3.1 Characterizing Fluctuating Stresses

Fluctuating stresses in machinery often take the form of a sinusoidal pattern because of the nature of some rotating machinery. However, other patterns, some quite irregular, do occur. It has been found that in periodic patterns exhibiting a single maximum and a single minimum of force, the shape of the wave is not important, but the peaks on both the high side (maximum) and the low side (minimum) are important. Thus F_{max} and F_{min} in a cycle of force can be used to characterize the force pattern. It is also true that ranging above and below some baseline can be equally effective in characterizing the force pattern. If the largest force is F_{max} and the smallest force is F_{min}, then a steady component and an alternating component can be constructed as follows:

$$F_m = \frac{F_{max} + F_{min}}{2} \qquad F_a = \left| \frac{F_{max} - F_{min}}{2} \right|$$

where F_m is the midrange steady component of force, and F_a is the amplitude of the alternating component of force.

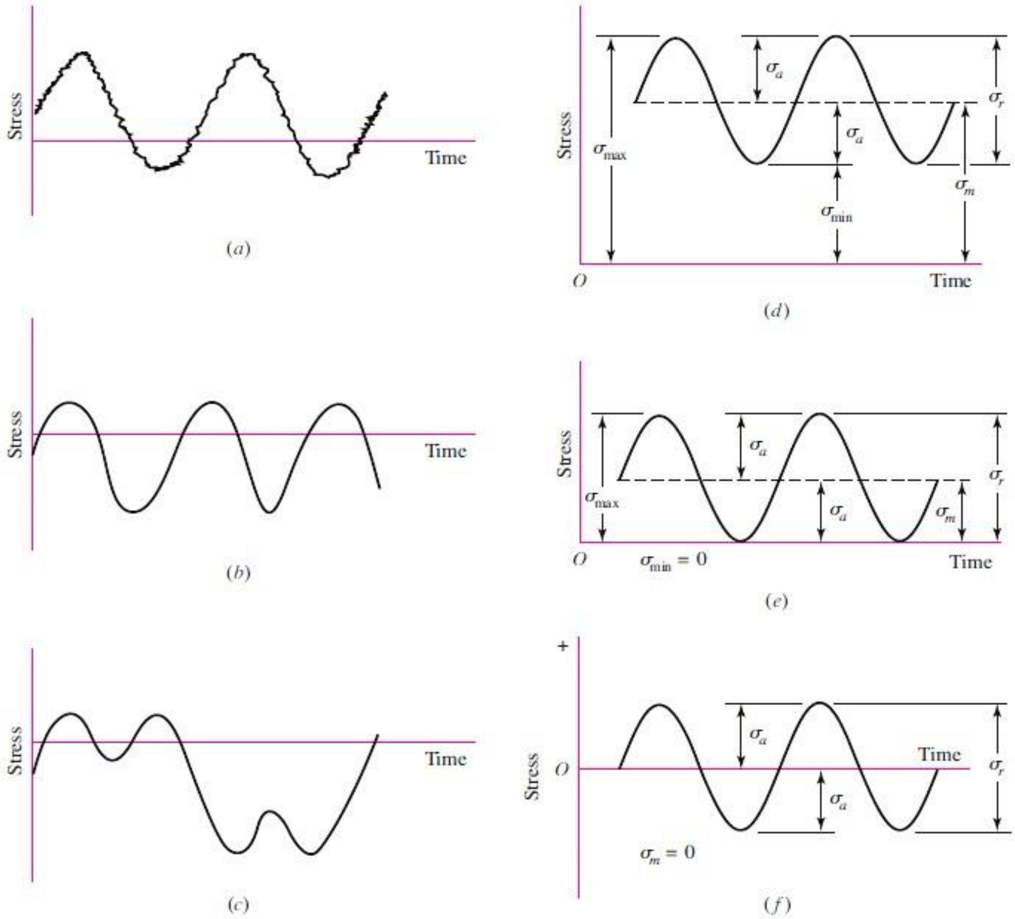


Figure (3–6)

Some stress-time relations: (*a*) fluctuating stress with high- frequency ripple; (*b* and *c*) non-sinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.

Figure (3–6) illustrates some of the various stress-time traces that occur. The components of stress, some of which are shown in Fig. (3–6*d*), are

σ_{min} = minimum stress
σ_{max} = maximum stress
σ_a = amplitude component
σ_m = midrange component
σ_r = range of stress
σ_s = static or steady stress

The steady, or static, stress is *not* the same as the midrange stress; in fact, it may have any value between σ_{min} and σ_{max}. The steady stress exists because of a fixed load or preload applied to the part, and it is usually independent of the varying portion of the load. A helical compression spring, for example, is always loaded into a space shorter than the free length of the spring. The stress created by this initial compression is called the steady, or static, component of the stress. It is not the same as the midrange stress. We shall have occasion to apply the subscripts of these components to shear stresses as well as normal stresses.

The following relations are evident from Fig. (3–6):

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right| \qquad 3\text{-}\ 14$$

In addition to Eq. (3–14), the *stress ratio*

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad 3\text{-}15$$

and the *amplitude ratio*

$$A = \frac{\sigma_a}{\sigma_m} \qquad 3\text{-}16$$

are also defined and used in connection with fluctuating stresses.

