4.1 Miscellaneous Shaft Components

➢ ***Setscrews***

Unlike bolts and cap screws, which depend on tension to develop a clamping force, the setscrew depends on compression to develop the clamping force. The resistance to axial motion of the collar or hub relative to the shaft is called *holding power*. This holding power, which is really a force resistance, is due to frictional resistance of the contacting portions of the collar and shaft as well as any slight penetration of the setscrew into the shaft.

Figure (4–6) shows the point types available with socket setscrews. These are also manufactured with screwdriver slots and with square heads.

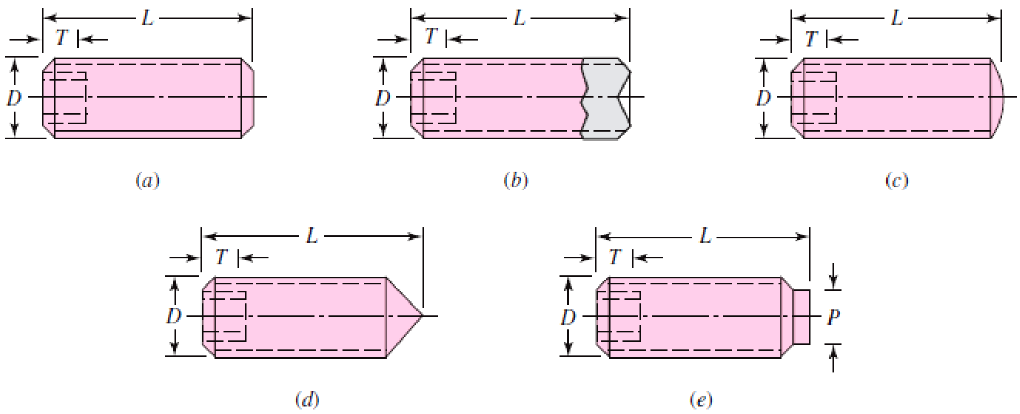


Figure (4–6)

Socket setscrews: (*a*) flat point; (*b*) cup point; (*c*) oval point; (*d*) cone point; (*e*) half-dog point.

Typical factors of safety are 1.5 to 2.0 for static loads and 4 to 8 for various dynamic loads. Setscrews should have a length of about half of the shaft diameter.

➢ ***Keys and Pins***

Keys and pins are used on shafts to secure rotating elements, such as gears, pulleys, or other wheels. Keys are used to enable the transmission of torque from the shaft to the shaft-supported element. Pins are used for axial positioning and for the transfer of torque or thrust or both.

Figure (4–7) shows a variety of keys and pins. Pins are useful when the principal loading is shear and when both torsion and thrust are present. Taper pins are sized according to the diameter at the large end. The diameter at the small end is

$$d = D - 0.0208L$$

where

d = diameter at small end, in., D = diameter at large end, in., and L = length, in.

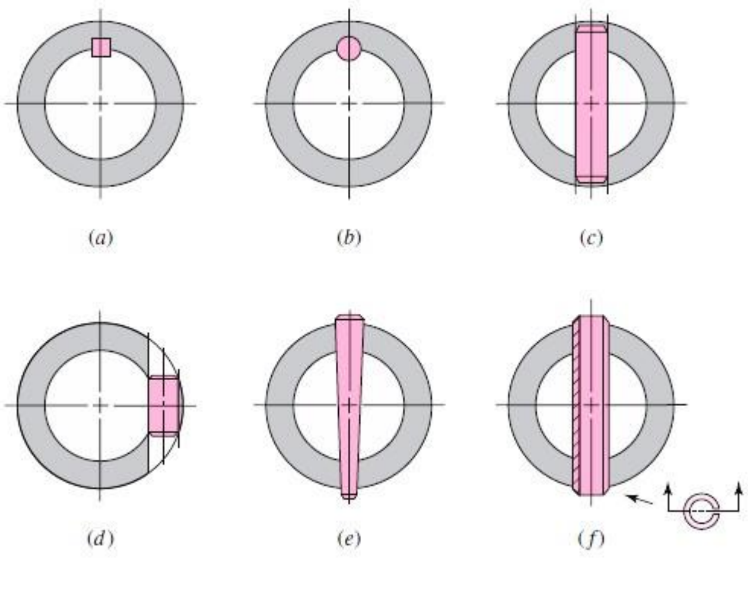


Figure (4–7)

(a) Square key; (*b*) round key; (*c* and *d*) round pins; (*e*) taper pin; (f) split tubular spring pin. The pins in parts (*e*) and (*f*) are shown longer than necessary, to illustrate the chamfer on the ends, but their lengths should be kept smaller than the hub diameters to prevent injuries due to projections on rotating parts.

