

1-8. Stress Concentration

In the development of the basic stress equations for tension, compression, bending, and torsion, it was assumed that no geometric irregularities occurred in the member under consideration. But it is quite difficult to design a machine without permitting some changes in the cross sections of the members. Rotating shafts must have shoulders designed on them so that the bearings can be properly seated and so that they will take thrust loads; and the shafts must have key slots machined into them for securing pulleys and gears. A bolt has a head on one end and screw threads on the other end, both of which account for abrupt changes in the cross section. Other parts require holes, oil grooves, and notches of various kinds. Any discontinuity in a machine part alters the stress distribution in the neighborhood of the discontinuity so that the elementary stress equations no longer describe the state of stress in the part at these locations. Such discontinuities are called *stress raisers*, and the regions in which they occur are called areas of *stress concentration*.

The distribution of elastic stress across a section of a member may be uniform as in a bar in tension, linear as a beam in bending, or even rapid and curvaceous as in a sharply curved beam. Stress concentrations can arise from some irregularity not inherent in the member, such as tool marks, holes, notches, grooves, or threads. The *nominal stress* is said to exist if the member is free of the stress raiser. This definition is not always honored, so check the definition on the stress-concentration chart or table you are using.

A *theoretical, or geometric, stress-concentration factor* K_t or K_{ts} is used to relate the actual maximum stress at the discontinuity to the nominal stress. The factors are defined by the equations

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \quad K_{ts} = \frac{\tau_{\max}}{\tau_0} \quad 1-30$$

where K_t is used for normal stresses and K_{ts} for shear stresses. The nominal stress σ_0 or τ_0 is more difficult to define. Generally, it is the stress calculated by using the elementary stress equations and the net area, or net cross section. But sometimes the gross cross section is

used instead, and so it is always wise to double check your source of K_t or K_{ts} before calculating the maximum stress.

The subscript t in K_t means that this stress-concentration factor depends for its value only on the *geometry* of the part. That is, the particular material used has no effect on the value of K_t . This is why it is called a *theoretical* stress-concentration factor.

The analysis of geometric shapes to determine stress-concentration factors is a difficult problem, and not many solutions can be found. Most stress-concentration factors are found by using experimental techniques. Though the finite-element method has been used, the fact that the elements are indeed finite prevents finding the true maximum stress. Experimental approaches generally used include photoelasticity, grid methods, brittle-coating methods, and electrical strain-gauge methods. Of course, the grid and strain-gauge methods both suffer from the same drawback as the finite-element method.

Stress-concentration factors for a variety of geometries may be found in the following charts.

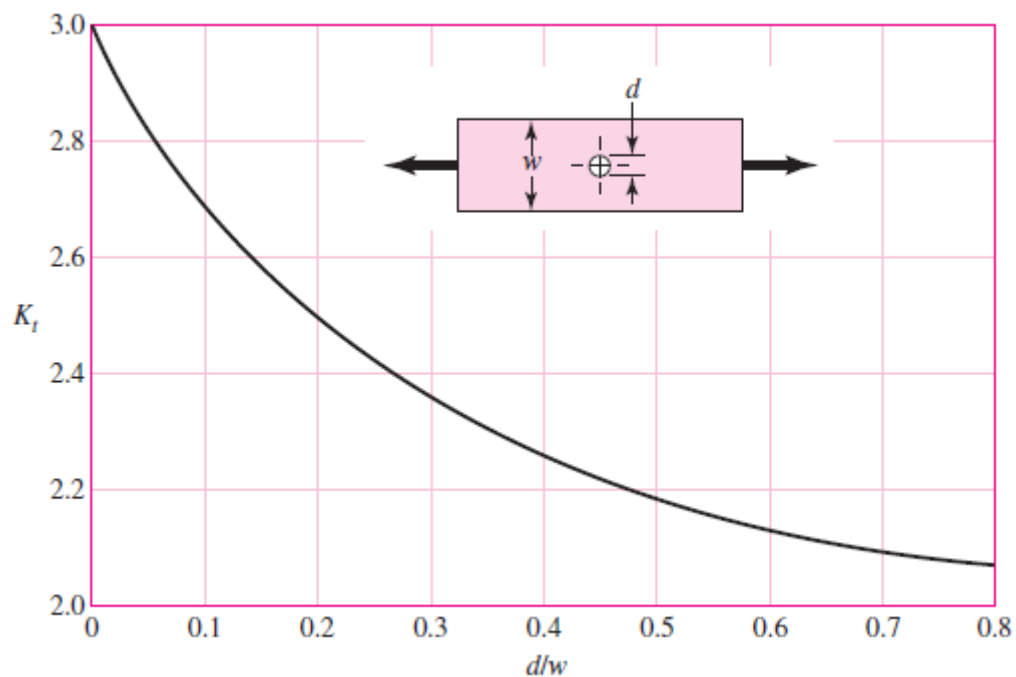


Figure (1-8)

Bar in tension or simple compression with a transverse hole.

