

➤ *Shear Joints with Eccentric Loading*

Integral to the analysis of a shear joint is locating the center of relative motion between the two members. In Fig. (5–17) let A_1 to A_5 be the respective cross-sectional areas of a group of five pins, or hot-driven rivets, or tight-fitting shoulder bolts. Under this assumption the rotational pivot point lies at the centroid of the cross-sectional area pattern of the pins, rivets, or bolts. Using statics, we learn that the centroid G is located by the coordinates \bar{x} and \bar{y} , where x_i and y_i are the distances to the i^{th} area center:

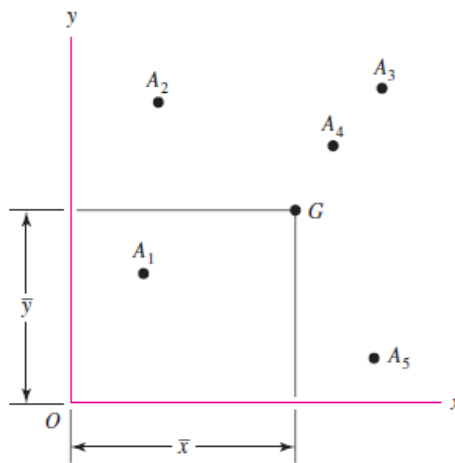


Figure (5–17)
Centroid of pins, rivets, or bolts

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i x_i}{\sum_1^n A_i}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i y_i}{\sum_1^n A_i}$$

5-17

In many instances the centroid can be located by symmetry.

An example of eccentric loading of fasteners is shown in Fig. (5–18). This is a portion of a machine frame containing a beam subjected to the action of a bending load. In this case, the beam is

fastened to vertical members at the ends with specially prepared load-sharing bolts. You will recognize the schematic representation in Fig. (5–18*b*) as a statically indeterminate beam with both ends fixed and with moment and shear reactions at each end.

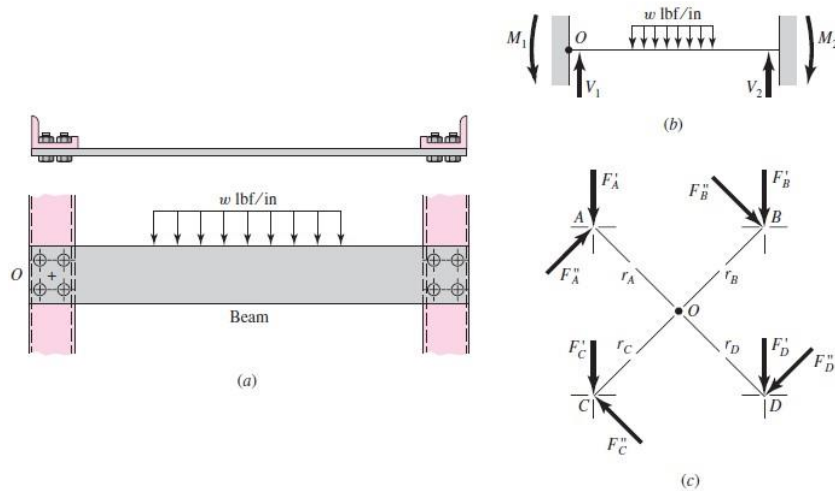


Figure (5–18)

- (a) Beam bolted at both ends with distributed load; (b) free-body diagram of beam; (c) enlarged view of bolt group centered at O showing primary and secondary resultant shear forces

For convenience, the centers of the bolts at the left end of the beam are drawn to a larger scale in Fig. (5–18*c*). Point O represents the centroid of the group, and it is assumed in this example that all the bolts are of the same diameter. Note that the forces shown in Fig. (5–18*c*) are the *resultant* forces acting on the pins with a net force and moment equal and opposite to the *reaction* loads V_1 and M_1

acting at O . The total load taken by each bolt will be calculated in three steps. In the first step the shear V_1 is divided equally among the bolts so that each bolt takes $F' = V_1/n$, where n refers to the number of bolts in the group and the force F' is called the *direct load*, or *primary shear*. It is noted that an equal distribution of the direct load to the bolts assumes an absolutely rigid member. The arrangement of the bolts or the shape and size of the members sometimes justifies the use of another assumption as to the division of the load. The direct loads F' are shown as vectors on the loading diagram (Fig. 5-18c).