Equations (3–14) utilize symbols σ_a and σ_m as the stress components at the location under scrutiny. This means, in the absence of a notch, σ_a and σ_m are equal to the nominal stresses σ_{ao} and σ_{mo} induced by loads F_a and F_m, respectively; in the presence of a notch they are ($K_f\,\sigma_{ao}$) and ($K_f\,\sigma_{mo}$), respectively, as long as the material remains without plastic strain. In other words, the fatigue stress concentration factor K_f is applied to *both* components.

3.2 Fatigue Failure Criteria for Fluctuating Stress

Five criteria of failure are diagrammed in Fig. (3–7): the *Soderberg*, the *modified Goodman*, the *Gerber*, the *ASME-elliptic*, and *yielding*. The diagram shows that only the Soderberg criterion guards against any yielding, but is biased low.

Considering the modified Goodman line as a criterion, point *A* represents a limiting point with an alternating strength S_a and midrange strength S_m. The slope of the load line shown is defined as $r = S_a/S_m$.

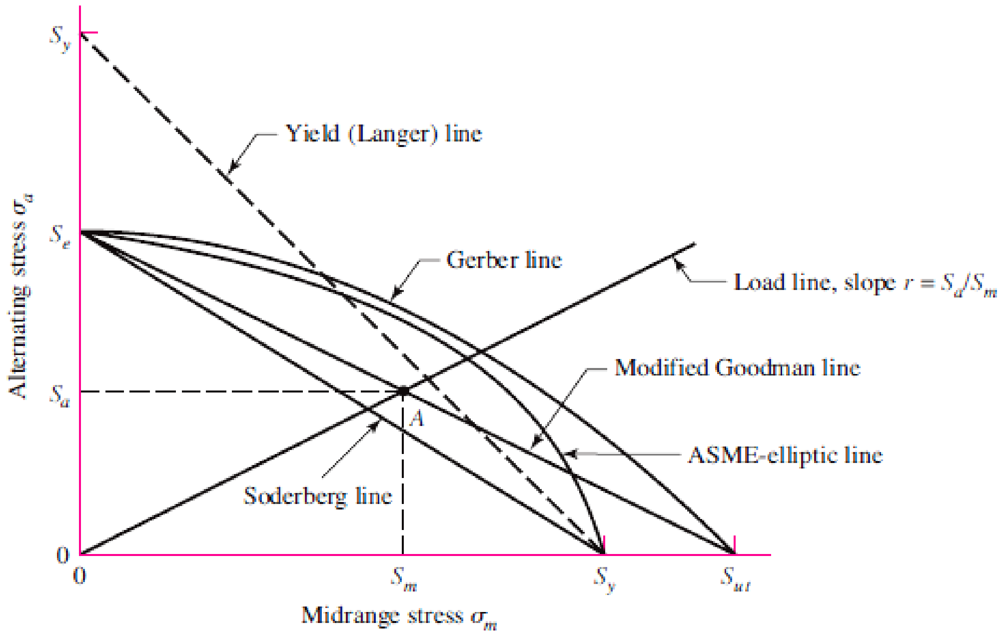


Figure (3–7)

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point *A* on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.

The line equations for every criterion are :

Soderberg line is: $\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$ 3-17

modified Goodman : $\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ 3-18

The Gerber failure criterion is: $\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ 3-19

ASME-elliptic is : $\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ 3-20

The Langer first-cycle-yielding criterion is used in connection with the fatigue curve:

$$S_a + S_m = S_y$$

3-21

The stresses $n\sigma_a$ and $n\sigma_m$ can replace S_a and S_m, where n is the design factor or factor of safety. Then, the last Eqs. become

Soderberg: $$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1 \qquad \text{3-22}$$

mod-Goodman : $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad \text{3-23}$$

Gerber : $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad \text{3-24}$$

ASME-elliptic: $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad \text{3-25}$$

Langer static yield: $$\sigma_a + \sigma_m = \frac{S_y}{n} \qquad \text{3-26}$$

The Gerber and ASME-elliptic are emphasized for fatigue failure criterion and the Langer for first-cycle yielding. Conservative designers often use the modified Goodman criterion.

The failure criteria are used in conjunction with a load line, $r = S_a/S_m = \sigma_a/\sigma_m$. Principal intersections are tabulated in Tables (3–5 to 3–7). Formal expressions for fatigue factor of safety are given in the lower panel of Tables (3–5 to 3–7). The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and fatigue criteria. The first column gives the intersecting equations and the second column the intersection coordinates.

There are two ways to proceed with a typical analysis. One method is to assume that fatigue occurs first and use one of Eqs. (3–22) to (3–25) to determine n or size, depending on the task. Most often fatigue is the governing failure mode. Then follow with a static check. If static failure governs then the analysis is repeated using Eq. (3–6). Alternatively, one could use the tables. Determine the load line and establish which criterion the load line intersects first and use the corresponding equations in the tables. Some examples will help solidify the ideas just discussed.

