is plotted *above* the σ axis because it tends to rotate the element clockwise. In Fig. (1–2) we create a coordinate system with normal stresses plotted along the abscissa and shear stresses plotted as the ordinates. On the abscissa, tensile (positive) normal stresses are plotted to the right of the origin *O* and compressive (negative) normal stresses to the left. On the ordinate, clockwise (cw) shear stresses are plotted up; counterclockwise (ccw) shear stresses are plotted down.

Using the stress state of Fig. (1–1*a*), we plot Mohr's circle, Fig. (1–2), by first looking at the right surface of the element containing σ_x to establish the sign of σ_x and the cw or ccw direction of the shear stress. The right face is called the *x face* where $\phi = 0^\circ$. If

σ_x is positive and the shear stress τ_{xy} is ccw as shown in Fig. (1–1*a*), we can establish point *A* with coordinates (σ_x , τ_{xy}^{ccw}) in Fig. (1–2).

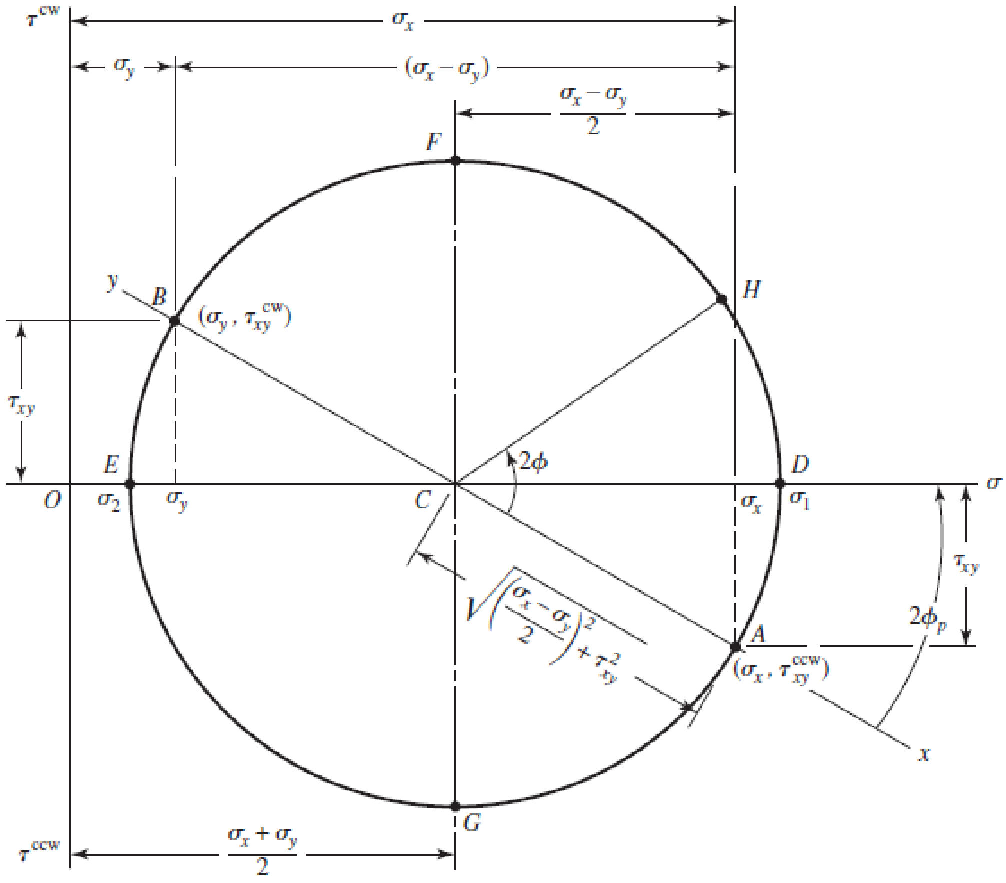


Figure (1-2)
Mohr's circle diagram.