The *moment load*, or *secondary shear*, is the additional load on each bolt due to the moment M_1 . If r_A, r_B, r_C , etc., are the radial distances from the centroid to the center of each bolt, the moment and moment loads are related as follows:

$$M_1 = F''_A r_A + F''_B r_B + F''_C r_C + \dots \quad a$$

where the F'' are the moment loads. The force taken by each bolt depends upon its radial distance from the centroid; that is, the bolt farthest from the centroid takes the greatest load, while the nearest bolt takes the smallest. We can therefore write

$$\frac{F''_A}{r_A} = \frac{F''_B}{r_B} = \frac{F''_C}{r_C} \quad b$$

where again, the diameters of the bolts are assumed equal. If not, then one replaces F'' in Eq. (b) with the shear stresses $\tau'' = 4F''/\pi d^2$ for each bolt. Solving Eqs. (a) and (b) simultaneously, we obtain:

$$F''_n = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \dots} \quad 5-18$$

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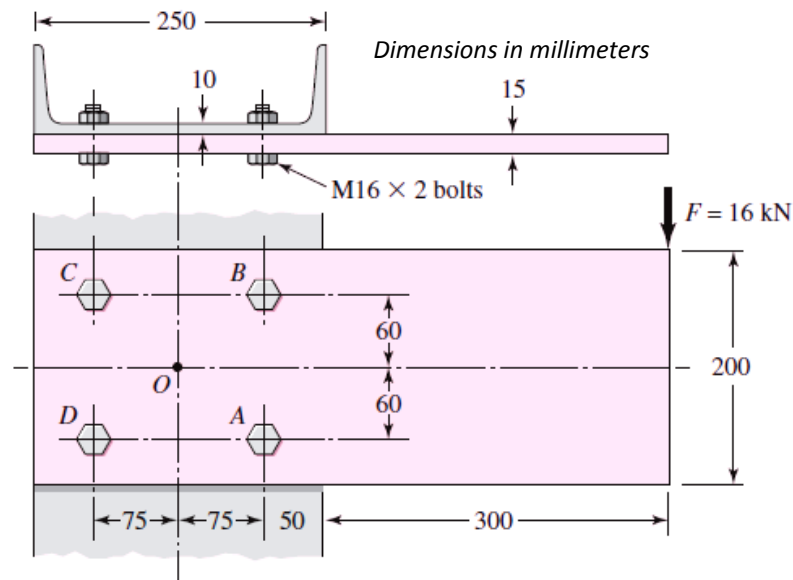
where the subscript n refers to the particular bolt whose load is to be found. These moment loads are also shown as vectors on the loading diagram.

In the third step the direct and moment loads are added vectorially to obtain the resultant load on each bolt. Since all the bolts or rivets are usually the same size, only that bolt having the maximum load need be considered. When the maximum load is found, the strength may be determined by using the various methods already described.

EXAMPLE 5-3

Shown in figure is a 15- by 200-mm rectangular steel bar cantilevered to a 250-mm steel channel using four tightly fitted bolts located at A , B , C , and D . For a load of $F = 16$ kN, find

- The resultant load on each bolt
- The maximum shear stress in each bolt
- The maximum bearing stress
- The critical bending stress in the bar

