

1. Fatigue Failure Resulting from Variable Loading

In most testing of those properties of materials that relate to the stress-strain diagram, the load is applied gradually, to give sufficient time for the strain to fully develop. Furthermore, the specimen is tested to destruction, and so the stresses are applied only once. Testing of this kind is applicable, to what are known as *static conditions*; such conditions closely approximate the actual conditions to which many structural and machine members are subjected.

The condition frequently arises, however, in which the stresses vary with time or they fluctuate between different levels. For example, a particular fiber on the surface of a rotating shaft subjected to the action of bending loads undergoes both tension and compression for each revolution of the shaft. If the shaft is part of an electric motor rotating at 1725 rev/min, the fiber is stressed in tension and compression 1725 times each minute. If, in addition, the shaft is also axially loaded (as it would be, for example, by a helical or worm gear), an axial component of stress is superposed upon the bending component. In this case, some stress is always present in any one fiber, but now the *level* of stress is fluctuating. These and other kinds of loading occurring in machine members produce stresses that are called *variable, repeated, alternating, or fluctuating stresses*.

Often, machine members are found to have failed under the action of repeated or fluctuating stresses; yet the most careful analysis reveals that the actual maximum stresses were well below the ultimate strength of the material, and quite frequently even below the yield strength. The most distinguishing characteristic of these failures is that the stresses have been repeated a very large number of times. Hence the failure is called a *fatigue failure*.

When machine parts fail statically, they usually develop a very large deflection, because the stress has exceeded the yield strength, and the part is replaced before fracture actually occurs. Thus many static failures give visible warning in advance. But a fatigue failure gives no warning! It is sudden and total, and hence dangerous. It is relatively simple to design against a static failure, because our knowledge is comprehensive. Fatigue is a much more complicated phenomenon, only partially understood, and the engineer seeking competence must acquire as much knowledge of the subject as possible.

Fatigue failure is due to crack formation and propagation. A fatigue crack will typically initiate at a discontinuity in the material where the cyclic stress is a maximum. Discontinuities can arise because of:

- Design of rapid changes in cross section, keyways, holes, etc. where stress concentrations occur
- Elements that roll and/or slide against each other (bearings, gears, cams, etc.) under high contact pressure, developing concentrated subsurface contact stresses that can cause surface pitting or spalling after many cycles of the load
- Carelessness in locations of stamp marks, tool marks, scratches, and burrs; poor joint design; improper assembly; and other fabrication faults
- Composition of the material itself as processed by rolling, forging, casting, extrusion, drawing, heat treatment, etc. Microscopic and submicroscopic surface and subsurface discontinuities arise, such as inclusions of foreign material, alloy segregation, voids, hard precipitated particles, and crystal discontinuities

Various conditions that can accelerate crack initiation include residual tensile stresses, elevated temperatures, temperature cycling, a corrosive environment, and high-frequency cycling.

Approach to Fatigue Failure in Analysis and Design

As noted in the previous section, there are a great many factors to be considered, even for very simple load cases. The methods of fatigue failure analysis represent a combination of engineering and science. Often science fails to provide the complete answers that are needed. But the airplane must still be made to fly—safely. And the automobile must be manufactured with a reliability that will ensure a long and trouble free life and at the same time produce profits for the stockholders of the industry. Thus, while science has not yet completely explained the complete mechanism of fatigue, the engineer must still design things that will not fail. In a sense this is a classic example of the true meaning of engineering as contrasted with science. Engineers use science to solve their problems if the science is available. But available or not, the problem must be solved, and whatever form the solution takes under these conditions is called *engineering*.

3.1 The Stress-Life Method

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction.

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. The results are plotted as an $S-N$ diagram (Fig. 3-1). This chart may be plotted on semilog paper or on log-log paper. In the case of ferrous metals and alloys, the graph becomes horizontal after the material has been stressed for a certain number of cycles.

