1. Shafts and Shaft Components

A *shaft* is a rotating member, usually of circular cross section, used to transmit power or motion. It provides the axis of rotation, or oscillation, of elements such as gears, pulleys, flywheels, cranks, sprockets, and the like and controls the geometry of their motion. An *axle* is a nonrotating member that carries no torque and is used to support rotating wheels, pulleys, and the like. A non-rotating axle can readily be designed and analyzed as a static beam, and will not subject to fatigue loading.

4.1 Shaft Materials

Necessary strength to resist loading stresses affects the choice of materials and their treatments. Many shafts are made from low carbon, cold-drawn or hot-rolled steel, such as ANSI 1020-1050 steels. Cold drawn steel is usually used for diameters under about 3 inches. The nominal diameter of the bar can be left unmachined in areas that do not require fitting of components. Hot rolled steel should be machined all over. For large shafts requiring much material removal, the residual stresses may tend to cause warping. If concentricity is important, it may be necessary to rough machine, then heat treat to remove residual stresses and increase the strength, then finish machine to the final dimensions.

4.2 Shaft Layout

The general layout of a shaft to accommodate shaft elements, e.g. gears, bearings, and pulleys, must be specified early in the design process in order to perform a free body force analysis and to obtain shear-moment diagrams. The geometry of a shaft is generally that of a stepped cylinder. The use of shaft shoulders is an excellent means of axially locating the shaft elements and to carry any thrust loads. Figure (4–1) shows an example of a stepped shaft supporting the gear of a worm-gear speed reducer. Each shoulder in the shaft serves a specific purpose, which you should attempt to determine by observation.

The geometric configuration of a shaft to be designed is often simply a revision of existing models in which a limited number of changes must be made. If there is no existing design to use as a starter, then the determination of the shaft layout may have many solutions. This problem is illustrated by the two examples of Fig. (4–2). In Fig. (4–2*a*) a geared countershaft is to be supported by two bearings. In Fig. (4–2*c*) a fanshaft is to be configured. The solutions shown in Fig. (4–2*b*) and (7–2*d*) are not necessarily the best ones, but they do illustrate how the shaft-mounted devices are fixed and located in the axial direction, and how provision is made for torque transfer from one element to another.

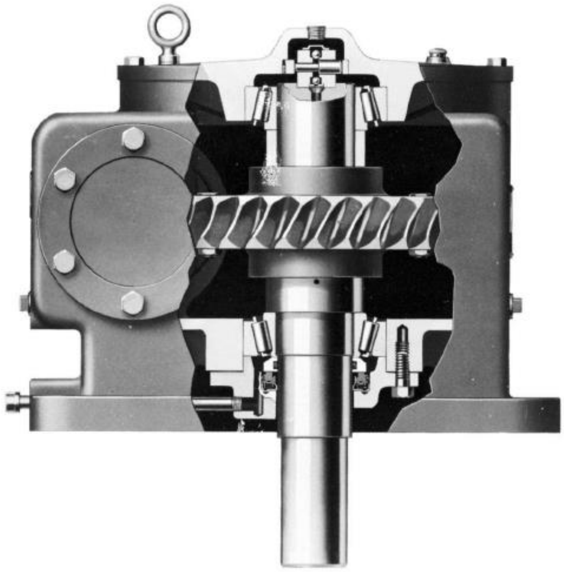

Figure (4–1)
A vertical worm-gear speed reducer.