Table (3–5)

Amplitude and steady coordinates of strength and important intersections in first quadrant for

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r\,S_e S_{ut}}{r\,S_{ut} + S_e}$ $S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r\,S_y}{1+r}$ $S_m = \frac{S_y}{1+r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = \frac{(S_y - S_e)\,S_{ut}}{S_{ut} - S_e}$ $S_a = S_y - S_m,\ r_{crit} = S_a/S_m$
Fatigue factor of safety $n_f = \dfrac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$	

modified Goodman and ***Langer*** failure criteria

Table (3–6)

Amplitude and steady coordinates of strength and important intersections in first quadrant for

Gerber and ***Langer*** failure criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r\,S_{ut}}\right)^2}\,\right]$ $S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r\,S_y}{1+r}$ $S_m = \frac{S_y}{1+r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\,\right]$ $S_a = S_y - S_m,\ r_{crit} = S_a/S_m$
Fatigue factor of safety $n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \quad \sigma_m > 0$	

Table (3–7)

Amplitude and steady coordinates of strength and important intersections in first quadrant for

ASME-elliptic and ***Langer*** failure criteria

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0, \ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a, \ r_{crit} = S_a/S_m$
Fatigue factor of safety $n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$	

EXAMPLE 3–7

A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Solution

From Table (3–4), S_{ut} = 100 kpsi and S_y = 84 kpsi.
$F_a = F_m$ = 8 kip.

$k_a = 2.7(100)^{-0.265} = 0.797$
k_b = 1 (axial loading)
$k_c = 0.85$

$k_d = k_e = k_f = 1$

$S_e = 0.797(1)0.85(1)(1)(1)0.5(100) = 33.9$ kpsi

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f\, \sigma_{ao} = 1.85(4.53) = 8.38 \text{ kpsi} = \sigma_m$$

(a) Let us calculate the factors of safety first. From the bottom panel from Table (3–6) the factor of safety for fatigue is

$$n_f = \frac{1}{2}\left(\frac{100}{8.38}\right)^2\left(\frac{8.38}{33.9}\right)\left\{-1+\sqrt{1+\left[\frac{2(8.38)33.9}{100(8.38)}\right]^2}\right\} = 3.66$$

From Eq. (3–26) the factor of safety guarding against first-cycle yield is

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{84}{8.38 + 8.38} = 5.01$$

Thus, we see that fatigue will occur first and the factor of safety is (3.68). This can be seen in Fig. (3–8) where the load line intersects the Gerber fatigue curve first at point *B*. If the plots are created to true scale it would be seen that $n_f = OB/OA$.

From the first panel of Table (3–6), $r = \sigma_a/\sigma_m = 1$,

$$S_a = \frac{(1)^2 100^2}{2(33.9)}\left\{-1+\sqrt{1+\left[\frac{2(33.9)}{(1)100}\right]^2}\right\} = 30.7 \text{ kpsi}$$

$$S_m = \frac{S_a}{r} = \frac{30.7}{1} = 30.7 \text{ kpsi}$$

As a check on the previous result,

$n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 30.7/8.38 = 3.66$ and we see total agreement.

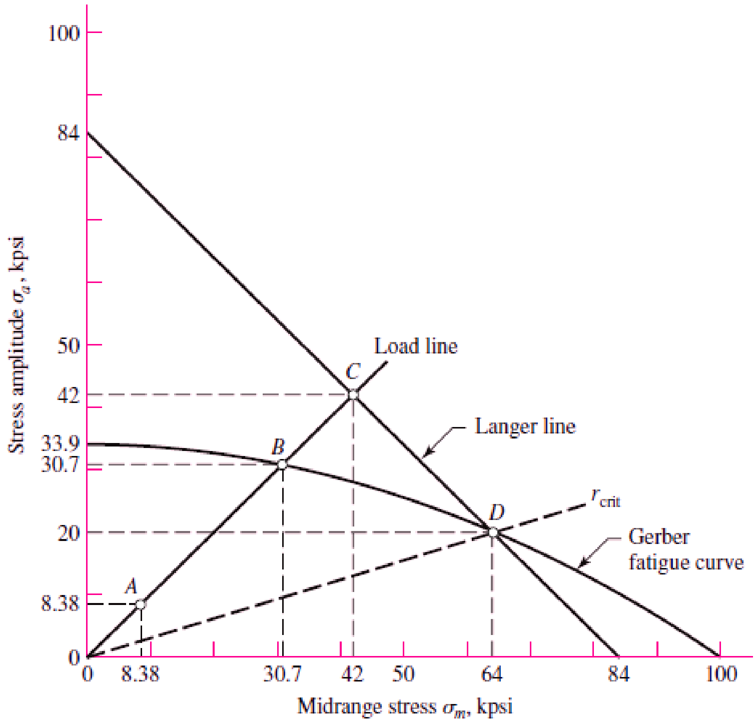


Figure (3–8)

Principal points *A*, *B*, *C*, and *D* on the designer's diagram drawn for ***Gerber***, ***Langer***, and *load line*

We could have detected that fatigue failure would occur first without drawing Fig. (3–8) by calculating r_{crit}. From the third row third column panel of Table (3–6), the intersection point between fatigue and first-cycle yield is

$$S_m = \frac{100^2}{2(33.9)}\left[1 - \sqrt{1 + \left(\frac{2(33.9)}{100}\right)^2\left(1 - \frac{84}{33.9}\right)}\right] = 64.0 \text{ kpsi}$$

$$S_a = S_y - S_m = 84 - 64 = 20 \text{ kpsi}$$

The critical slope is thus

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{20}{64} = 0.312$$

which is less than the actual load line of $r = 1$. This indicates that fatigue occurs before first-cycle-yield.