$\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

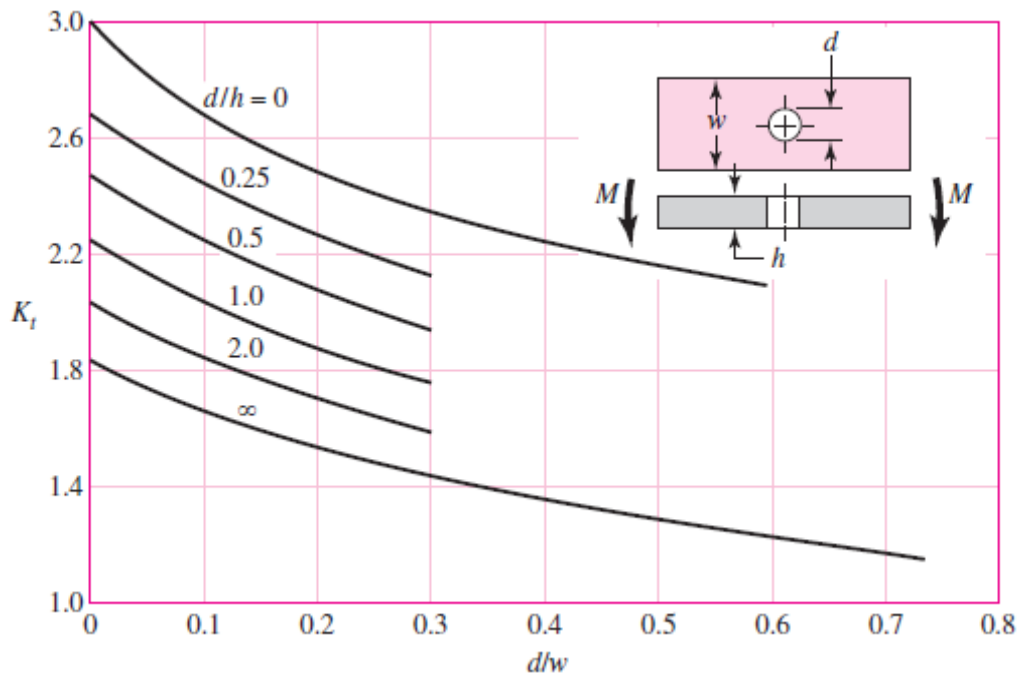


Figure (1-9)
 Rectangular bar with a transverse hole in bending.
 $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