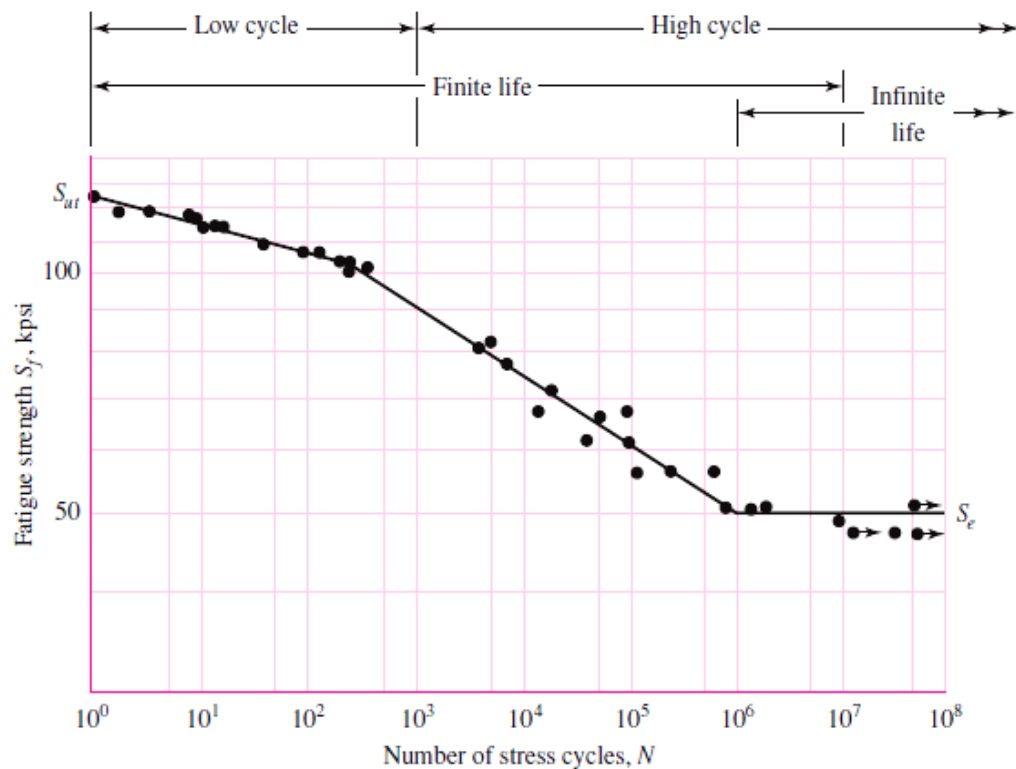


Figure (3-1)

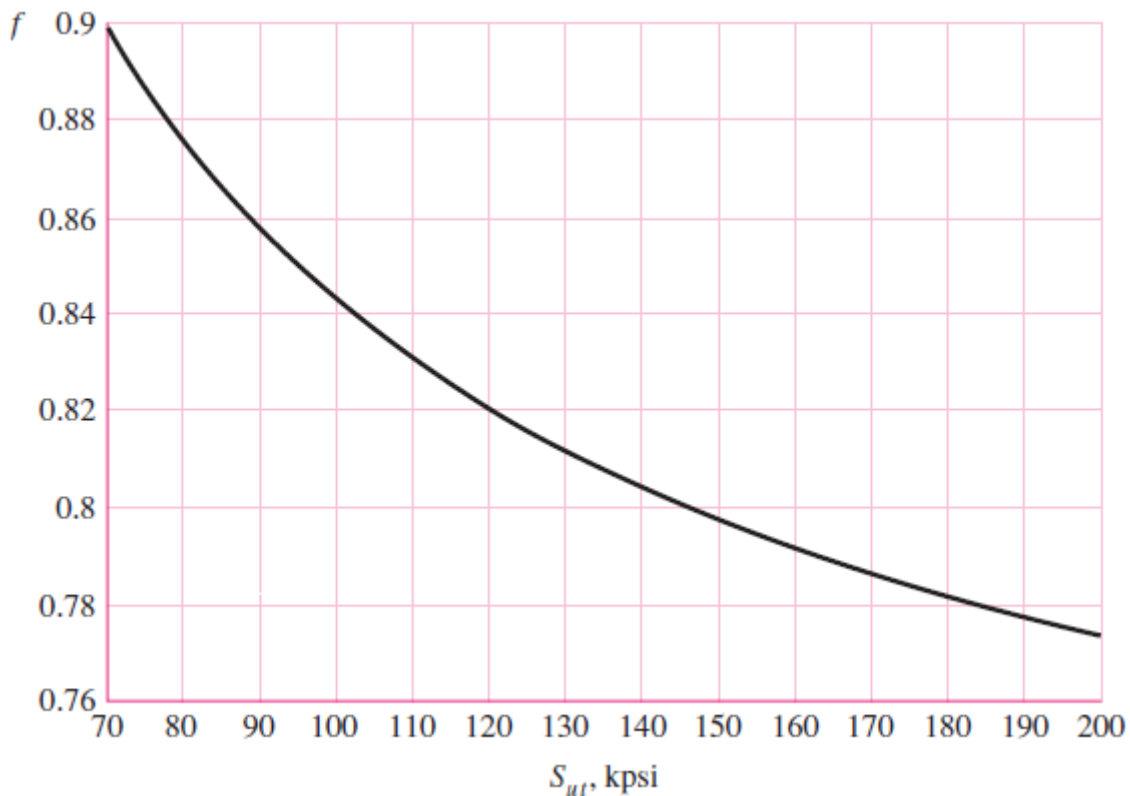
An $S-N$ diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel, normalized; $S_{ut}=116$ kpsi.