(b) Repeating the same procedure for the ASME-elliptic line, for fatigue

$$n_f = \sqrt{\frac{1}{(8.38/33.9)^2 + (8.38/84)^2}} = 3.75$$

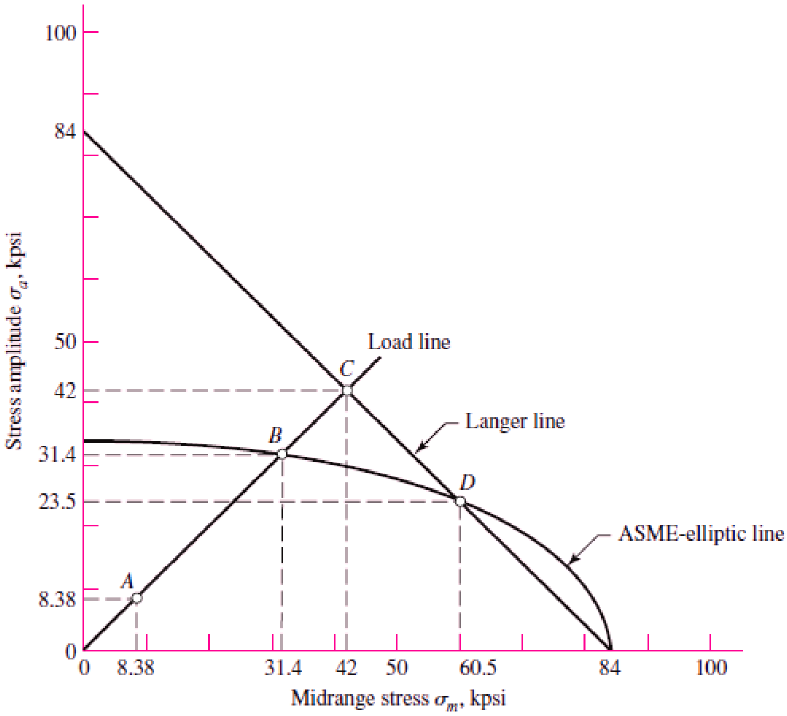


Figure (3–9)

Principal points *A*, *B*, *C*, and *D* on the designer's diagram drawn for

ASME-elliptic, ***Langer***, and *load line*

Again, this is less than $n_y = 5.01$ and fatigue is predicted to occur first. From the first row second column panel of Table (3–7), with $r = 1$, we obtain the coordinates S_a and S_m of point *B* in Fig. (3–9) as

$$S_a = \sqrt{\frac{(1)^2 33.9^2 (84)^2}{33.9^2 + (1)^2 84^2}} = 31.4 \text{ kpsi}, \quad S_m = \frac{S_a}{r} = \frac{31.4}{1} = 31.4 \text{ kpsi}$$

The fatigue factor of safety, $n_f = S_a/\sigma_a = 31.4/8.38 = 3.75$

As before, r_{crit}. From the third row second column panel of Table (3–7),

$$S_a = \frac{2(84)33.9^2}{33.9^2 + 84^2} = 23.5 \text{ kpsi}, \quad S_m = S_y - S_a = 84 - 23.5 = 60.5 \text{ kpsi}$$

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{23.5}{60.5} = 0.388$$

which again is less than $r = 1$, verifying that fatigue occurs first with $n_f = 3.75$.

The Gerber and the ASME-elliptic fatigue failure criteria are very close to each other and are used interchangeably. The ANSI/ASME Standard B106.1M–1985 uses ASME-elliptic for shafting.

For many *brittle* materials, the first quadrant fatigue failure criteria follows a concave upward Smith-Dolan locus represented by

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \qquad \text{3-27}$$

or as a design equation,

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \qquad \text{3-28}$$

For a radial load line of slope r, we substitute S_a/r for S_m in Eq. (3–27) and solve for S_a , obtaining

$$S_a = \frac{rS_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}}\right] \qquad \text{3-29}$$

The most likely domain of designer use is in the range from $-S_{ut} \leq \sigma_m \leq S_{ut}$. The locus in the first quadrant is Goodman, Smith-Dolan, or something in between. The portion of the second quadrant that is used is represented by a straight line between the points $-S_{ut}$, S_{ut} and 0, S_e, which has the equation

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1\right)S_m \qquad -S_{ut} \leq S_m \leq 0 \quad \text{(for cast iron)} \qquad \text{3-30}$$

EXAMPLE 3–8

A grade 30 gray cast iron (S_{ut} = 31 kpsi, S_{uc} = 109 kpsi, $k_a k_b S_e'$ = 14 kpsi, and k_c for axial loading is 0.9) is subjected to a load F applied to a 1 by 3/8 -in cross-section link with a 1/4 -in-diameter hole drilled in the center as depicted in Fig. (3–10*a*). The surfaces are machined. In the neighborhood of the hole, what is the factor of safety guarding against failure under the following conditions:

(a) The load F = 1000 lbf tensile, steady.
(b) The load is 1000 lbf repeatedly applied.
(c) The load fluctuates between (−1000 lbf and 300 lbf) without column action.
Use the Smith-Dolan fatigue locus.

Solution

$S_e = (k_a k_b S'_e)k_c = 14(0.9) = 12.6$ kpsi.