Next, we look at the top *y face*, where $\phi = 90^\circ$, which contains σ_y , and repeat the process to obtain point *B* with coordinates (σ_y , τ_{xy}^{ccw}) as shown in Fig. (1–2). The two states of stress for the element are $\Delta\phi = 90^\circ$ from each other on the element so they will be $2\Delta\phi = 180^\circ$ from each other on Mohr's circle. Points *A* and *B* are the same vertical distance from the σ axis. Thus, *AB* must be on the diameter of the circle, and the center of the circle *C* is where *AB* intersects the σ axis. With points *A* and *B* on the circle, and center *C*, the complete circle can then be drawn. Note that the extended ends of line *AB* are labeled *x* and *y* as references to the normal to the surfaces for which points *A* and *B* represent the stresses. The entire Mohr's circle represents the state of stress at a *single* point in a structure. Each point on the circle represents the stress state for a *specific* surface intersecting the point in the structure. Each pair of points on the circle 180° apart represent the state of stress on an element whose surfaces are 90° apart. Once the circle is drawn, the states of stress can be visualized for various surfaces intersecting the point being analyzed. For example, the principal stresses σ_1 and σ_2 are points *D* and *E*, respectively, and their values obviously agree with Eq. (1–6). We also see that the shear stresses are zero on the surfaces containing σ_1 and σ_2. The two extreme-value shear stresses, one clockwise and one counterclockwise, occur at *F* and *G* with magnitudes equal to the radius of the circle. The surfaces at *F* and *G* each also contain normal stresses of $(\sigma_x + \sigma_y)/2$ as noted earlier in Eq. (1–5). Finally, the state of stress on an arbitrary surface located at an angle ϕ counterclockwise from the *x* face is point *H*.

At one time, Mohr's circle was used graphically where it was drawn to scale very accurately and values were measured by using a scale and protractor. Here, we are strictly using Mohr's circle as a visualization aid and will use a semi-graphical approach, calculating values from the properties of the circle. This is illustrated by the following example.

EXAMPLE 1–1

A stress element has σ_x = 80 MPa and τ_{xy} = 50 MPa cw, as shown in Fig. (1–3*a*).

(a) Using Mohr's circle, find the principal stresses and directions, and show these on a stress element correctly aligned with respect to the *xy* coordinates. Draw another stress element to show τ_1 and τ_2, find the corresponding normal stresses, and label the drawing completely.

(b) Repeat part *a* using the transformation equations only.

Solution

(a) In the semi-graphical approach used here, we first make an approximate freehand sketch of Mohr's circle and then use the geometry of the figure to obtain the desired information.

Draw the σ and τ axes first (Fig. 1–3*b*) and from the *x* face locate σ_x = 80 MPa along the σ axis. On the *x* face of the element, we see that the shear stress is 50 MPa in the cw direction. Thus, for the x face, this establishes point A (80, 50cw) MPa. Corresponding to the y face, the stress is $\sigma = 0$ and $\tau = 50$ MPa in the ccw direction. This locates point B (0, 50ccw) MPa. The line AB forms the diameter of the required circle, which can now be drawn. The intersection of the circle with the σ axis defines σ_1 and σ_2 as shown. Now, noting the triangle ACD, indicate on the sketch the length of the legs AD and CD as 50 and 40 MPa, respectively. The length of the hypotenuse AC is

$$\tau_1 = \sqrt{(50)^2 + (40)^2} = 64.0 \text{ MPa} \qquad \textit{Ans.}$$

and this should be labeled on the sketch too. Since intersection C is 40 MPa from the origin, the principal stresses are now found to be

$$\sigma_1 = 40 + 64 = 104 \text{ MPa and } \sigma_2 = 40 - 64 = -24 \text{ MPa} \qquad \textit{Ans.}$$