The ordinate of the $S-N$ diagram is called the *fatigue strength* S_f ; a statement of this strength value must always be accompanied by a statement of the number of cycles N to which it corresponds.

In the case of the steels, a knee occurs in the graph, and beyond this knee failure will not occur, no matter how great the number of cycles. The strength corresponding to the knee is called the *endurance limit* (S_e), or the *fatigue limit*. The graph of Fig. (3-1) never does become horizontal for nonferrous metals and alloys, and hence these materials do not have an endurance limit.

The body of knowledge available on fatigue failure from $N = 1$ to $N = 1000$ cycles is generally classified as *low-cycle fatigue*, as indicated in Fig. (3-1). *High-cycle fatigue*, then, is concerned with failure corresponding to stress cycles greater than 10^3 cycles.

Also a *finite-life region* and an *infinite-life region* are distinguished. The boundary between these regions cannot be clearly defined except for a specific material; but it lies somewhere between 10^6 and 10^7 cycles for steels, as shown in the figure.



Fatigue strength fraction, f , of S_{ut} at 103 cycles for $S_e = S'_e = 0.5S_{ut}$.

$$a = \frac{(f S_{ut})^2}{S_e} \quad b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right)$$

If a completely reversed stress σ_a is given, the number of cycles-to-failure can be expressed as

$$N = \left(\frac{\sigma_a}{a} \right)^{1/b}$$

3.2 The endurance Limit

The determination of endurance limits by fatigue testing is now routine, though a lengthy procedure. Generally, stress testing is preferred to strain testing for endurance limits.

There are great quantities of data in the literature on the results of rotating-beam tests and simple tension tests of specimens taken from the same bar or ingot. The endurance limit ranges from about 40 to 60 percent of the tensile strength for steels up to about 210 kpsi (1450 MPa). For steels, the endurance limit may be estimated as

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \quad 3-1$$

where S_{ut} is the *minimum* tensile strength. The prime mark on S'_e in this equation refers to the *rotating-beam specimen*.

When designs include detailed heat-treating specifications to obtain specific microstructures, it is possible to use an estimate of the endurance limit based on test data for the particular microstructure; such estimates are much more reliable and indeed should be used.

3.3 Endurance Limit Modifying Factors

Joseph Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. A Marin equation is written as

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \quad 3-2$$

Where

k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

When endurance tests of parts are not available, estimations are made by applying Marin factors to the endurance limit.

➤ Surface Factor k_a

$$k_a = a S_{ut}^b \quad 3-3$$

where S_{ut} is the minimum tensile strength and a and b are to be found in the following table.