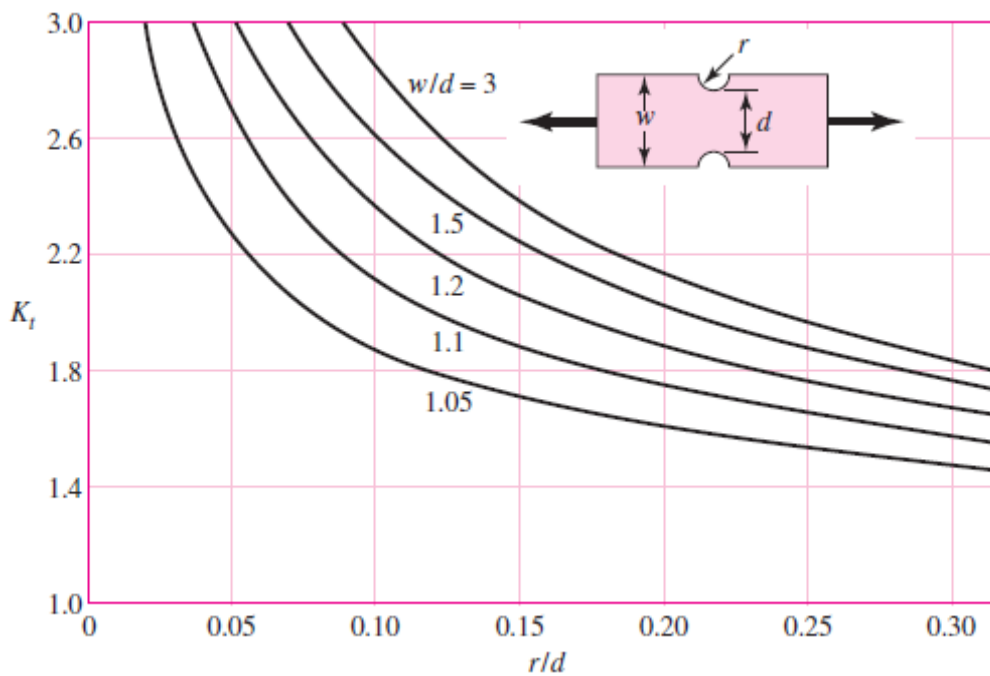


Figure (1-10)
 Notched rectangular bar in tension or simple compression.
 $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

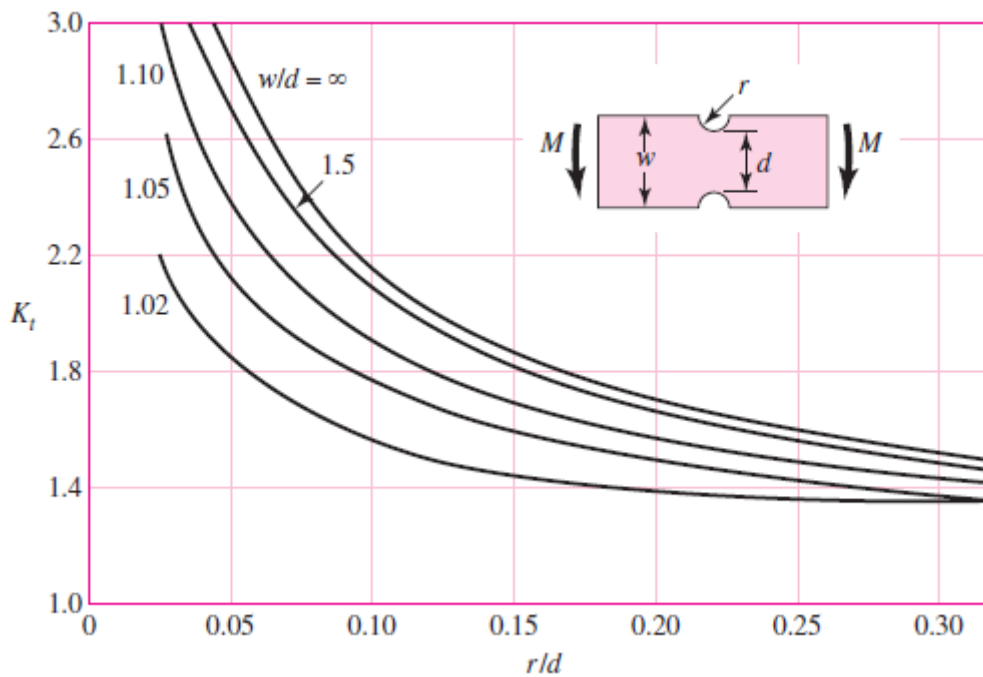


Figure (1-11)
Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and t is the thickness.