The angle 2ϕ from the x axis *cw* to σ_1 is

$$2\phi_p = \tan^{-1}\frac{50}{40} = 51.3^\circ \qquad \textit{Ans.}$$

To draw the principal stress element (Fig. 1–3*c*), sketch the x and y axes parallel to the original axes. The angle ϕ_p on the stress element must be measured in the *same* direction as is the angle $2\phi_p$ on the Mohr circle. Thus, from x measure 25.7° (half of 51.3°) clockwise to locate the σ_1 axis. The σ_2 axis is 90° from the σ_1 axis and the stress element can now be completed and labeled as shown. Note that there are *no* shear stresses on this element. The two maximum shear stresses occur at points E and F in Fig. (1–3*b*). The two normal stresses corresponding to these shear stresses are each 40 MPa, as indicated. Point E is 38.7° ccw from point A on Mohr's circle. Therefore, in Fig. (1–3*d*), draw a stress element oriented 19.3° (half of 38.7°) ccw from x. The element should then be labeled with magnitudes and directions as shown. In constructing these stress elements it is important to indicate the x and y directions of the original reference system. This completes the link between the original machine element and the orientation of its principal stresses.

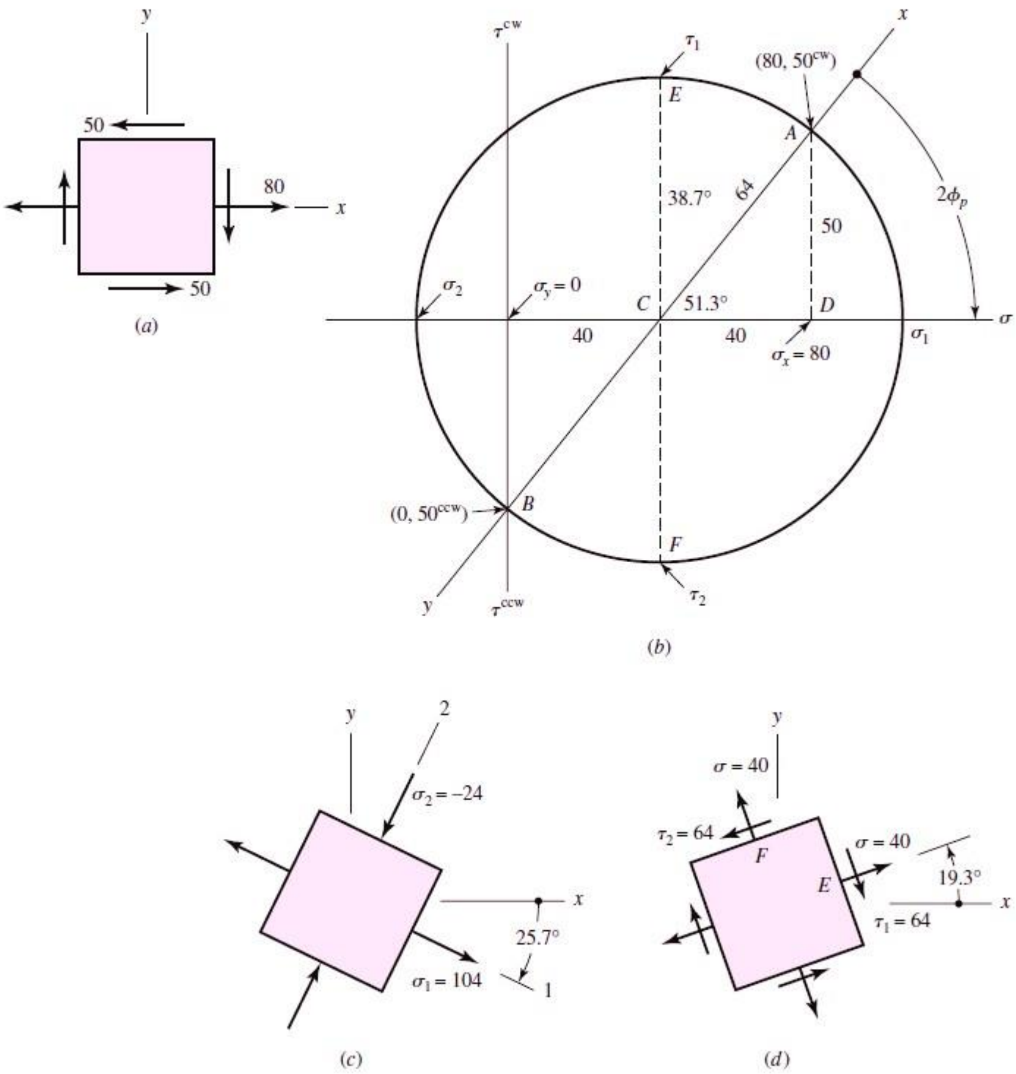


Figure (1-3)
All stresses in MPa.