For less important applications, a dowel pin or a drive pin can be used. A large variety of these are listed in manufacturers' catalogs. The square key, shown in Fig. (4–7*a*), is also available in rectangular sizes. The shaft diameter determines standard sizes for width, height, and key depth. The designer chooses an appropriate key length to carry the torsional load. Failure of the key can be by direct shear, or by bearing stress. The maximum length of a key is limited by the hub length of the attached element, and should generally not exceed about 1.5 times the shaft diameter to avoid excessive twisting with the angular deflection of the shaft. Multiple keys may be used as necessary to carry greater loads, typically oriented at 90^{o} from one another. Excessive safety factors should be avoided in key design, since it is desirable in an overload situation for the key to fail, rather than more costly components.

Stock key material is typically made from low carbon cold-rolled steel, and is manufactured such that its dimensions never exceed the nominal dimension. This allows standard cutter sizes to be used for the keyseats. A setscrew is sometimes used along with a key to hold the hub axially, and to minimize rotational backlash when the shaft rotates in both directions.

The gib-head key, in Fig. (4–8*a*), is tapered so that, when firmly driven, it acts to prevent relative axial motion. This also gives the advantage that the hub position can be adjusted for the best axial location. The head makes removal possible without access to the other end, but the projection may be hazardous.

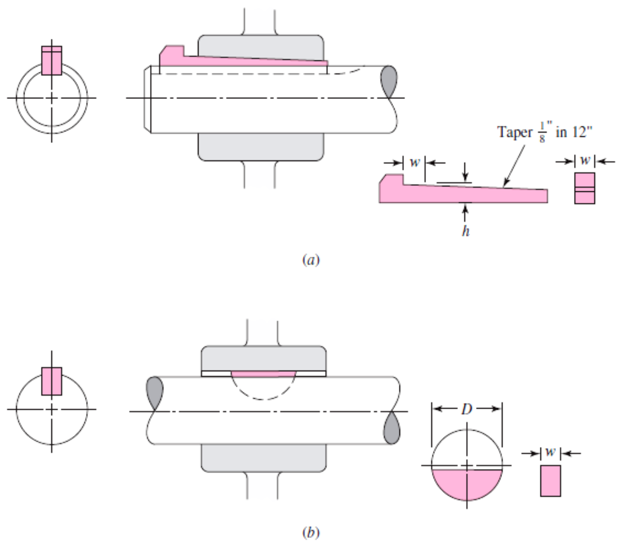


Figure (4–8)

(a) Gib-head key; (*b*) Woodruff key.

The Woodruff key, shown in Fig. (4–8*b*), is of general usefulness, especially when a wheel is to be positioned against a shaft shoulder, since the keyslot need not be machined into the shoulder stress-concentration region. The use of the Woodruff key also yields better concentricity after assembly of the wheel and shaft. This is especially important at high speeds, as, for example, with a turbine wheel and shaft. Woodruff keys are particularly useful in smaller shafts where their deeper penetration helps prevent key rolling.

➢ ***Retaining Rings***

A retaining ring is frequently used instead of a shaft shoulder or a sleeve to axially position a component on a shaft or in a housing bore. As shown in Fig. (4–9), a groove is cut in the shaft or bore to receive the spring retainer. For sizes, dimensions, and axial load ratings, the manufacturers' catalogs should be consulted.