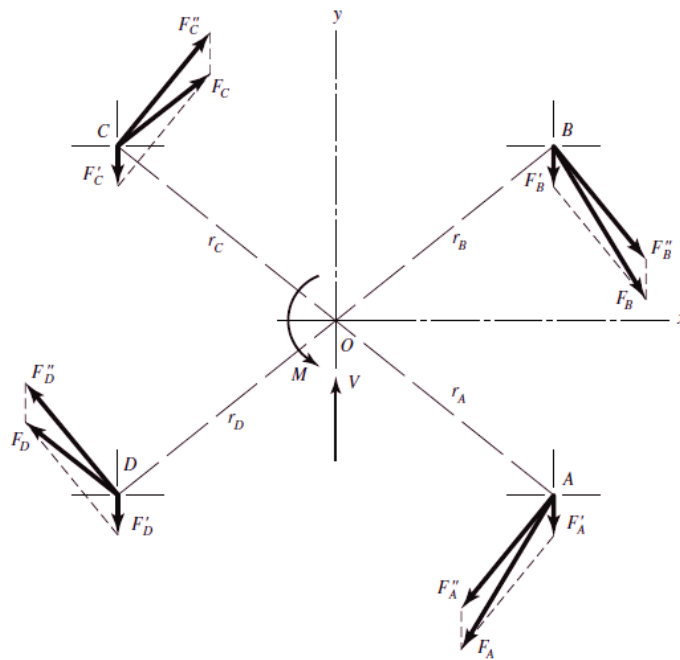


Solution

(a) Point O , the centroid of the bolt group, is found by symmetry. If a free-body diagram of the beam were constructed, the shear reaction V would pass through O and the moment reactions M would be about O . These reactions are

$$V = 16 \text{ kN} \qquad M = 16(425) = 6800 \text{ N}\cdot\text{m}$$

In the following figure, the bolt group has been drawn to a larger scale and the reactions are shown.



The distance from the centroid to the center of each bolt is

$$r = [(60)^2 + (75)^2]^{1/2} = 96 \text{ mm}$$

The primary shear load per bolt is

$$F' = \frac{V}{n} = \frac{16}{4} = 4 \text{ kN}$$

Since the secondary shear forces are equal, Eq. (5-19) becomes

$$F'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{6800}{4(96.0)} = 17.7 \text{ kN}$$

The primary and secondary shear forces are plotted to scale and the resultants obtained by using the parallelogram rule. The magnitudes are found by measurement (or analysis) to be

$$F_A = F_B = 21.0 \text{ kN}$$

(HW)

$$F_C = F_D = 14.8 \text{ kN}$$

(b) Bolts A and B are critical because they carry the largest shear

load. Does this shear act on the threaded portion of the bolt, or on

the unthreaded portion? The bolt length will be 25 mm plus the height of the nut plus about 2 mm for a washer. From net-tables, the nut height is 14.8 mm. Including two threads beyond the nut, this adds up to a length of 43.8 mm, and so a bolt 46 mm long will be needed. From Eq. (5-12) we compute the thread length as $L_T = 38$ mm. Thus the unthreaded portion of the bolt is $46 - 38 = 8$ mm long. This is less than the 15 mm for the plate in Fig. 8-28, and so the bolt will tend to shear across its minor diameter. Therefore, from table (5-1), the shear-stress area is $A_s = 144 \text{ mm}^2$, and so the shear stress is

$$\tau = \frac{F}{A_s} = \frac{21.0(10)^3}{144} = 146 \text{ MPa}$$

(c) The channel is thinner than the bar, and so the largest bearing stress is due to the pressing of the bolt against the channel web. The bearing area is $A_b = td = 10(16) = 160 \text{ mm}^2$. Thus the bearing stress

$$\sigma = \frac{F}{A_b} = \frac{21.0(10)^3}{160} = 131 \text{ MPa}$$

(d) The critical bending stress in the bar is assumed to occur in a section parallel to the y axis and through bolts A and B . At this section the bending moment is

$$M = 16(300 + 50) = 5600 \text{ N}\cdot\text{m}$$

The second moment of area through this section is obtained by the use of the transfer formula, as follows:

$$\begin{aligned} I &= I_{\text{bar}} - 2(I_{\text{holes}} + \bar{d}^2 A) \\ &= \frac{15(200)^3}{12} - 2 \left[\frac{15(16)^3}{12} + (60)^2(15)(16) \right] = 8.26(10)^6 \text{ mm}^4 \end{aligned}$$

Then:

$$\sigma = \frac{Mc}{I} = \frac{5600(100)}{8.26(10)^6} (10)^3 = 67.8 \text{ MPa}$$

Table (5-1)
 Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads

Nominal Major Diameter d mm	Coarse-Pitch Series			Fine-Pitch Series		
	Pitch p mm	Tensile-Stress Area A_t mm ²	Minor-Diameter Area A_r mm ²	Pitch p mm	Tensile-Stress Area A_t mm ²	Minor-Diameter Area A_r mm ²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

The equations and data used to develop this table have been obtained from ANSI B1.1-1974 and B18.3.1-1978. The minor diameter was found from the equation $d_r = d - 1.226869 p$, and the pitch diameter from $d_p = d - 0.649519 p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