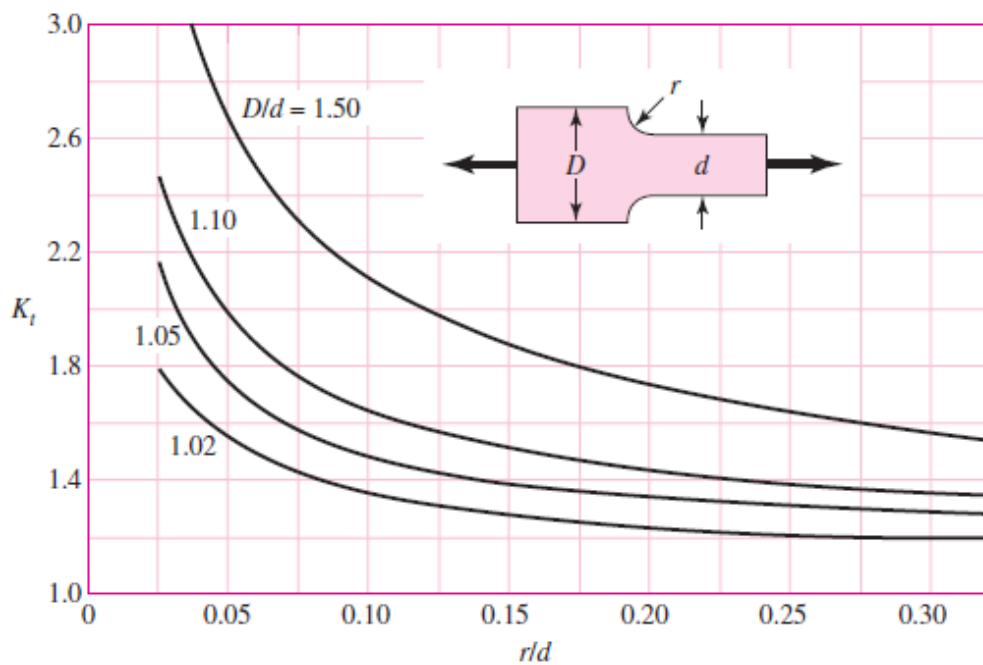


Figure (1-12)
Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

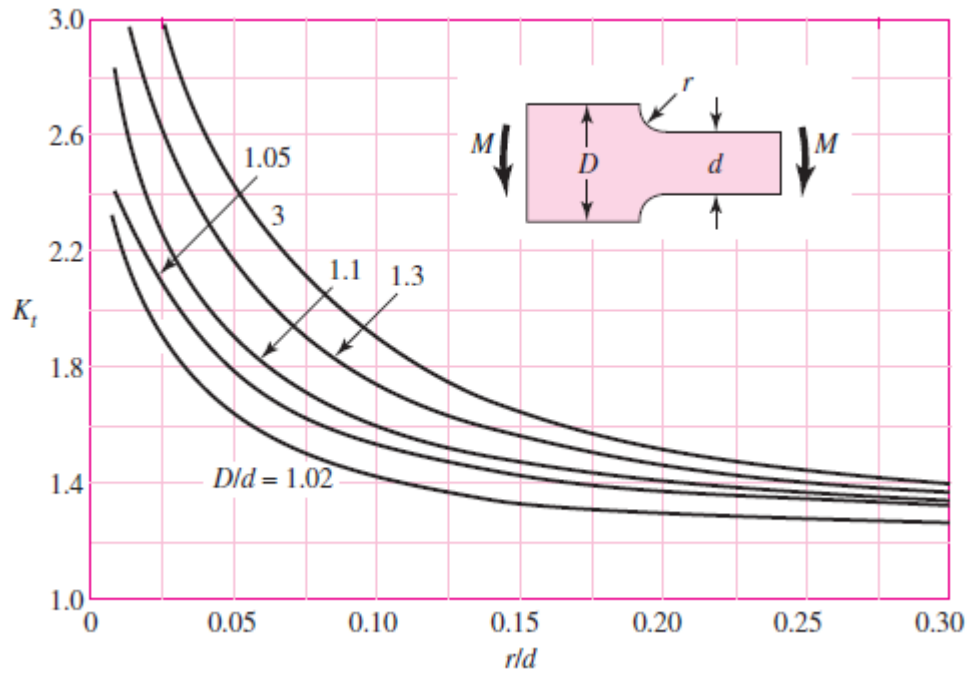


Figure (1-13)

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.

