1. Failures Resulting from Static Loading

A *static load* is a stationary force or couple applied to a member. To be stationary, the force or couple must be unchanging in magnitude, point or points of application, and direction. A static load can produce axial tension or compression, a shear load, a bending load, a torsional load, or any combination of these. To be considered static, the load cannot change in any manner.

In this part we consider the relations between strength and static loading in order to make the decisions concerning material and its treatment, fabrication, and geometry for satisfying the requirements of functionality, safety, reliability, competitiveness, usability, manufacturability, and marketability. How far we go down this list is related to the scope of the examples.

Stress Concentration

Stress concentration (see Sec. 1–8) is a highly localized effect. In some instances it may be due to a surface scratch. If the material is ductile and the load static, the design load may cause yielding in the critical location in the notch. This yielding can involve strain strengthening of the material and an increase in yield strength at the small critical notch location. Since the loads are static and the material is ductile, that part can carry the loads satisfactorily with no general yielding. In these cases the designer sets the geometric (theoretical) stress concentration factor K_t to unity.

When using this rule for ductile materials with static loads, be careful to assure yourself that the material is not susceptible to brittle fracture in the environment of use.

Brittle materials do not exhibit a plastic range. A brittle material "feels" the stress concentration factor K_t or K_{ts}.

An exception to this rule is a brittle material that inherently contains microdiscontinuity stress concentration, worse than the macrodiscontinuity that the designer has in mind. Sand molding introduces sand particles, air, and water vapor bubbles. The grain structure of cast iron contains graphite flakes (with little strength), which are literally cracks introduced during the solidification process. When a tensile test on a cast iron is performed, the strength reported in the literature *includes* this stress concentration. In such cases K_t or K_{ts} need not be applied.

2-1. Failure Theories

Unfortunately, there is no universal theory of failure for the general case of material properties and stress state. Instead, over the years several hypotheses have been formulated and tested, leading to today's accepted practices. Being accepted, we will characterize these "practices" as *theories* as most designers do.

Structural metal behavior is typically classified as being ductile or brittle, although under special situations, a material normally considered ductile can fail in a brittle manner. Ductile materials are normally classified such that $\varepsilon_f \geq 0.05$ and have an identifiable yield strength that is often the same in compression as in tension ($S_{yt} = S_{yc} = S_y$). Brittle materials, $\varepsilon_f < 0.05$, do not exhibit an identifiable yield strength, and are typically classified by ultimate tensile and compressive strengths, S_{ut} and S_{uc}, respectively (where S_{uc} is given as a positive quantity). The generally accepted theories are:

Ductile materials (yield criteria)

- Maximum shear stress (MSS)
- Distortion energy (DE)
- Ductile Coulomb-Mohr (DCM)

Brittle materials (fracture criteria)

- Maximum normal stress (MNS)
- Brittle Coulomb-Mohr (BCM)
- Modified Mohr (MM)

2-2. Maximum-Shear-Stress Theory for Ductile Materials (MSS)

The *maximum-shear-stress theory* predicts that *yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield*.

The maximum-shear-stress theory predicts yielding when

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{S_y}{2n}$$

Where S_y is the yielding stress, and n is the factor of safety. Note that this implies that the yield strength in shear is given by

$$S_{sy} = 0.5S_y$$

The MSS theory is also referred to as the *Tresca* or *Guest theory*. It is an acceptable theory but conservative predictor of failure; and since engineers are conservative by nature, it is quite often used.

2-3. Distortion-Energy Theory for Ductile Materials (DE)

The distortion-energy theory predicts that yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material.

The distortion-energy theory is also called:

- The von Mises or von Mises–Hencky theory
- The shear-energy theory
- The octahedral-shear-stress theory

The distortion-energy theory predicts yielding when

$$\sigma' = \frac{S_y}{n}$$

where σ′ is usually called the *von Mises stress*, named after Dr. R. von Mises, who contributed to the theory; and

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

The shear yield strength predicted by the distortion-energy theory is

$$S_{sy} = 0.577S_y$$

EXAMPLE 2-1

A hot-rolled steel has a yield strength of $S_{yt} = S_{yc} = 100$ kpsi and a true strain at fracture of $\varepsilon_f = 0.55$. Estimate the factor of safety for the following principal stress states:

(*a*) 70, 70, 0 Mpa
(*b*) 30, 70, 0 Mpa.
(*c*) 0, 70, −30 Mpa.
(*d*) 0, −30, −70 Mpa.
(*e*) 30, 30, 30 Mpa.

Solution

2-4. Coulomb-Mohr Theory for Ductile Materials (DCM)

A variation of Mohr's theory, called the *Coulomb-Mohr theory* or the *internal-friction theory*.

Not all materials have compressive strengths equal to their corresponding tensile values. For example, the yield strength of magnesium alloys in compression may be as little as 50 percent of their yield strength in tension. The ultimate strength of gray cast irons in compression varies from 3 to 4 times greater than the ultimate tensile strength. So, this theory can be used to predict failure for materials whose strengths in tension and compression are not equal; this is can be expressed as a design equation with a factor of safety, *n*, as

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

where either yield strength or ultimate strength can be used.
The torsional yield strength occurs when $\tau_{max} = S_{sy}$; then

$$S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}}$$

EXAMPLE 2-2

A 25-mm-diameter shaft is statically torqued to 230 N·m. It is made of cast 195-T6 aluminum, with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. It is machined to final diameter. Estimate the factor of safety of the shaft.