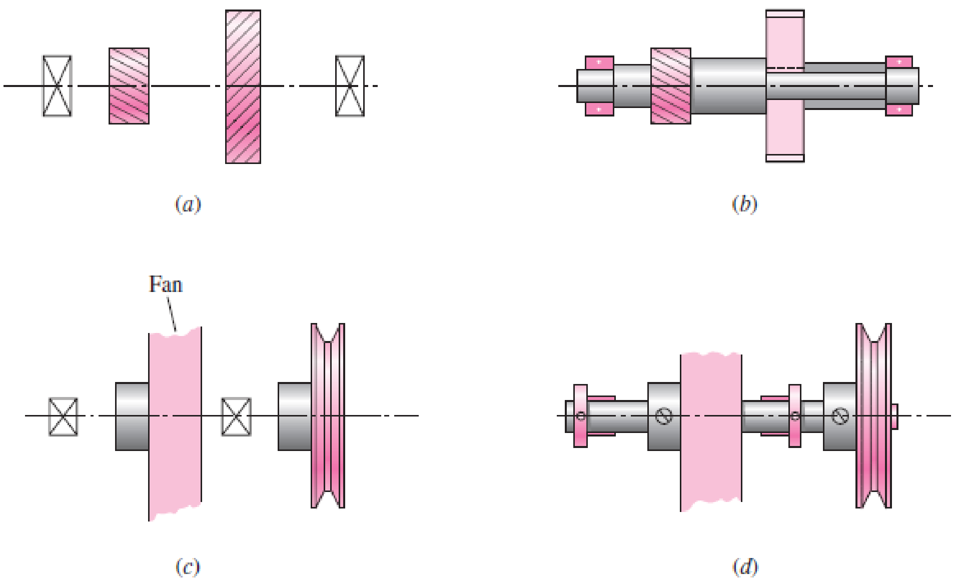

Figure (4–2)
(a) Choose a shaft configuration to support and locate the two gears and two bearings. (b) Solution uses an integral pinion, three shaft shoulders, key and keyway, and sleeve. The housing locates the bearings on their outer rings and receives the thrust loads. (c) Choose fanshaft configuration. (d) Solution uses sleeve bearings, a straight-through shaft, locating collars, and setscrews for collars, fan pulley, and fan itself. The fan housing supports the sleeve bearings.

There are no absolute rules for specifying the general layout, but the following guidelines may be helpful.

➢ *Axial Layout of Components*

The axial positioning of components is often dictated by the layout of the housing and other meshing components. In general, it is best to support load-carrying components between bearings, such as in Fig. (4–2*a*), rather than cantilevered outboard of the bearings, such as in Fig. (4–2*c*). Pulleys and sprockets often need to be mounted outboard for ease of installation of the belt or chain. The length of the cantilever should be kept short to minimize the deflection.

Only two bearings should be used in most cases. For extremely long shafts carrying several load-bearing components, it may be necessary to provide more than two bearing supports. In this case, particular care must be given to the alignment of the bearings.

In cases where axial loads are very small, it may be feasible to do without the shoulders entirely, and rely on press fits, pins, or collars with setscrews to maintain an axial location. See Fig. (4–2*b*) and (4–2*d*) for examples of some of these means of axial location.

➢ *Supporting Axial Loads*

In cases where axial loads are not trivial, it is necessary to provide a means to transfer the axial loads into the shaft, then through a bearing to the ground. This will be particularly necessary with helical or bevel gears, or tapered roller bearings, as each of these produces axial force components. Often, the same means of providing axial location, e.g., shoulders, retaining rings, and pins, will be used to also transmit the axial load into the shaft.

It is generally best to have only one bearing carry the axial load, to allow greater tolerances on shaft length dimensions, and to prevent binding if the shaft expands due to temperature changes. This is particularly important for long shafts. Figures (4–3 & 4–4) show examples of shafts with only one bearing carrying the axial load against a shoulder, while the other bearing is simply press-fit onto the shaft with no shoulder.

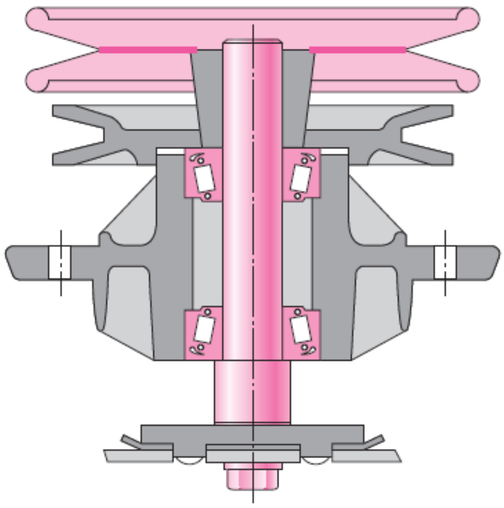

Figure (4–3)

Tapered roller bearings used in a mowing machine spindle. This design represents good practice for the situation in which one or more torque-transfer elements must be mounted outboard.

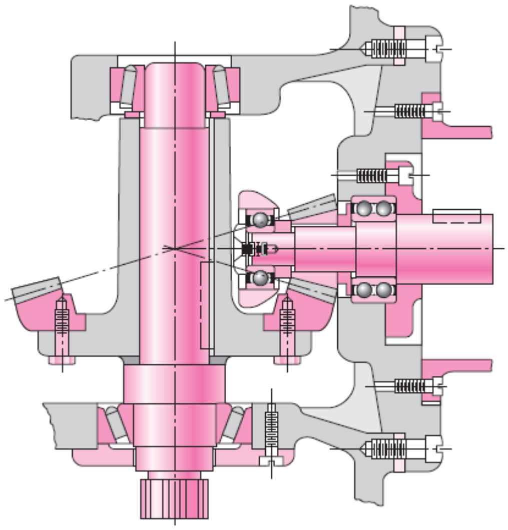

Figure (4–4)

A bevel-gear drive in which both pinion and gear are straddle-mounted.