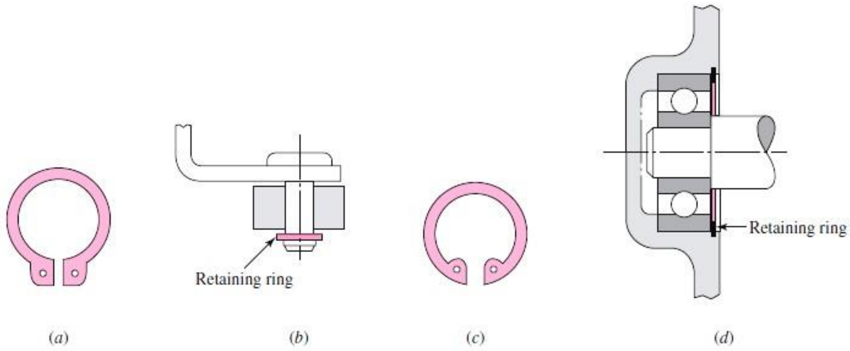


Figure (4–9)
Typical uses for retaining rings. (*a*) External ring and (*b*) its application; (*c*) internal ring and (*d*) its application.

For the rings to seat nicely in the bottom of the groove, and support axial loads against the sides of the groove, the radius in the bottom of the groove must be reasonably sharp, typically about one-tenth of the groove width. This causes comparatively high values for stress concentration factors, around 5 for bending and axial, and 3 for torsion. Care should be taken in using retaining rings, particularly in locations with high bending stresses.

1. Screws, Fasteners, and the Design of Nonpermanent Joints

The helical-thread screw was undoubtably an extremely important mechanical invention. It is the basis of power screws, which change angular motion to linear motion to transmit power or to develop large forces (presses, jacks, etc.), and threaded fasteners, an important element in nonpermanent joints.

5.1 Thread Standards and Definitions

The terminology of screw threads, illustrated in Fig. (5–1), is explained as follows:

The *pitch* is the distance between adjacent thread forms measured parallel to the thread axis. The pitch in U.S. units is the reciprocal of the number of thread forms per inch N.

The *major diameter* (d) is the largest diameter of a screw thread.

The *minor* (or root) *diameter* (d_r) is the smallest diameter of a screw thread.

The pitch diameter (d_p) is a theoretical diameter between the major and minor diameters.

The *lead* (l), not shown, is the distance the nut moves parallel to the screw axis when the nut is given one turn. For a single thread, as in Fig. (5–1), the lead is the same as the pitch.

A *multiple-threaded* product is one having two or more threads cut beside each other (imagine two or more strings wound side by side around a pencil). Standardized products such as screws, bolts, and nuts all have single threads; a *double-threaded* screw has a lead equal to twice the pitch, a *triple-threaded* screw has a lead equal to 3 times the pitch, and so on.

All threads are made according to the *right-hand rule* unless otherwise noted.

The *American National (Unified)* thread standard has been approved in Great Britain for use on all standard threaded products. The thread angle is 60^o and the crests of the thread may be either flat or rounded.

Figure (5–2) shows the thread geometry of the metric M and MJ profiles. The M profile replaces the inch class and is the basic ISO 68 profile with 60^o symmetric threads. The MJ profile has a rounded fillet at the root of the external thread and a larger minor diameter of both the internal and external threads. This profile is especially useful where high fatigue strength is required.

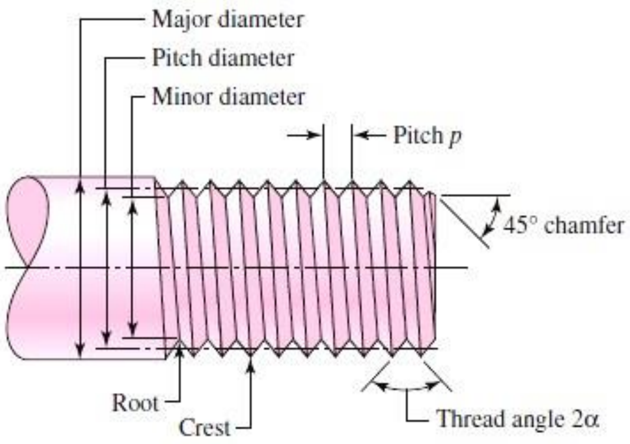


Figure (5–1)
Terminology of screw threads. Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation

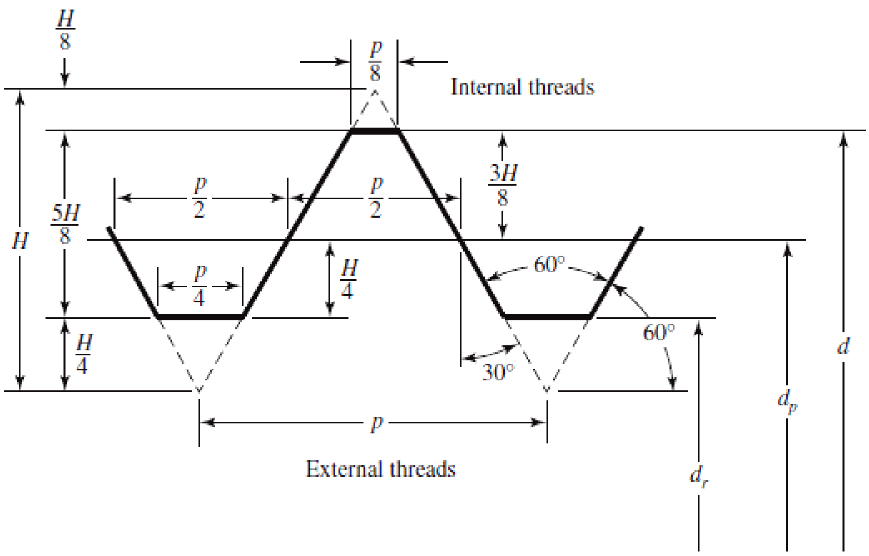


Figure (5–2)
Basic profile for metric M and MJ threads; d = major diameter d_r = minor diameter, d_p= pitch diameter, p = pitch, $H = \sqrt{3}/2\ p$

Two major Unified thread series are in common use: UN and UNR. The difference between these is simply that a root radius must be used in the UNR series. Because of reduced thread stress-concentration factors, UNR series threads have improved fatigue strengths. Unified threads are specified by stating the nominal major diameter, the number of threads per inch, and the thread series, for example, 5/8 in-18 UNRF or 0.625 in-18 UNRF.

Metric threads are specified by writing the diameter and pitch in millimeters, in that order. Thus, M12 × 1.75 is a thread having a nominal major diameter of 12 mm and a pitch of 1.75 mm. Note that the letter M, which precedes the diameter, is the clue to the metric designation.

Square and Acme threads, shown in Fig. (5–3*a* and *b*), respectively, are used on screws when power is to be transmitted.

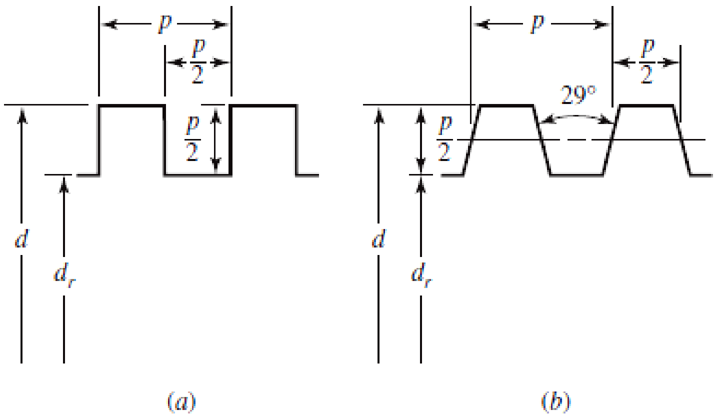


Figure (5–3)

(a) Square thread; (*b*) Acme thread.

Modifications are frequently made to both Acme and square threads. For instance, the square thread is sometimes modified by cutting the space between the teeth so as to have an included thread angle of 10 to 15^{o}. This is not difficult, since these threads are usually cut with a single-point tool anyhow; the modification retains most of the high efficiency inherent in square threads and makes the cutting simpler. Acme threads are sometimes modified to a stub form by making the teeth shorter. This results in a larger minor diameter and a somewhat stronger screw.

5.2 The Mechanics of Power Screws

A power screw is a device used in machinery to change angular motion into linear motion, and, usually, to transmit power. Familiar applications include the lead screws of lathes, and the screws for vises, presses, and jacks.

An application of power screws to a power-driven jack is shown in Fig. (5–4).