Table (3–1)
Parameters for Marin surface modification factor, Eq. (3–3)

Surface finish	Factor a		Exponent b
	S_{ut} , kpsi	S_{ut} , MPa	
<i>Ground</i>	1.34	1.58	−0.085
<i>Machined or cold-drawn</i>	2.7	4.51	−0.265
<i>Hot-rolled</i>	14.4	57.7	−0.718
<i>As-forged</i>	39.9	272	−0.995

EXAMPLE 3-1

A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a .

Solution

From Table (3-1), $a = 4.51$ and $b = -0.265$. Then, from Eq. (3-3)

$$k_a = 4.51(520)^{-0.265} = 0.860 \quad \text{Ans.}$$

Again, it is important to note that this is an approximation as the data is typically quite scattered. Furthermore, this is not a correction to take lightly. For example, if in the previous example the steel was forged, the correction factor would be 0.540, a significant reduction of strength.

➤ Size Factor k_b

For round shafts in bending and torsion when rotating, k_b may be expressed as

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases} \quad 3-4$$

The effective size of a round corresponding to a non-rotating solid or hollow round,

$$d_e = 0.37d \quad 3-5$$

For a rectangular section of dimensions $h \times b$

$$d_e = 0.808(hb)^{1/2} \quad 3-6$$

For axial loading there is no size effect, so

$$k_b = 1 \quad 3-7$$

EXAMPLE 3–2

A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor k_b if the shaft is used in

- (a) A rotating mode.
- (b) A non-rotating mode.

Solution

(a) From Eq. (3–4)

$$k_b = (d/7.62)^{-0.107} = (32/7.62)^{-0.107} = 0.858 \quad \text{Ans.}$$

(b) From Eq. (3–5),

$$d_e = 0.37d = 0.37(32) = 11.84 \text{ mm}$$

Then, from Eq. (3–4)

$$k_b = (d/7.62)^{-0.107} = (11.84/7.62)^{-0.107} = 0.954 \quad \text{Ans.}$$

➤ *Loading Factor k_c*

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with S_{ur} . The average values of the load factor are specified as

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^* \end{cases} \quad 3-8$$

* The latter is used only for pure torsional fatigue loading. When torsion is combined with other stresses, such as bending, $k_c = 1$.

➤ *Temperature Factor k_d*

When operating temperatures are below room temperature, brittle fracture is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, yielding should be investigated first because the yield strength drops

off so rapidly with temperature; see Fig. (3-2). Any stress will induce creep in a material operating at high temperatures; so this factor must be considered too.

Finally, it may be true that there is no fatigue limit for materials operating at high temperatures. Because of the reduced fatigue resistance, the failure process is, to some extent, dependent on time.

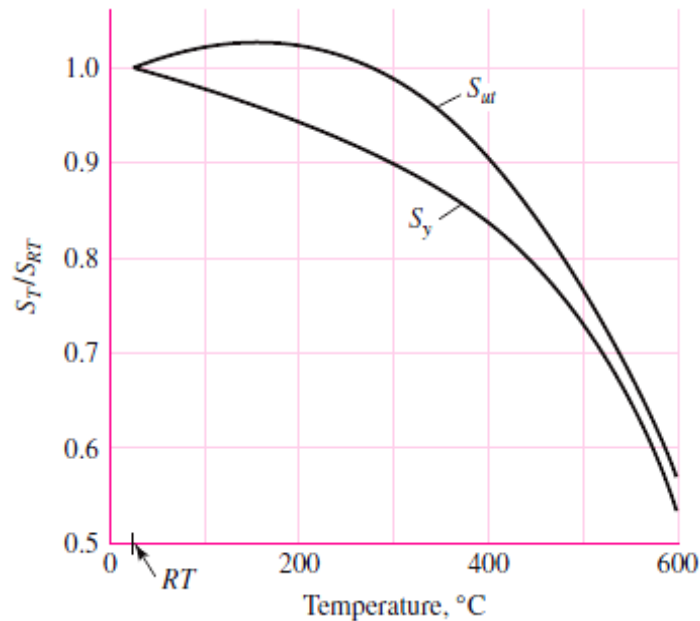


Figure (3-2)

A plot of the results of 145 tests of 21 carbon and alloy steels showing the effect of operating temperature on the yield strength S_y and the ultimate strength S_{ut} . The ordinate is the ratio of the strength at the operating temperature (S_T) to the strength at room temperature (S_{RT}).