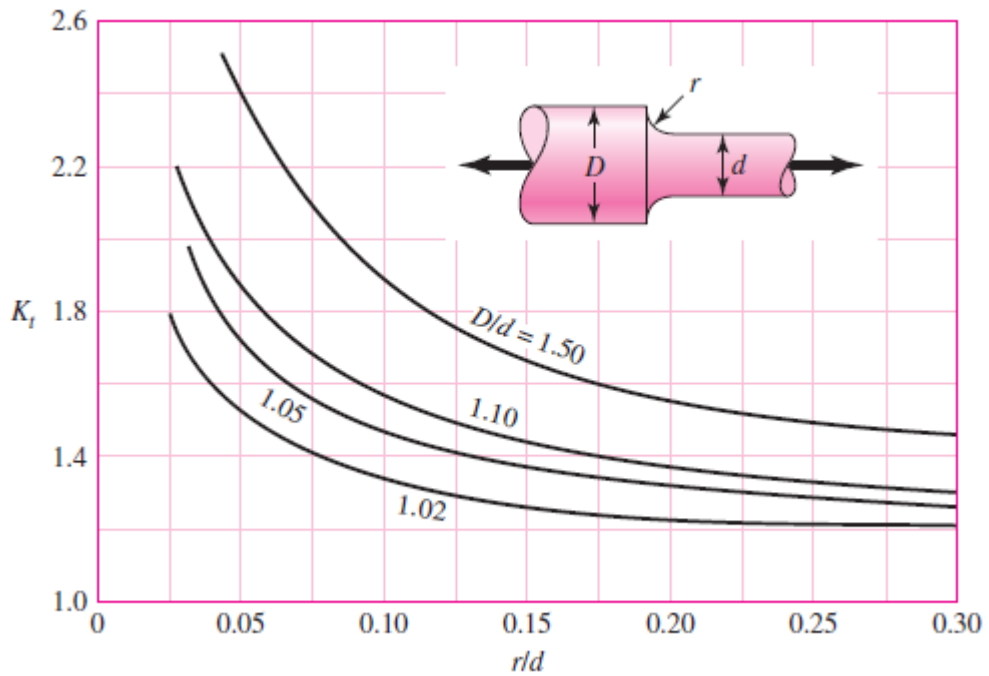


Figure (1-14)

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

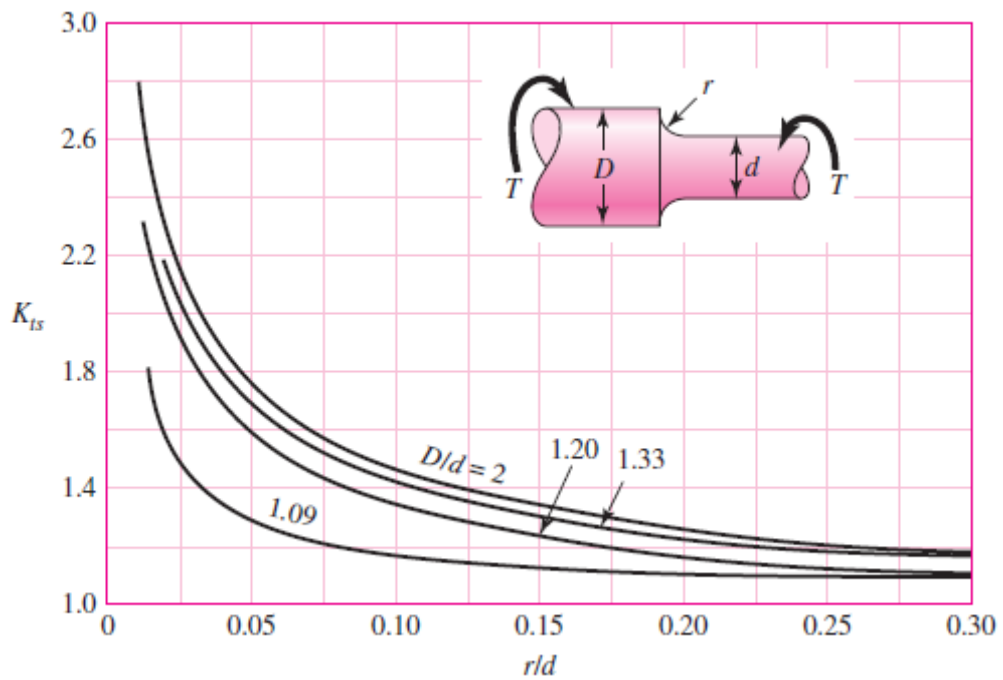


Figure (1-15)