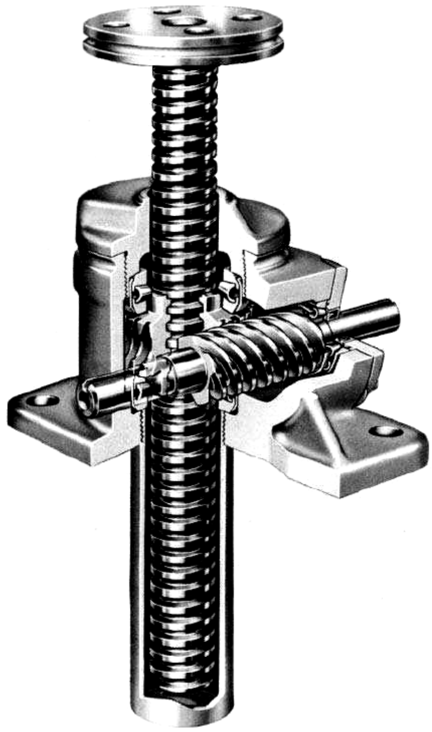


Figure (5–4)
The Joyce worm-gear screw jack.

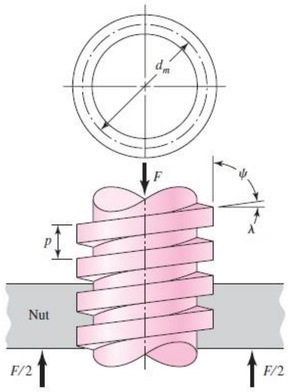
In Fig. (5–5) a square-threaded power screw with single thread having a mean diameter d_m, a pitch p, a lead angle λ, and a helix angle ψ is loaded by the axial compressive force F. We wish to find an expression for the torque required to raise this load, and another expression for the torque required to lower the load.

First, imagine that a single thread of the screw is unrolled or developed (Fig. 5–6) for exactly a single turn. Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean-thread-diameter circle and whose height is the lead. The angle λ, in Figs. (5–5) and (5–6), is the lead angle of the thread. We represent the summation of all the unit axial forces acting upon the normal thread area by F. To raise the load, a force P_R acts to the right (Fig. 5–6*a*), and to lower the load, P_L acts to the left (Fig. 5–6*b*). The friction force is the product of the coefficient of friction f with the normal force N, and acts to oppose the motion. The system is in equilibrium under the action of these forces, and hence, for raising the load, we have

$$\sum F_H = P_R - N\sin\lambda - fN\cos\lambda = 0 \qquad a$$

$$\sum F_V = F + fN\sin\lambda - N\cos\lambda = 0$$

In a similar manner, for lowering the load, we have



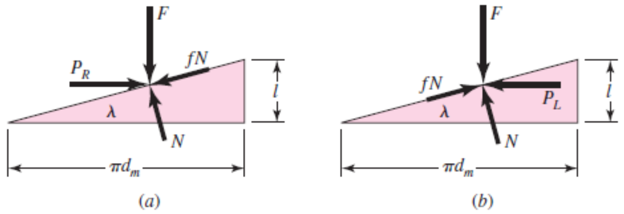


Figure (5–6)

Figure (5–5)

Force diagrams: (*a*) lifting the load; (*b*) lowering the load

Portion of a power screw

$$\sum FH = -PL - N \sin\lambda + f\, N \cos\lambda = 0$$
$$\sum FV = F - f\, N \sin\lambda - N \cos\lambda = 0 \qquad b$$

Since we are not interested in the normal force *N*, we eliminate it from each of these sets of equations and solve the result for *P*. For raising the load, this gives

$$P_R = F(\sin\lambda + f\cos\lambda) / (\cos\lambda - f\sin\lambda) \qquad c$$

and for lowering the load,

$$P_L = F(f\cos\lambda - \sin\lambda) / (\cos\lambda + f\sin\lambda) \qquad d$$

Next, divide the numerator and the denominator of these equations by cos λ and use the relation $\tan\lambda = l/\pi d_m$ (Fig. 5–6). We then have, respectively,

$$P_R = F\,[(l/\pi dm) + f\,] / [1 - (f\, l/\pi d_m)] \qquad e$$

$$P_L = F\,[\,f - (l/\pi d_m)] / [1 + (f\, l/\pi d_m)] \qquad f$$

Finally, noting that the torque is the product of the force *P* and the mean radius $d_m/2$, for raising the load we can write:

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) \qquad 5\text{-}1$$

where T_R is the torque required for two purposes: to overcome thread friction and to raise the load.