Table (3-2) has been obtained from Fig. (3-2) by using only the tensile-strength data. Note that the table represents 145 tests of 21 different carbon and alloy steels. A fourth-order polynomial curve fit to the data underlying Fig. (3-2) gives

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

where $70 \leq T_F \leq 1000$ F.

Table (3–2)

Effect of operating temperature on the tensile strength of steel. (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature)

Temperature °C	S_T/S_{RT}	Temperature °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549	<i>Data source : Fig. (3–2)</i>	

Two types of problems arise when temperature is a consideration. If the rotating beam endurance limit is known at room temperature, then use

$$k_d = S_T/S_{RT} \qquad 3-10$$

from Table (3–2) or Eq. (3–9) and proceed as usual. If the rotating-beam endurance limit is not given, then compute it using Eq. (3–1) and the temperature-corrected tensile strength obtained by using the factor from Table (3–2). Then use $k_d = 1$.

EXAMPLE 3-3

A 1035 steel has a tensile strength of 70 kpsi and is to be used for a part that sees 450°F in service. Estimate the Marin temperature modification factor and $(S_e)_{450^\circ}$ if

- (a) The room-temperature endurance limit by test is $(S'_e)_{70^\circ} = 39$ kpsi
- (b) Only the tensile strength at room temperature is known

Solution

- (a) First, from Eq. (3-9),

$$k_d = 0.975 + 0.432(10^{-3})(450) - 0.115(10^{-5})(450^2) + 0.104(10^{-8})(450^3) - 0.595(10^{-12})(450^4) = 1.007$$

Thus,

$$(S_e)_{450^\circ} = k_d (S'_e)_{70^\circ} = 1.007(39) = 39.3 \text{ kpsi} \quad \text{Ans.}$$

- (b) Interpolating from Table (3-2) gives

$$(S_T/S_{RT})_{450^\circ} = 1.018 + (0.995 - 1.018) \frac{450 - 400}{500 - 400} = 1.007$$

Thus, the tensile strength at 450°F is estimated as

$$(S_{ut})_{450^\circ} = (S_T/S_{RT})_{450^\circ} (S_{ut})_{70^\circ} = 1.007(70) = 70.5 \text{ kpsi}$$

From Eq. (3-1) then,

$$(S_e)_{450^\circ} = 0.5 (S_{ut})_{450^\circ} = 0.5(70.5) = 35.2 \text{ kpsi}$$

Part *a* gives the better estimate due to actual testing of the particular material.

➤ **Reliability Factor k_e**

The reliability modification factor can be determined from the following table.

Table (3–3)

Reliability factors k_e corresponding to 8 percent standard deviation of the endurance limit

Reliability,%	Reliability factors k_e
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620
99.99999	0.584

➤ **Miscellaneous-Effects Factor k_f**

Though the factor k_f is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for, because actual values of k_f are not always available.

Residual stresses may either improve the endurance limit or affect it adversely. Generally, if the residual stress in the surface of the part is compression, the endurance limit is improved. Fatigue failures appear to be tensile failures, or at least to be caused by tensile stress, and so anything that reduces tensile stress will also reduce the possibility of a fatigue failure. Operations such as shot peening, hammering, and cold rolling build compressive stresses into the surface of the part and improve the endurance limit significantly. Of course, the material must not be worked to exhaustion. The endurance limits of parts that are made from rolled or drawn sheets or bars, as well as parts that are forged, may be affected by the so-called *directional characteristics* of the operation. Rolled or drawn parts, for example, have an endurance limit in the transverse direction that may be 10 to 20 percent less than the endurance limit in the longitudinal direction.

Corrosion, electrolytic plating, metal spraying, cyclic frequency and frottage corrosion may also have an effect on the endurance limit.