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

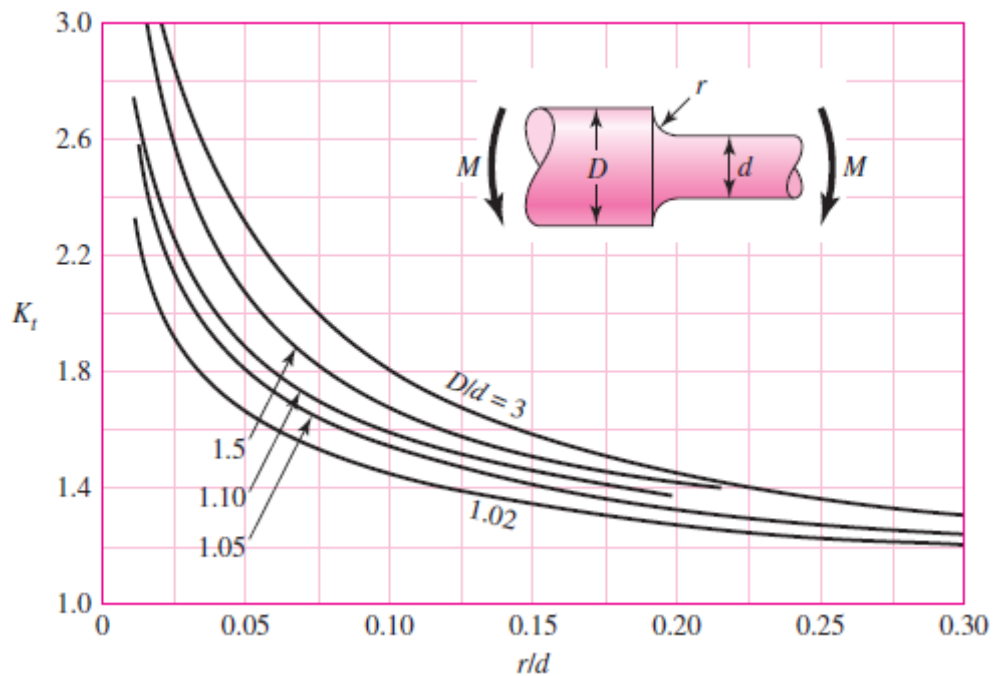


Figure (1-16)

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

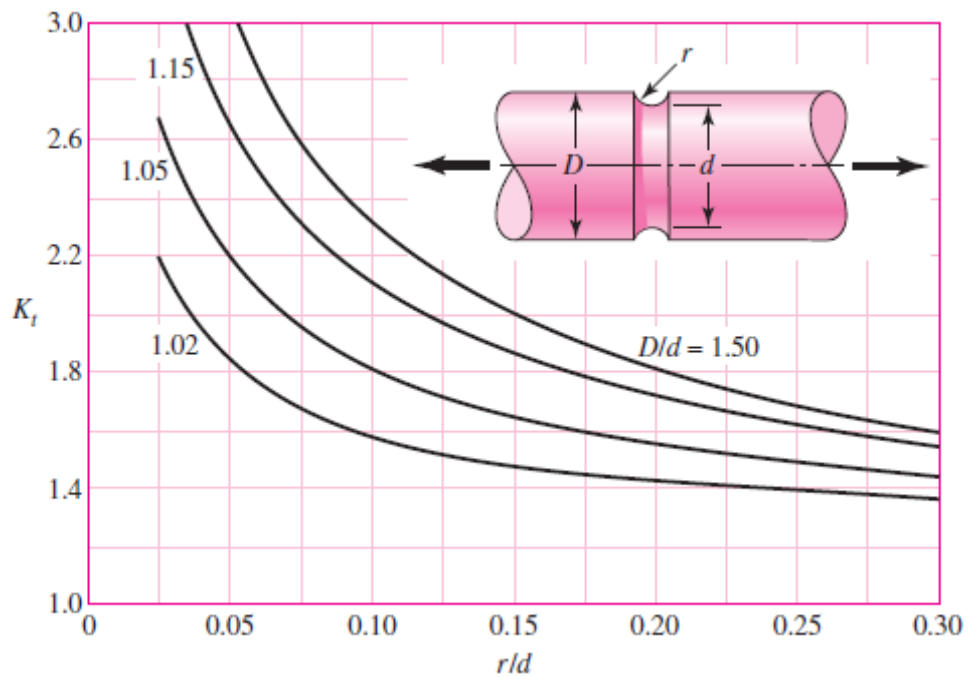


Figure (1-17)
Grooved round bar in tension. $\sigma_0 = F/A$, where
 $A = \pi d^2/4$.