The torque required to lower the load,

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) \qquad 5\text{-}2$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort. In such cases, the torque T_L from Eq. (5–2) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be *self-locking*. Thus the condition for self-locking is

$$\pi f d_m > l$$

Dividing both sides of this inequality by πd_m. Recognizing that $l/\pi d_m = \tan\lambda$, we get

$$f > \tan\lambda \qquad 5\text{-}3$$

This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle. An expression for efficiency is also useful in the evaluation of power screws. If we let $f = 0$ in Eq. (5–1), we obtain

$$T_o = Fl / 2\pi$$

which, since thread friction has been eliminated, is the torque required only to raise the load. The efficiency is therefore

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R}$$

The preceding equations have been developed for square threads where the normal thread loads are parallel to the axis of the screw. In the case of Acme or other threads, the normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ. Since lead angles are small, this inclination can be neglected and only the effect of the thread angle (Fig. 5–7*a*) considered. The effect of the angle α is to increase the frictional force by the wedging action of the threads. Therefore the frictional terms in Eq. (5–1) must be

divided by cos α. For raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right) \qquad \text{5-4}$$

In using Eq. (5–5), remember that it is an approximation because the effect of the lead angle has been neglected.

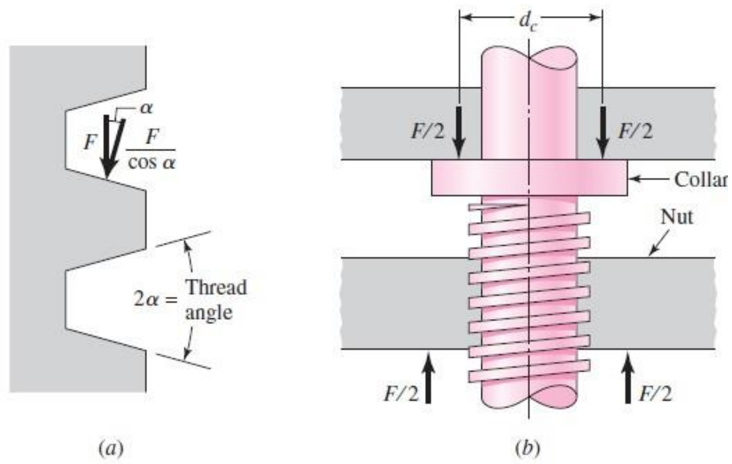


Figure (5–7)
(a) Normal thread force is increased because of angle α;
(b) thrust collar has frictional diameter d_c

For power screws, the Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine and permits the use of a split nut, which can be adjusted to take up for wear.

Usually a third component of torque must be applied in power-screw applications. When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. Figure (5–7*b*) shows a typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter d_c. If f_c is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2} \qquad \text{5-5}$$

For large collars, the torque should probably be computed in a manner similar to that employed for disk clutches.

Nominal body stresses in power screws can be related to thread parameters as follows. The maximum nominal shear stress τ in torsion of the screw body can be expressed as

$$\tau = \frac{16T}{\pi d_r^3} \qquad \text{5-6}$$

The axial stress σ in the body of the screw due to load *F* is

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2} \qquad \text{5-7}$$

Nominal thread stresses in power screws can be related to thread parameters as follows. The bearing stress in Fig. (5–8), σ_B, is

$$\sigma_B = -\frac{F}{\pi d_m n_t p/2} = -\frac{2F}{\pi d_m n_t p} \qquad \text{5-8}$$

where n_t is the number of engaged threads.