3.4 Stress Concentration and Notch Sensitivity

It turns out that some materials are not fully sensitive to the presence of notches and hence, for these, a reduced value of K_t can be used. For these materials, the maximum stress is, in fact,

$$\sigma_{\max} = K_f \sigma_o \quad \text{or} \quad \tau_{\max} = K_{fs} \tau_o \quad 3-11$$

where K_f is a reduced value of K_t and σ_o is the nominal stress. The factor K_f is commonly called a *fatigue stress-concentration factor*, and hence the subscript f . So it is convenient to think of K_f as a stress-concentration factor reduced from K_t because of lessened sensitivity to notches. The resulting factor is defined by the equation

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

by

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

Notch sensitivity q is defined
the equation

where q is usually between zero and unity. Equation (2–12) shows that if $q = 0$, then $K_f = 1$, and the material has no sensitivity to notches at all. On the other hand, if $q = 1$, then $K_f = K_t$, and the material has full notch sensitivity. In analysis or design work, find K_t first, from the geometry of the part. Then specify the material, find q , and solve for K_f from the equation

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \quad 3-13$$

For steels and 2024 aluminum alloys, use Fig. (3–3) to find q for bending and axial loading. For shear loading, use Fig. (3–4).

The notch sensitivity of the cast irons is very low, varying

from 0 to about 0.2, depending upon the tensile strength. To be on the conservative side,

$$q = 0.2 \quad \text{for all grades of cast iron}$$

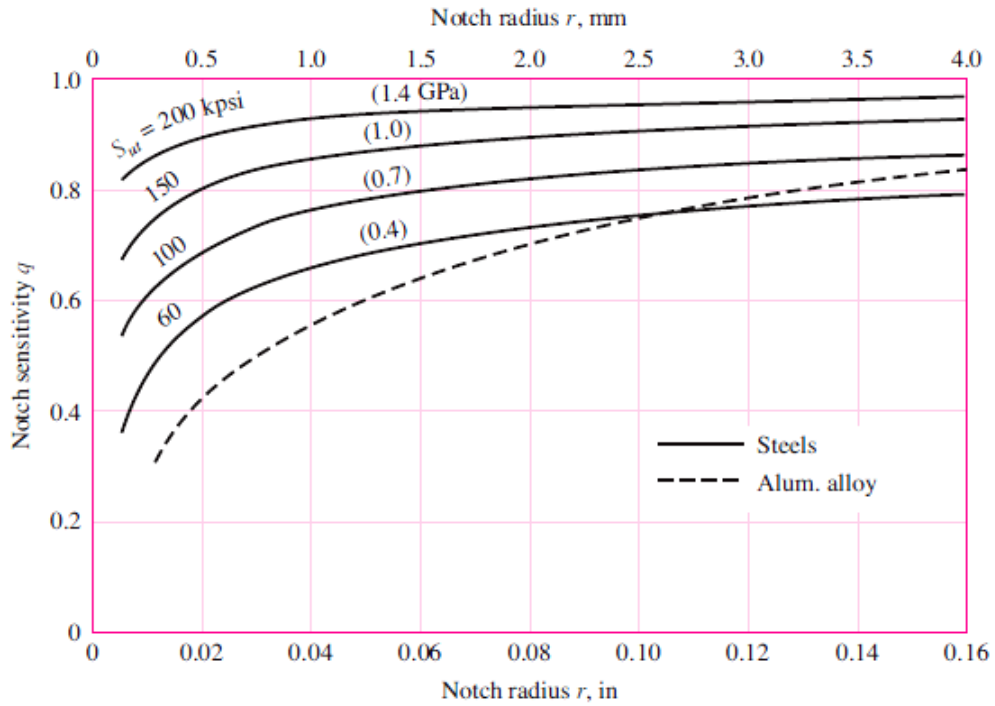


Figure (3-3)

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values

of q corresponding to the $r = 0.16$ -in (4-mm)