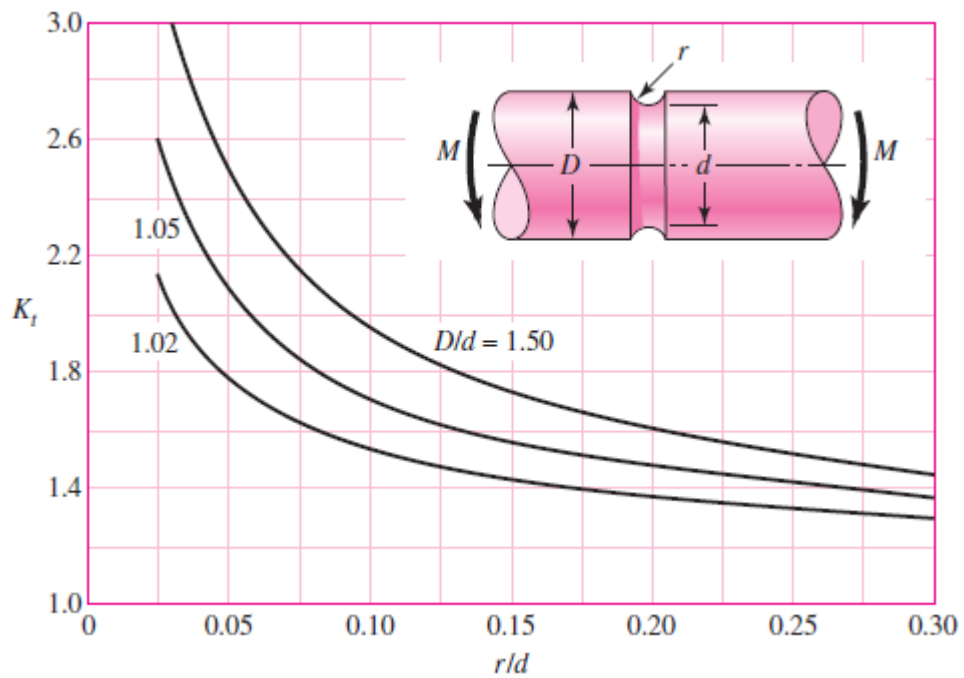


Figure (1-18)
Grooved round bar in bending. $\sigma_0 = Mc/I$, where
 $c = d/2$ and $I = \pi d^4/64$.

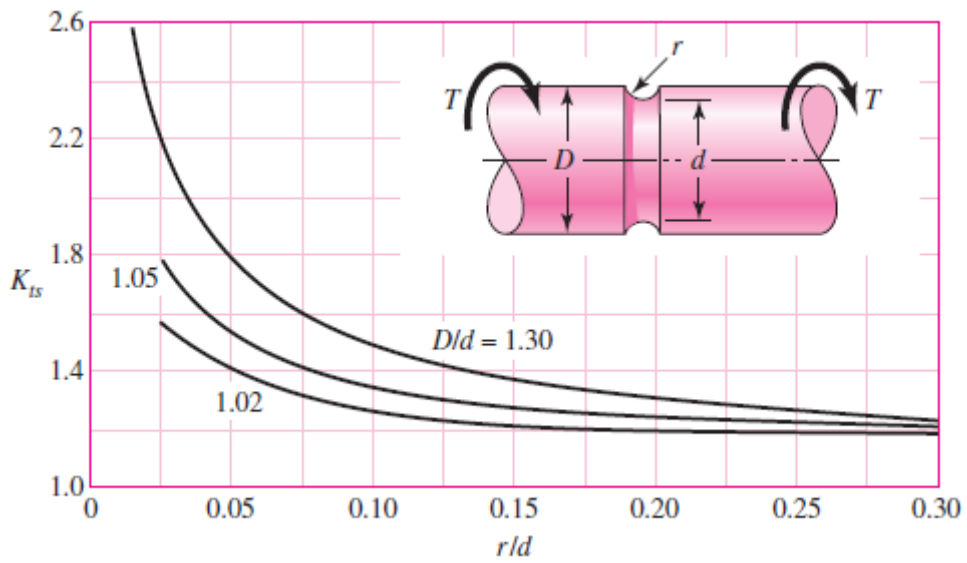


Figure (1-19)
Grooved round bar in torsion. $\tau_0 = Tc/J$, where
 $c = d/2$ and $J = \pi d^4/32$.