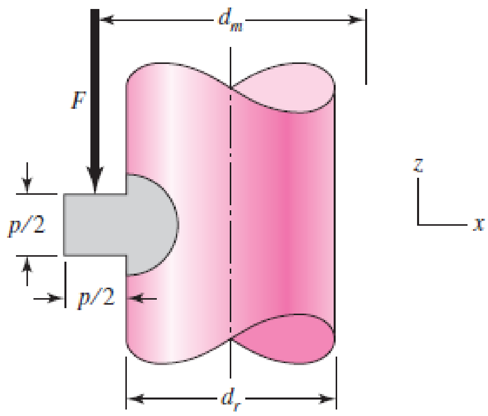


Figure (5–8)

Geometry of square thread useful in finding bending and transverse shear stresses at the thread root

The bending stress at the root of the thread σ_b is found from:

$$\frac{I}{c} = \frac{(\pi d_r n_t)(p/2)^2}{6} = \frac{\pi}{24} d_r n_t p^2 \qquad M = \frac{Fp}{4}$$

$$\sigma_b = \frac{M}{I/c} = \frac{Fp}{4} \frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \qquad \text{5-9}$$

The transverse shear stress τ at the center of the root of the thread due to load *F* is

$$\tau = \frac{3V}{2A} = \frac{3}{2} \frac{F}{\pi d_r n_t p/2} = \frac{3F}{\pi d_r n_t p} \qquad \text{5-10}$$

and at the top of the root it is zero. The von Mises stress σ′ at the top of the root "plane" is found by first identifying the orthogonal normal stresses and the shear stresses. From the coordinate system of Fig. (5–8), we note

$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = 0$$

$$\sigma_y = 0 \qquad \tau_{yz} = \frac{16T}{\pi d_r^3}$$

$$\sigma_z = -\frac{4F}{\pi d_r^2} \qquad \tau_{zx} = 0$$

The screw-thread form is complicated from an analysis viewpoint. The tensile-stress area A_t , comes from experiment [see tables (5–1) & 5–2)] . A power screw lifting a load is in compression and its thread pitch is *shortened* by elastic deformation. Its engaging nut is in tension and its thread pitch is *lengthened*. The engaged threads cannot share the load equally. Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load. In estimating thread stresses by the equations above, substituting $0.38F$ for F and setting n_t to 1 will give the largest level of stresses in the thread-nut combination.

EXAMPLE 5–1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. (5–4). The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

(a) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
(b) Find the torque required to raise and lower the load.
(c) Find the efficiency during lifting the load.
(d) Find the body stresses, torsional and compressive.
(e) Find the bearing stress.
(f) Find the thread stresses bending at the root, shear at the root, and von Mises stress and maximum shear stress at the same location.

Solution

(a) From Fig. (5–3*a*) the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

(b) Using Eqs. (5–1) and (5–6), the torque required to turn the screw against the load is

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)}\right] + \frac{6.4(0.08)40}{2}$$

$$= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m}$$

Using Eqs. (5–2) and (5–6), we find the load-lowering torque is

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)}\right] + \frac{6.4(0.08)(40)}{2}$$

$$= -0.466 + 10.24 = 9.77 \text{ N} \cdot \text{m}$$

The minus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(c) The overall efficiency in raising the load is

$$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

(d) The body shear stress τ due to torsional moment T_R at the outside of the screw body is

$$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(e) The bearing stress σ_B is, with one thread carrying 0.38F:

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(f) The thread-root bending stress σb with one thread carrying 0.38F is:

$$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$

The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part
(*d*) of 6.07 MPa. The three-dimensional stresses, after Fig. (5–8), noting the *y* coordinate is into the page, are

$\sigma_x = 41.5$ MPa	$\tau_{xy} = 0$
$\sigma_y = 0$	$\tau_{yz} = 6.07$ MPa
$\sigma_z = -10.39$ MPa	$\tau_{zx} = 0$

$$\sigma' = (1/\sqrt{2})\{(41.5-0)^2+[0-(-10.39)]^2+(-10.39-41.5)^2+6(6.07)^2\}^{1/2}$$

$$= 48.7 \text{ MPa}$$

Alternatively, you can determine the principal stresses and then the von Mises stress noting that there are no shear stresses on the *x* face. This means that σ_x is a principal stress. The remaining principal stresses are:

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79, -13.18 \text{ MPa}$$

$$\sigma' = \left\{\frac{[41.5-2.79]^2 + [2.79-(-13.18)]^2 + [-13.18-41.5]^2}{2}\right\}^{1/2}$$

$$= 48.7 \text{ MPa}$$