$K_t = 2.45$ (HW)

The notch sensitivity (q) for cast iron is 0.2
$K_f = 1 + q(K_t - 1) = 1 + 0.2(2.45 - 1) = 1.29$

(*a*) $\sigma_a = \frac{K_f F_a}{A} = \frac{1.29(0)}{0.281} = 0 \qquad \sigma_m = \frac{K_f F_m}{A} = \frac{1.29(1000)}{0.281}(10^{-3}) = 4.59 \text{ kpsi}$

then, the factor of safety guarding against failure

$$n = \frac{S_{ut}}{\sigma_m} = \frac{31.0}{4.59} = 6.75$$

$$\sigma_a = \sigma_m = \frac{K_f F_a}{A} = \frac{1.29(500)}{0.281}(10^{-3}) = 2.30 \text{ kpsi}$$

$$r = \frac{\sigma_a}{\sigma_m} = 1$$

(*b*) $F_a = F_m = \frac{F}{2} = \frac{1000}{2} = 500 \text{ lbf}$

From Eq. (3-29)

$$S_a = \frac{(1)31 + 12.6}{2}\left[-1 + \sqrt{1 + \frac{4(1)31(12.6)}{[(1)31 + 12.6]^2}}\right] = 7.63 \text{ kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{7.63}{2.30} = 3.32$$

(*c*) $F_a = \frac{1}{2}|300 - (-1000)| = 650 \text{ lbf} \qquad \sigma_a = \frac{1.29(650)}{0.281}(10^{-3}) = 2.98 \text{ kpsi}$

$$F_m = \frac{1}{2}[300 + (-1000)] = -350 \text{ lbf} \qquad \sigma_m = \frac{1.29(-350)}{0.281}(10^{-3}) = -1.61 \text{ kpsi}$$

$$r = \frac{\sigma_a}{\sigma_m} = \frac{3.0}{-1.61} = -1.86$$

From Eq. (3-30), $S_a = S_e + (S_e/S_{ut} - 1)S_m$ and $S_m = S_a/r$. It follows that

$$S_a = \frac{S_e}{1 - \frac{1}{r}\left(\frac{S_e}{S_{ut}} - 1\right)} = \frac{12.6}{1 - \frac{1}{-1.86}\left(\frac{12.6}{31} - 1\right)} = 18.5 \text{ kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{18.5}{2.98} = 6.20$$

Figure (3–10*b*) shows the portion of the designer's fatigue diagram that was constructed.

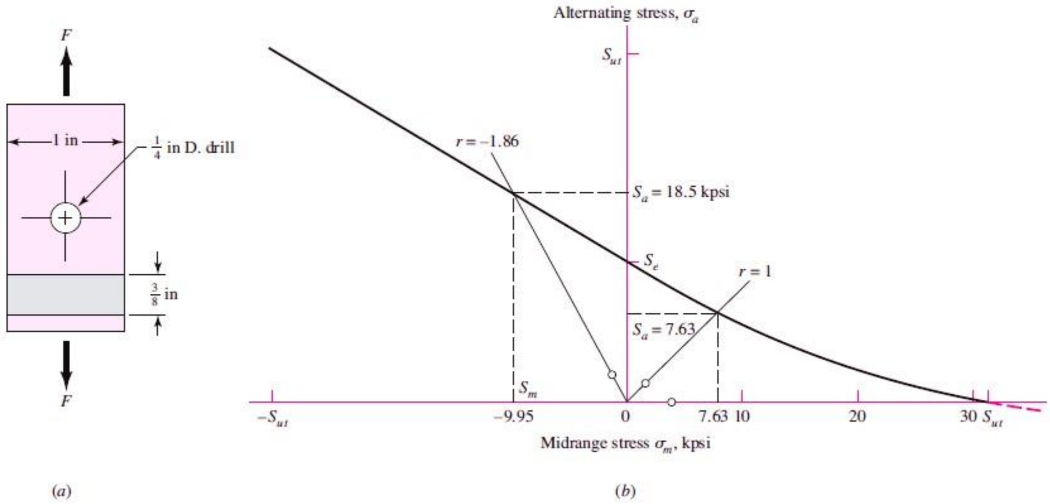


Figure (3–10)

The grade 30 cast-iron part in axial fatigue with (*a*) its geometry displayed and

(*b*) its designer's fatigue diagram

3.3 Torsional Fatigue Strength under Fluctuating Stresses

Use the same equations as apply for $\sigma_m \geq 0$, except replace σ_m and σ_a with τ_m and τ_a , use $k_c = 0.59$ for S_e, replace S_{ut} with $S_{su} = 0.67S_{ut}$, and replace S_y with $S_{sy} = 0.577S_y$.

3.4 Combinations of Loading Modes

It may be helpful to think of fatigue problems as being in three categories:

- Completely reversing simple loads
- Fluctuating simple loads
- *Combinations of loading modes*

For the last one, calculate von Mises stresses for alternating and midrange stress states, σ'_a and σ'_m . When determining S_e , do not use k_c nor divide by K_f or K_{fs} . Apply K_f and/or K_{fs} directly to each specific alternating and midrange stress. If axial stress is present divide the alternating axial stress by k_c = 0.85. For the special case of combined bending, torsional shear, and axial stresses

$$\sigma'_a = \left\{ \left[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial}\frac{(\sigma_a)_{axial}}{0.85} \right]^2 + 3\left[(K_{fs})_{torsion}(\tau_a)_{torsion}\right]^2 \right\}^{1/2}$$

$$\sigma'_m = \left\{ \left[(K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axial}(\sigma_m)_{axial} \right]^2 + 3\left[(K_{fs})_{torsion}(\tau_m)_{torsion}\right]^2 \right\}^{1/2}$$

Then apply stresses to fatigue criterion.