➢ ***Providing for Torque Transmission***

Most shafts serve to transmit torque from an input gear or pulley, through the shaft, to an output gear or pulley. Of course, the shaft itself must be sized to support the torsional stress and torsional deflection. It is also necessary to provide a means of transmitting the torque between the shaft and the gears. Common torque-transfer elements are:

- Keys
- Splines
- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

In addition to transmitting the torque, many of these devices are designed to fail if the torque exceeds acceptable operating limits, protecting more expensive components.

One of the most effective and economical means of transmitting moderate to high levels of torque is through a *key* that fits in a groove in the shaft and gear.

Splines are essentially stubby gear teeth formed on the outside of the shaft and on the inside of the hub of the load-transmitting component. Splines are generally much more expensive to manufacture than keys, and are usually not necessary for simple torque transmission. They are typically used to transfer high torques. For cases of low torque transmission, various means of transmitting torque are available. These include *pins*, *setscrews* in hubs, *tapered fits*, and *press fits*.

Press and shrink fits for securing hubs to shafts are used both for torque transfer and for preserving axial location. The resulting stress-concentration factor is usually quite small.

Tapered fits between the shaft and the shaft-mounted device, such as a wheel, are often used on the overhanging end of a shaft. Screw threads at the shaft end then permit the use of a nut to lock the wheel tightly to the shaft. This approach is useful because it can be disassembled, but it does not provide good axial location of the wheel on the shaft.

At the early stages of the shaft layout, the important thing is to select an appropriate means of transmitting torque, and to determine how it affects the overall shaft layout. It is necessary to know where the shaft discontinuities, such as keyways, holes, and splines, will be in order to determine critical locations for analysis.

- ***Assembly and Disassembly***

Consideration should be given to the method of assembling the components onto the shaft, and the shaft assembly into the frame. This generally requires the largest diameter in the center of the shaft, with progressively smaller diameters towards the ends to allow components to be slid on from the ends. If a shoulder is needed on both sides of a component, one of them must be created by such means as a retaining ring or by a sleeve between two components. The gearbox itself will need means to physically position the shaft into its bearings, and the bearings into the frame. This is typically accomplished by providing access through the housing to the bearing at one end of the shaft. See Fig. (4–5) for examples.

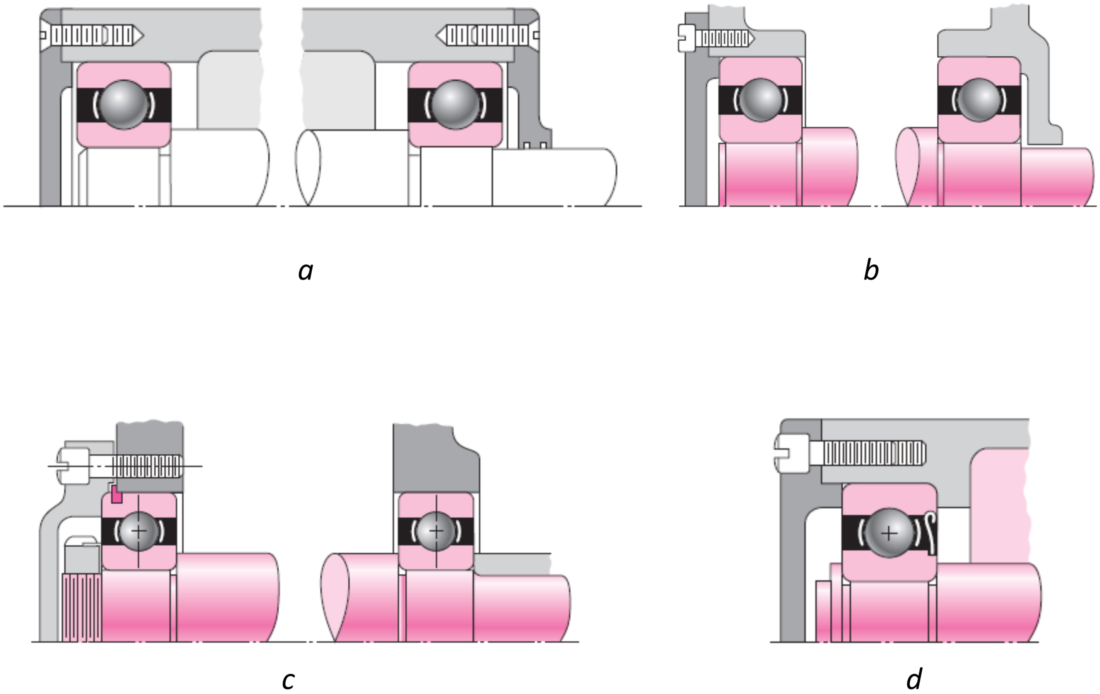

Figure (4–5)