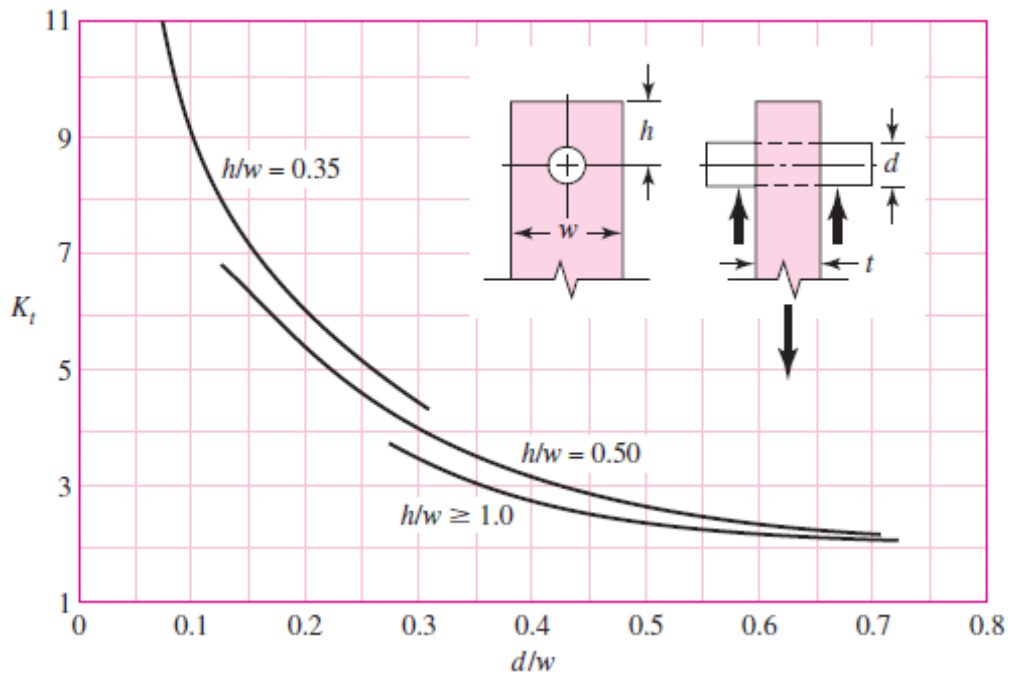


Figure (1-20)
Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$,
where $A = (w - d)t$.

EXAMPLE 1-3

Considering the stress concentration at point A shown in Fig. (1-21), determine the maximum normal and shear stresses at A if $F = 200$ lbf.

Solution

$$D/d = \frac{1.5}{1} = 1.5$$

$$r/d = \frac{1/8}{1} = 0.125$$

From Fig. (1-15):

$$K_{ts} \doteq 1.39$$

From Fig. (1-16):

$$K_t \doteq 1.60$$

$$\sigma_A = K_t \frac{Mc}{I} = \frac{32K_t M}{\pi d^3} = \frac{32(1.6)(200)(14)}{\pi(1^3)} = 45\,630 \text{ psi}$$

$$\tau_A = K_{ts} \frac{Tc}{J} = \frac{16K_{ts} T}{\pi d^3} = \frac{16(1.39)(200)(15)}{\pi(1^3)} = 21\,240 \text{ psi}$$

$$\begin{aligned} \sigma_{\max} &= \frac{\sigma_A}{2} + \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \frac{45.63}{2} + \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2} \\ &= 54.0 \text{ kpsi} \quad \text{Ans.} \end{aligned}$$

$$\tau_{\max} = \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2} = 31.2 \text{ kpsi}$$

Ans.

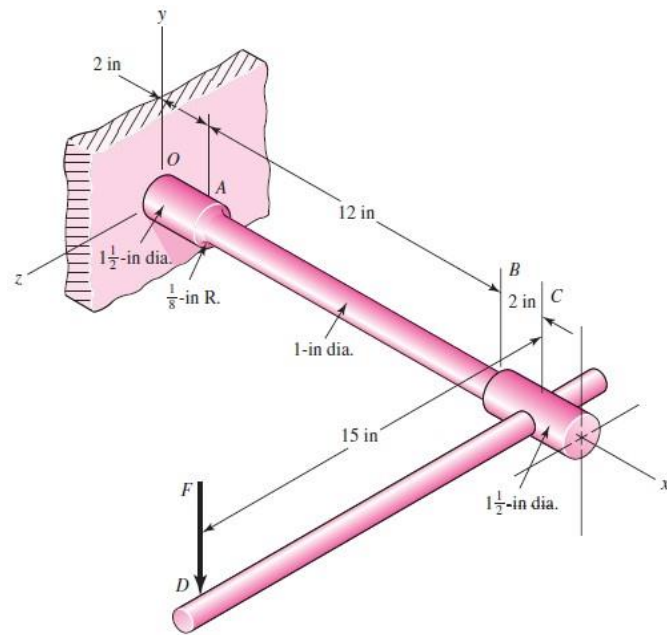


Figure (1-21)