EXAMPLE 3–9

A rotating shaft is made of (42×4 mm) AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

(a) The shaft is subjected to a completely reversed torque of 120 N·m in phase with a completely reversed bending moment of 150 N·m.

(b) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N·m and a steady bending moment of 150 N·m.

Solution

Here we follow the procedure of estimating the strengths and then the stresses, followed by relating the two.

From Table (3–4): S_{ut} = 440 MPa and S_y = 370 MPa.

$$S_e' = 0.5(440) = 220 \text{ MPa.}$$

$$k_a = 4.51(440)^{-0.265} = 0.899$$

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{42}{7.62}\right)^{-0.107} = 0.833$$

The remaining Marin factors are all unity, so the modified endurance strength S_e is

$$S_e = 0.899(0.833)220 = 165 \text{ MPa}$$

(*a*)

K_t = 2.366 for bending; and K_{ts}= 1.75 for torsion (HW)

Thus, for bending,

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4) = \frac{\pi(0.798)}{32(42)}[(42)^4 - (34)^4] = 3.31\ (10^3)\text{mm}^3$$

and for torsion

$$J_{\text{net}} = \frac{\pi A}{32}(D^4 - d^4) = \frac{\pi(0.89)}{32}[(42)^4 - (34)^4] = 155\ (10^3)\text{mm}^4$$

q = 0.78 for bending and q = 0.96 for torsion (HW)

$K_f = 1 + q(K_t - 1) = 1 + 0.78(2.366 - 1) = 2.07$
$K_{fs} = 1 + 0.96(1.75 - 1) = 1.72$

The alternating bending stress is now found to be

$$\sigma_{xa} = K_f \frac{M}{Z_{\text{net}}} = 2.07 \frac{150}{3.31(10^{-6})} = 93.8(10^6)\text{Pa} = 93.8 \text{ MPa}$$

and the alternating torsional stress is

$$\tau_{xya} = K_{fs} \frac{TD}{2J_{\text{net}}} = 1.72 \frac{120(42)(10^{-3})}{2(155)(10^{-9})} = 28.0(10^6)\text{Pa} = 28.0 \text{ MPa}$$

The midrange von Mises component σ'_m is zero. The alternating component σ'_a is given by:

$$\sigma'_a = \left(\sigma_{xa}^2 + 3\tau_{xya}^2\right)^{1/2} = [93.8^2 + 3(28^2)]^{1/2} = 105.6 \text{ MPa}$$

Since $S_e = S_a$, the fatigue factor of safety n_f is

$$n_f = \frac{S_a}{\sigma'_a} = \frac{165}{105.6} = 1.56$$

The first-cycle yield factor of safety is

$$n_y = \frac{S_y}{\sigma'_a} = \frac{370}{105.6} = 3.50$$

This means that there is no localized yielding; so, the threat is from fatigue. See Fig. (3–11).

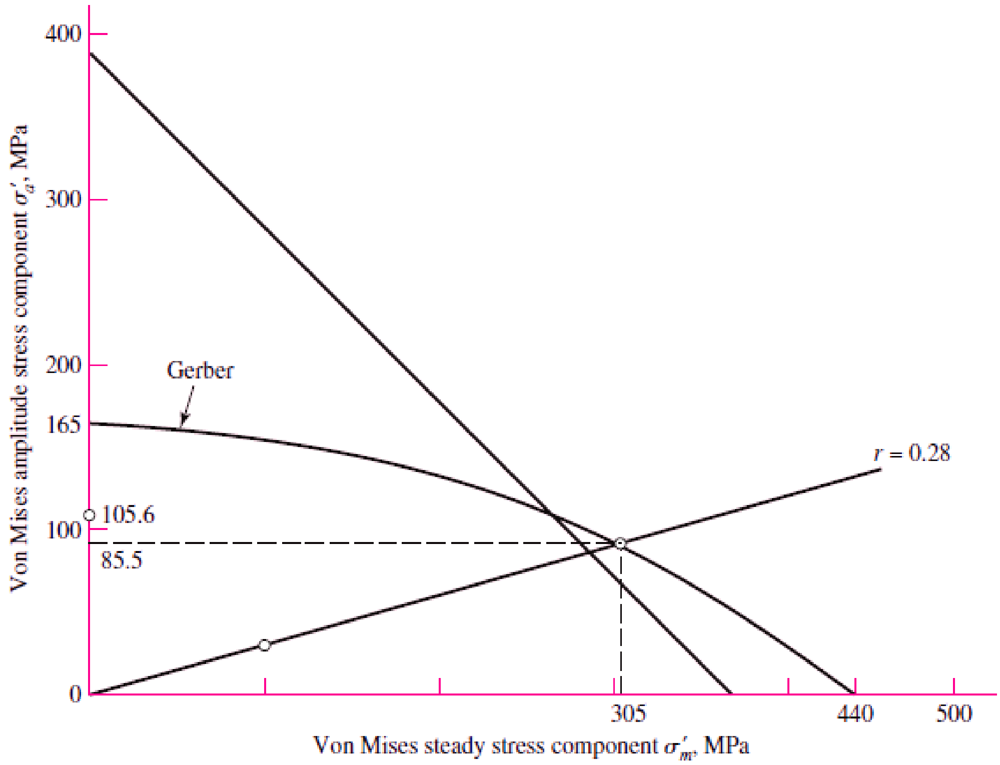


Figure (3–11)