(*a*) Arrangement showing bearing inner rings press-fitted to shaft while outer rings float in the housing. The axial clearance should be sufficient only to allow for machinery vibrations. Note the labyrinth seal on the right. **(*b*)** Similar to the arrangement of (*a*) except that the outer bearing rings are preloaded. **(*c*)** In this arrangement the inner ring of the left-hand bearing is locked to the shaft between a nut and a shaft shoulder. The locknut and washer are AFBMA standard. The snap ring in the outer race is used to positively locate the shaft assembly in the axial direction. Note the floating right-hand bearing and the grinding runout grooves in the shaft. **(*d*)** This arrangement is similar to (*c*) in that the left-hand bearing positions the entire shaft assembly. In this case the inner ring is secured to the shaft using a snap ring. Note the use of a shield to prevent dirt generated from within the machine from entering the bearing.

When components are to be press-fit to the shaft, the shaft should be designed so that it is not necessary to press the component down a long length of shaft. This may require an extra change in diameter, but it will reduce manufacturing and assembly cost by only requiring the close tolerance for a short length. Consideration should also be given to the necessity of disassembling the components from the shaft. This requires consideration of issues such as accessibility of retaining rings, space for pullers to access bearings, openings in the housing to allow pressing the shaft or bearings out, etc.

4.3 Shaft Design for Stress

Bending, torsion, and axial stresses may be present in both midrange and alternating components. For analysis, it is simple enough to combine the different types of stresses into alternating and midrange von Mises stresses, as shown in the fatigue part. It is sometimes convenient to customize the equations specifically for shaft applications. Axial loads are usually comparatively very small at critical locations where bending and torsion dominate, so they will be left out of the following equations.

Neglecting axial loads, the resulting equations for several of the commonly used failure curves are summarized below. The names given to each set of equations identifies the significant failure theory, followed by a fatigue failure locus name. For example, DE-Gerber indicates the stresses are combined using the distortion energy (DE) theory, and the Gerber criteria is used for the fatigue failure.

DE-Goodman

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

DE-ASME Elliptic

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}$$

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$$

DE-Soderberg

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{yt}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{yt}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

To check for yielding,

$$\sigma'_{max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2}$$

$$= \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2}$$

$$n_y = \frac{S_y}{\sigma'_{max}}$$

For a quick, conservative check, an estimate for σ'_{max} can be obtained by simply adding σ'_a and σ'_m. $(\sigma'_a + \sigma'_m)$ will always be greater than or equal to σ'_{max}, and will therefore be conservative.

EXAMPLE 4–1

At a machined shaft shoulder the small diameter *d* is 1.100 in, the large diameter *D* is 1.65 in, and the fillet radius is 0.11 in. The bending moment is 1260 lbf·in and the steady torsion moment is 1100 lbf·in. The heat-treated steel shaft has an ultimate strength of S_{ut} = 105 kpsi and a yield strength of S_y = 82 kpsi. The reliability goal is 0.99.

(a) Determine the fatigue factor of safety of the design using each of the fatigue failure criteria described in this section.
(b) Determine the yielding factor of safety.

Solution

(a) $D/d = 1.65/1.100 = 1.50$, $r/d = 0.11/1.100 = 0.10$, then,

$K_t = 1.68$, $K_{ts} = 1.42$, $q = 0.85$ & $q_{\text{shear}} = 0.92$ (HW).

$K_f = 1 + 0.85(1.68 - 1) = 1.58$ $\qquad$ $K_{fs} = 1 + 0.92(1.42 - 1) = 1.39$

$k_a = 0.787$, $k_b = 0.870$, $k_c = k_d = k_f = 1$, $k_e = 0.814$ (HW)

then,

$$S_e = 0.787(0.870)0.814(0.5)(105) = 29.3 \text{ kpsi}$$

$$M_a = 1260 \text{ lbf·in}, \; T_m = 1100 \text{ lbf·in}, \; M_m = T_a = 0$$

Then

$n = 1.62$	DE-Goodman
$n = 1.87$	DE-Gerber
$n = 1.88$	DE-ASME Elliptic
$n = 1.56$	DE-Soderberg

(b) For the yielding factor of safety, determine an equivalent von Mises maximum stress

$$\sigma'_{\text{max}} = 18.300 \text{ kpsi, then, } n_y = 4.48$$

For comparison, a quick and very conservative check on yielding can be obtained by replacing $\sigma'_{\max}$ with $\sigma'_a + \sigma'_m$. This just saves the extra time of calculating $\sigma'_{\max}$ if σ'_a and σ'_m have already been determined. For this example,

$$n_y = S_y/(\sigma'_a + \sigma'_m) = 82.000/(\underline{15.235 + 10.134}) = 3.23 \ (\underline{\text{HW}})$$

which is quite *conservative* compared with $n_y = 4.48$.