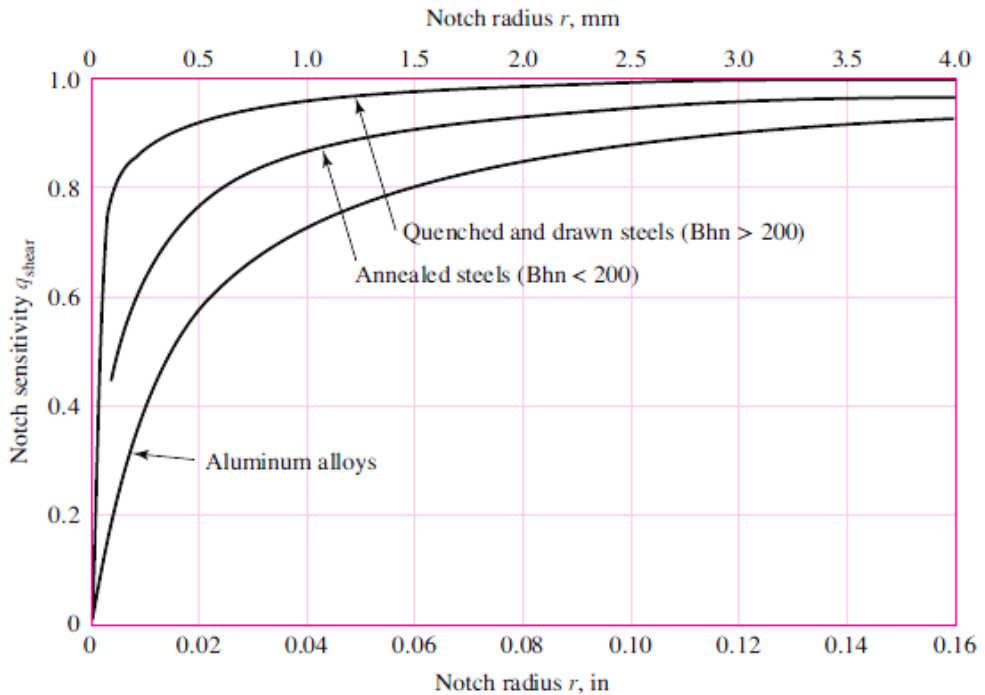


Figure (3-4)

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ -in (4-mm)

EXAMPLE 3–4

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f .

Solution

From Fig. (1–16), using $D/d = 38/32 = 1.1875$, $r/d = 3/32 = 0.09375$, we read the graph to find ($K_t = 1.65$)

From Fig. (3–3), for $S_{ut} = 690$ MPa and $r = 3$ mm, ($q = 0.84$). Thus, from Eq. (3–13)

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55 \quad \text{Ans.}$$

EXAMPLE 3–5

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit.

Solution

From Table (3–4), $S_{ut} = 50$ kpsi at 70°F. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Table (3–2):

$$(S_T/S_{RT})_{550^\circ} = (0.995 + 0.963)/2 = 0.979$$

The ultimate strength at 550°F is then

$$(S_{ut})_{550^\circ} = (S_T/S_{RT})_{550^\circ} (S_{ut})_{70^\circ} = 0.979(50) = 49 \text{ kpsi}$$

The rotating-beam specimen endurance limit at 550°F is then estimated from Eq. (3–1) as

$$S'_e = 0.5(49) = 24.5 \text{ kpsi}$$

Next, we determine the Marin factors. For the machined surface, Eq. (3–3) with Table (3–1) gives

$$k = aS^b = 2.70(49)^{-0.265} = 0.963$$

a *ut*

For axial loading, from Eq. (3–7), the size factor $k_b = 1$, and from Eq. (3–8) the loading factor is $k_c = 0.85$. The temperature factor $k_d = 1$, since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table (3–3), $k_e = 0.814$. Finally, since no other conditions were given, the miscellaneous factor is $k_f = 1$. The endurance limit for the part is estimated by Eq. (3–2) as

$$S_e = k_a k_b k_c k_d k_e k_f S'_e = 0.963(1)(0.85)(1)(0.814)(1)24.5 = 16.3 \text{ kpsi} \quad \text{Ans.}$$

Table (3–4)

Deterministic ASTM minimum tensile and yield strengths for some hot-rolled (HR) and cold-drawn (CD) steels. [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm (3/4 to 1 1/4 in). These strengths are suitable for use with the design factor, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications]

UNS No.	SAE and/or AISI No.	Processing	Tensile Strength, MPa (kpsi)	Yield Strength, MPa (kpsi)	Elongation in 2 in, %	Reduction in Area, %	Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

EXAMPLE 3-6

Figure (3-5a) shows a rotating shaft simply supported in ball bearings at A and D and loaded by a non-rotating force F of 6.8 kN. Using ASTM “minimum” strengths, estimate the endurance limit and the reversing bending stress.