Designer’s fatigue diagram

(b) This part asks us to find the factors of safety when the alternating component is due to pulsating torsion, and a steady component is due to both torsion and bending. We have

$$T_a = (160 - 20)/2 = 70 \text{ N·m and } T_m = (160 + 20)/2 = 90 \text{ N·m}$$

The corresponding amplitude and steady-stress components are

$$\tau_{xya} = K_{fs}\frac{T_a D}{2J_{\text{net}}} = 1.72\frac{70(42)(10^{-3})}{2(155)(10^{-9})} = 16.3(10^6)\text{Pa} = 16.3 \text{ MPa}$$
$$\tau_{xym} = K_{fs}\frac{T_m D}{2J_{\text{net}}} = 1.72\frac{90(42)(10^{-3})}{2(155)(10^{-9})} = 21.0(10^6)\text{Pa} = 21.0 \text{ MPa}$$

The steady bending stress component σ_{xm} is

$$\sigma_{xm} = K_f\frac{M_m}{Z_{\text{net}}} = 2.07\frac{150}{3.31(10^{-6})} = 93.8(10^6)\text{Pa} = 93.8 \text{ MPa}$$

The von Mises components σ'_a and σ'_m are

$\sigma'_a = [3(16.3)^2]^{1/2} = 28.2$ MPa

$\sigma'_m = [(93.82)^2 + 3(21)^2]^{1/2} = 100.6$ MPa

From Table (3–6), the fatigue factor of safety is

$$n_f = \frac{1}{2}\left(\frac{440}{100.6}\right)^2\frac{28.2}{165}\left\{-1 + \sqrt{1 + \left[\frac{2(100.6)165}{440(28.2)}\right]^2}\right\} = 3.03$$

From the same table, with $r = \sigma'_a/\sigma'_m = 28.2/100.6 = 0.28$, the strengths can be shown to be $S_a = 85.5$ MPa and $S_m = 305$ MPa. See the plot in Fig. (3–11).

The first-cycle yield factor of safety n_y is

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{370}{28.2 + 100.6} = 2.87$$

There is no notch yielding. The likelihood of failure may first come from first-cycle yielding at the notch. See the plot in Fig. (3–11).

Homework

(1) A 0.25-in drill rod was heat-treated and ground. Estimate the endurance strength if the rod is used in rotating bending, if S_{ut} = 242.6 kpsi. (*Ans.*/ S_e = 85.7 kpsi)

(2) Estimate S'_e for the following materials:

(a) AISI 1020 CD steel. (*Ans.*/ 34 kpsi)

(b) AISI 1080 HR steel. (*Ans.*/ 56 kpsi)

(c) 2024 T3 aluminum. (*Ans.*/ no endurance limit)

(d) AISI 4340 steel heat-treated to a tensile strength of 250 kpsi. (*Ans.*/ 100 kpsi)

(3) Estimate the endurance strength of a 32-mm-diameter rod of AISI 1035 steel having a machined finish and heat-treated to a tensile strength of 710 MPa. (*Ans.*/ S_e = 241 kpsi)

(4) Two steels are being considered for manufacture of as-forged connecting rods. One is AISI 4340 Cr-Mo-Ni steel capable of being heat-treated to a tensile strength of 260 kpsi. The other is a plain carbon steel AISI 1040 with an attainable S_{ut} of 113 kpsi. If each rod is to have a size giving an equivalent diameter d_e of 0.75-in, is there any advantage to use the alloy steel for fatigue application?
(*Ans.*/ S_e = 14.3 and 18.6 kpsi. Not only is AISI 1040 steel a contender, it has a superior endurance strength. Can you see why?)

(5) A rectangular bar is cut from an AISI 1018 cold-drawn steel flat. The bar is 60 mm wide by 10 mm thick and has a 12-mm hole drilled through the center as depicted in Fig. (1-8). The bar is concentrically loaded in push-pull fatigue by axial forces F_a, uniformly distributed across the width. Using a design factor of 1.8, estimate the largest force F_a that can be applied ignoring column action. (*Ans.*/ Largest force amplitude is 20.1 kN)

(6) A bar of steel has the minimum properties S_e = 276 MPa, S_y = 413 MPa, and S_{ut} = 551 MPa. The bar is subjected to a steady torsional stress of 103 MPa and an alternating bending stress of 172 MPa. Find the factor of safety guarding against a static failure, and the factor of safety guarding against a fatigue failure. For the fatigue analysis use: (*a*) Modified Goodman criterion. (*b*) Gerber criterion. (*c*) ASME-elliptic criterion. (*Ans.*/ n_y = 1.67, n_f = 1.06, 1.31, 1.32)

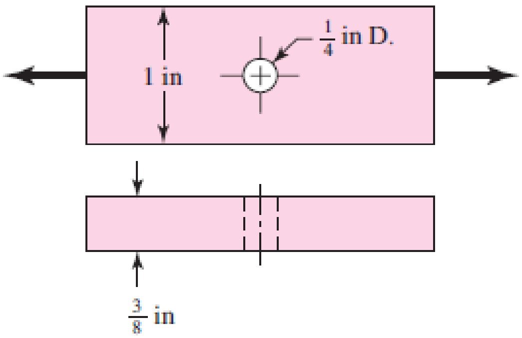
(7) Repeat question (*6*) but with a steady torsional stress of 138 MPa and an alternating bending stress of 69 MPa. (*Ans./* n_y = 1.66, n_f = 1.46, 1.73, 1.59)