Solution

The maximum shear stress is given by

$$\tau = \frac{16T}{\pi d^3} = \frac{16(230)}{\pi\left[25\left(10^{-3}\right)\right]^3} = 75\left(10^6\right)\text{N/m}^2 = 75\,\text{MPa}$$

The two nonzero principal stresses are 75 and −75 MPa, making the ordered principal stresses $\sigma_1 = 75$, $\sigma_2 = 0$, and $\sigma_3 = -75$ MPa. From Eq. (2–6), for yield,

$$n = \frac{1}{\sigma_1/S_{yt} - \sigma_3/S_{yc}} = \frac{1}{75/160 - (-75)/170} = 1.10$$

Alternatively, from Eq. (2–7),

$$S_{sy} = \frac{S_{yt}S_{yc}}{S_{yt} + S_{yc}} = \frac{160(170)}{160 + 170} = 82.4\,\text{MPa}$$

and $\tau_{max} = 75$ MPa. Thus,

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{82.4}{75} = 1.10$$

2-5. Maximum-Normal-Stress Theory for Brittle Materials (MNS)

The maximum-normal-stress (MNS) theory states that *failure occurs whenever one of the three principal stresses equals or exceeds the strength*. Again we arrange the principal stresses for a general stress state in the ordered form $\sigma_1 \geq \sigma_2 \geq \sigma_3$. This theory then predicts that failure occurs whenever

$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc}$$

where S_{ut} and S_{uc} are the ultimate tensile and compressive strengths, respectively, given as positive quantities.

2-6. Modifications of the Mohr Theory for Brittle Materials

We will discuss two modifications of the Mohr theory for brittle materials: the Brittle- Coulomb-Mohr (BCM) theory and the modified Mohr (MM) theory. The equations provided for the theories will be restricted to plane stress and be of the design type incorporating the factor of safety.

***Brittle-Coulomb-Mohr* (*BCM*)**

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

***Modified Mohr* (*MM*)**

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

EXAMPLE 2-4

Consider the wrench in Ex. (2–3), Fig. (2–1), as made of cast iron, machined to dimension. The force F required to fracture this part can be regarded as the strength of the component part. If the material is cast iron, the tensile ultimate strength is 31 kpsi and the compressive ultimate strength is 109 kpsi, find the force F with
(a) Coulomb-Mohr failure model (*b*) Modified Mohr failure model.

2-7. Selection of Failure Criteria

For ductile behavior the preferred criterion is the distortion-energy theory, although some designers also apply the maximum-shear-stress theory because of its simplicity and conservative nature. In the rare case when $S_{yt} \neq S_{yc}$, the ductile Coulomb-Mohr method is employed.

For brittle behavior, the original Mohr hypothesis, constructed with tensile, compression, and torsion tests, with a curved failure locus is the best hypothesis we have. However, the difficulty of applying it without a computer leads engineers to choose modifications, namely, Coulomb Mohr, or modified Mohr. Figure (2–2) provides a summary flowchart for the selection of an effective procedure for analyzing or predicting failures from static loading for brittle or ductile behavior.

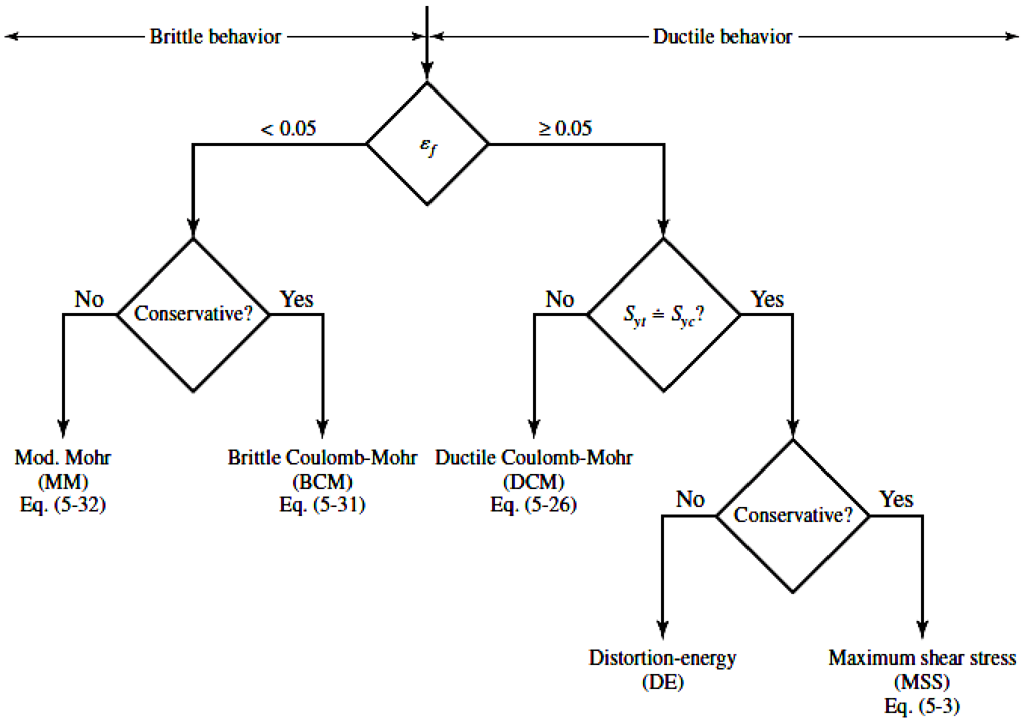

Figure (2-2)

Failure theory selection flowchart.

Homework

(1) The cantilevered bar shown in the figure is made of AISI 1006 cold-drawn steel with (S_y = 280 MPa) and is loaded by the forces F = 0.55 kN, P = 8 kN, and T = 30 N·m. Compute the factor of safety, based upon the distortion-energy theory, for stress elements at A. ($Ans./\ n = 2.77$)