Solution

From Fig. (3-5b) we learn that failure will probably occur at B rather than at C or at the point of maximum moment. Point B has a smaller cross section, a higher bending moment, and a higher stress-concentration factor than C , and the location of maximum moment has a larger size and no stress-concentration factor.

We shall solve the problem by first estimating the strength at point B , since the strength will be different elsewhere, and comparing this strength with the stress at the same point.

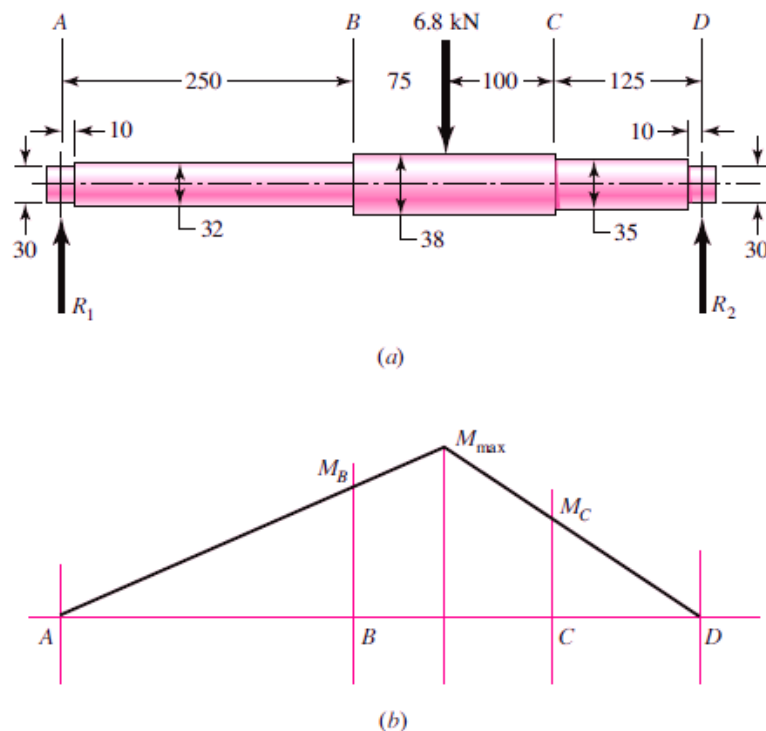


Figure (3-5)

(a) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel.

(b) Bending-moment diagram.

From Table (3–4) we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa.

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

$$k_a = 4.51(690)^{-0.265} = 0.798$$

$$k_b = (32/7.62)^{-0.107} = 0.858$$

$$k_c = k_d = k_e = k_f = 1$$

Then, $S_e = 0.798(0.858)345 = 236$ MPa *Ans.*

Same as Example (3–4), $K_f = 1.55$

The next step is to estimate the bending stress at point *B*. The bending moment is $M_B = 695.5$ N·m

Then, the reversing bending stress is,

$$\frac{M_B c}{I} = 335.1 \text{ MPa} \quad \text{Ans.}$$

This stress is greater than S_e and less than S_y . This means we have both finite life and no yielding on the first cycle
For finite life, The ultimate strength, $S_{ut} = 690 \text{ MPa} = 100 \text{ kpsi}$.
From Fig. 6–18, $f = 0.844$.

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

$$b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left[\frac{0.844(690)}{236} \right] = -0.1308$$

$$N = \left(\frac{\sigma_a}{a} \right)^{1/b} = \left(\frac{335.1}{1437} \right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$