(b) The transformation equations are programmable. From Eq. (1–3),

$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-50)}{80}\right) = -25.7^\circ, 64.3^\circ$$

From Eq. (1–2), for the first angle $\phi_p = -25.7^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(-25.7)] + (-50)\sin[2(-25.7)] = 104.03 \text{ MPa}$$

which confirms that 104.03 MPa is a principal stress. From Eq. (1–1), for $\phi_p = 64.3°$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(64.3)] + (-50)\sin[2(64.3)] = -24.03 \text{ MPa}$$

Substituting ϕ_p = 64.3° into Eq. (3–9) again yields τ = 0, indicating that −24.03 MPa is also a principal stress. Once the principal stresses are calculated they can be ordered such that $\sigma_1 \geq \sigma_2$. Thus,
σ_1 = 104.03 MPa and σ_2 = −24.03 MPa. *Ans*.

Since for σ_1 = 104.03 MPa, ϕ_p = −25.7°, and since ϕ is defined positive ccw in the transformation equations, we rotate *clockwise* 25.7° for the surface containing σ_1. We see in Fig. (1–3*c*) that this totally agrees with the semigraphical method.
To determine τ_1 and τ_2, we first use Eq. (1–4) to calculate ϕ_s :

$$\phi_s = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(-\frac{80}{2(-50)}\right) = 19.3°, 109.3°$$

For ϕ_s = 19.3°, Eqs. (1–1) and (1–2) yield

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(19.3)] + (-50)\sin[2(19.3)] = 40.0 \text{ MPa}$$

$$\tau = -\frac{80-0}{2}\sin[2(19.3)] + (-50)\cos[2(19.3)] = -64.0 \text{ MPa}$$

Ans.

Remember that Eqs. (1–1) and (1–2) are *coordinate* transformation equations. Imagine that we are rotating the *x*, *y* axes 19.3° counterclockwise and *y* will now point up and to the left. So a negative shear stress on the rotated *x* face will point down and to the right as shown in Fig. (1–3*d*). Thus again, results agree with the semigraphical method. For ϕ_s = 109.3°, Eqs. (1–1) and (1–2) give σ = 40.0 MPa and τ = +64.0 MPa.
Using the same logic for the coordinate transformation we find that results again agree with Fig. (1–3*d*).

Homework

For each of the plane stress states listed below, draw a Mohr's circle diagram properly labeled, find the principal normal and shear stresses, and determine the angle from the *x* axis to σ_1. Draw stress elements as in Fig. (1–3*c* and *d*) and label all details.

(1) $\sigma_x = 12$, $\sigma_y = 6$, $\tau_{xy} = 4$ cw
(2) $\sigma_x = 16$, $\sigma_y = 9$, $\tau_{xy} = 5$ ccw
(3) $\sigma_x = 10$, $\sigma_y = 24$, $\tau_{xy} = 6$ ccw
(4) $\sigma_x = 9$, $\sigma_y = 19$, $\tau_{xy} = 8$ cw
(5) $\sigma_x = -4$, $\sigma_y = 12$, $\tau_{xy} = 7$ ccw
(6) $\sigma_x = 6$, $\sigma_y = -5$, $\tau_{xy} = 8$ ccw
(7) $\sigma_x = -8$, $\sigma_y = 7$, $\tau_{xy} = 6$ cw
(8) $\sigma_x = 9$, $\sigma_y = -6$, $\tau_{xy} = 3$ cw
(*9*) $\sigma_x = 20$, $\sigma_y = -10$, $\tau_{xy} = 8$ cw
(*10*) $\sigma_x = 30$, $\sigma_y = -10$, $\tau_{xy} = 10$ ccw
(*11*) $\sigma_x = -10$, $\sigma_y = 18$, $\tau_{xy} = 9$ cw
(12) $\sigma_x = -12$, $\sigma_y = 22$, $\tau_{xy} = 12$ cw