(8) Repeat question (*6*) but with a steady torsional stress of 103 MPa, an alternating torsional stress of 69 MPa, and an alternating bending stress of 83 MPa. (*Ans./* n_y = 1.34, n_f= 1.18, 1.47, 1.47)

(9) Repeat question (*6*) but with an alternating torsional stress of 207 MPa. (*Ans./* n_y = 1.15, n_f = 0.77, 0.77, 0.77)

(10) Repeat question (*6*) but with an alternating torsional stress of 103 MPa and a steady bending stress of 103MPa.(*Ans./* n_y = 2, n_f = 1.2, 1.44, 1.44)

(11) The cold-drawn AISI 1018 steel bar shown in the figure is subjected to an axial load fluctuating between 800 and 3000 lbf. Estimate the factors of safety n_y and n_f using (*a*) a Gerber fatigue failure criterion, and (*b*) an ASME-elliptic fatigue failure criterion.

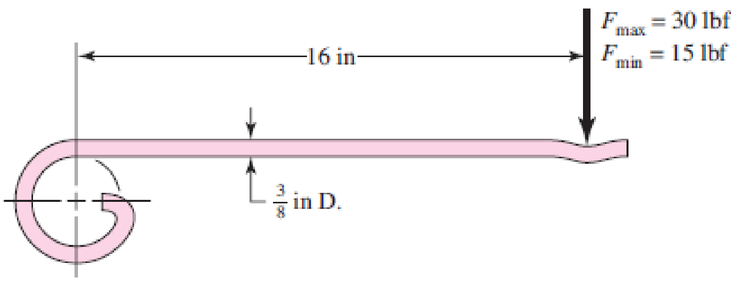


(*Ans./* n_f = 2.17, 2.28)

(12) Repeat question (*11*), with the load fluctuating between −800 and 3000 lbf. Assume no buckling. (*Ans./* n_f = 1.6, 1.62)

(13) Repeat question (*11*), with the load fluctuating between 800 and −3000 lbf. Assume no buckling. (*Ans./* n_f = 1.67, 1.67)

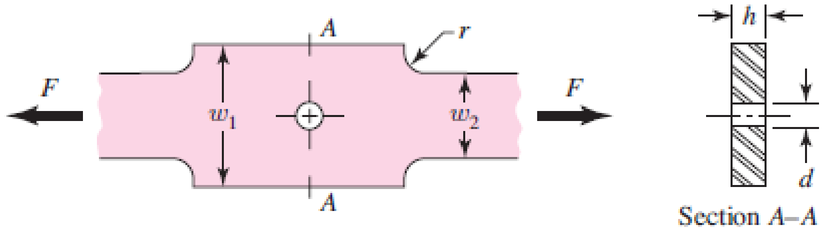
(14) The figure shows a formed round-wire cantilever spring (S_{ut} = 188.1 kpsi) subjected to a varying force. It is apparent from the



mounting details that there is no stress concentration. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. Estimate the factors of safety n_y and n_f using (*a*) Modified Goodman criterion, and (*b*) Gerber criterion.

(*Ans.*/ n_f = 0.955, 1.2)

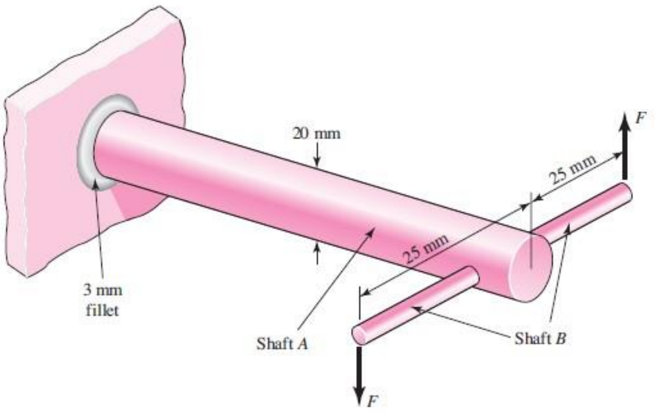
(15) The figure shows the free-body diagram of a connecting-link portion having stress concentration at three sections. The dimensions are r = 0.25 in, d = 0.75 in, h = 0.5 in, w_1 = 3.75 in, and w_2 = 2.5 in.



The forces F fluctuate between a tension of 4 kip and a compression of 16 kip. Neglect column action and find the least factor of safety if the material is cold-drawn AISI 1018 steel.

(*Ans.*/ *Fillet*: n_y = 4.22, n_f = 1.61; *Hole*: n_y = 5.06, n_f = 1.61; then: n = 1.61)

(16) In the figure shown, shaft *A*, made of AISI 1010 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces *F* via shaft *B*. A fatigue stress concentration K_{fs} of
1.6 is induced by the 3-mm fillet. The length of shaft *A* from the fixed support to the connection at shaft *B* is 1 m. The load *F* cycles from 0.5 to 2 kN.
(a) For shaft A, find the factor of safety for infinite life using the modified Goodman fatigue failure criterion.
(b) Repeat part (*a*) using the Gerber fatigue failure criterion.



(*Ans*./ n_f = 1.36, 1.7)