1-3. General Three-Dimensional Stress

As in the case of plane stress, a particular orientation of a stress element occurs in space for which all shear-stress components are zero. When an element has this particular orientation, the normals to the faces are mutually orthogonal and correspond to the principal directions, and the normal stresses associated with these faces are the principal stresses. Since there are three faces, there are three principal directions and three principal stresses σ_1, σ_2, and σ_3. For plane stress, the stress-free surface contains the third principal stress which is zero. The process in finding the three principal stresses from the six stress components σ_x , σ_y , σ_z , τ_{xy} , τ_{yz}, and τ_{zx} , involves finding the roots of the cubic equation

$$\begin{aligned}\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right)\sigma \\ - \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2\right) = 0 \qquad \text{1-8}\end{aligned}$$

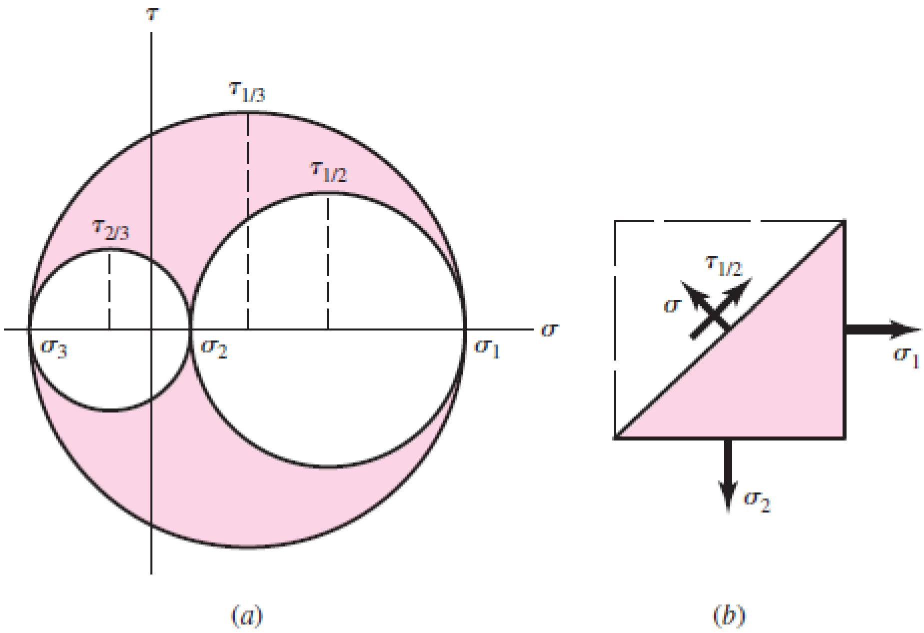


Figure (1-4)
Mohr's circles for three-dimensional stress

In plotting Mohr's circles for three-dimensional stress, the principal normal stresses are ordered so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Then the result appears as in Fig. (1–4*a*). The stress coordinates σ, τ for any arbitrarily located plane will always lie on the boundaries or within the shaded area. Figure (1–4*a*) also shows the three *principal shear stresses* $\tau_{1/2}$, $\tau_{2/3}$, and $\tau_{1/3}$. Each of these occurs on the two planes, one of which is shown in Fig. (3–12*b*). The figure shows that the principal shear stresses are given by the equations

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \qquad \text{1-9}$$

Of course, $\tau_{max} = \tau_{1/3}$ when the normal principal stresses are ordered ($\sigma_1 > \sigma_2 > \sigma_3$), so always order your principal stresses. Do this in any computer code you generate and you'll always generate τ_{max}.