Table (5–2)
Diameters and Area of Unified Screw Threads UNC and UNF









Size Designation	Nominal Major Diameter in	Coarse Series—UNC			Fine Series—UNF		
		Threads per Inch N	Tensile-Stress Area A_t , in ²	Minor-Diameter Area A_r , in ²	Threads per Inch N	Tensile-Stress Area A_t , in ²	Minor-Diameter Area A_r , in ²
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

This table was compiled from ANSI B1.1-1974. The minor diameter was found from the equation $d_r = d - 1.299038 p$, and the pitch diameter from $d_p = d - 0.649519 p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

Table (5–3)
Preferred Pitches for Acme Threads










d , in	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
p , in	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

Table (5-4)
SAE Specifications for Steel Bolts

SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
1	$\frac{1}{4}$ - $1\frac{1}{2}$	33	60	36	Low or medium carbon	
2	$\frac{1}{4}$ - $\frac{3}{4}$ $\frac{7}{8}$ - $1\frac{1}{2}$	55	74	57	Low or medium carbon	
		33	60	36		
4	$\frac{1}{4}$ - $1\frac{1}{2}$	65	115	100	Medium carbon, cold-drawn	
5	$\frac{1}{4}$ -1 $1\frac{1}{8}$ - $1\frac{1}{2}$	85	120	92	Medium carbon, Q&T	
		74	105	81		
5.2	$\frac{1}{4}$ -1	85	120	92	Low-carbon martensite, Q&T	
7	$\frac{1}{4}$ - $1\frac{1}{2}$	105	133	115	Medium-carbon alloy, Q&T	
8	$\frac{1}{4}$ - $1\frac{1}{2}$	120	150	130	Medium-carbon alloy, Q&T	
8.2	$\frac{1}{4}$ -1	120	150	130	Low-carbon martensite, Q&T	

*Minimum strengths are strengths exceeded by 99 percent of fasteners.








Table (5-5)
ASTM Specifications for Steel Bolts

ASTM Designation No.	Size Range, Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
A307	$\frac{1}{4}$ - $1\frac{1}{2}$	33	60	36	Low carbon	
A325, type 1	$\frac{1}{2}$ -1 $1\frac{1}{8}$ - $1\frac{1}{2}$	85 74	120 105	92 81	Medium carbon, Q&T	
A325, type 2	$\frac{1}{2}$ -1 $1\frac{1}{8}$ - $1\frac{1}{2}$	85 74	120 105	92 81	Low-carbon, martensite, Q&T	
A325, type 3	$\frac{1}{2}$ -1 $1\frac{1}{8}$ - $1\frac{1}{2}$	85 74	120 105	92 81	Weathering steel, Q&T	
A354, grade BC	$\frac{1}{4}$ - $2\frac{1}{2}$ $2\frac{3}{4}$ -4	105 95	125 115	109 99	Alloy steel, Q&T	
A354, grade BD	$\frac{1}{4}$ -4	120	150	130	Alloy steel, Q&T	
A449	$\frac{1}{4}$ -1 $1\frac{1}{8}$ - $1\frac{1}{2}$ $1\frac{3}{4}$ -3	85 74 55	120 105 90	92 81 58	Medium-carbon, Q&T	
A490, type 1	$\frac{1}{2}$ - $1\frac{1}{2}$	120	150	130	Alloy steel, Q&T	
A490, type 3	$\frac{1}{2}$ - $1\frac{1}{2}$	120	150	130	Weathering steel, Q&T	

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Table (5-6)

Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs*

Property Class	Size Range, Inclusive	Minimum Proof Strength* MPa	Minimum Tensile Strength* MPa	Minimum Yield Strength* MPa	Material	Head Marking
4.6	M5-M36	225	400	240	Low or medium carbon	
4.8	M1.6-M16	310	420	340	Low or medium carbon	
5.8	M5-M24	380	520	420	Low or medium carbon	
8.8	M16-M36	600	830	660	Medium carbon, Q&T	
9.8	M1.6-M16	650	900	720	Medium carbon, Q&T	
10.9	M5-M36	830	1040	940	Low-carbon martensite, Q&T	
12.9	M1.6-M36	970	1220	1100	Alloy, Q&T	

*The thread length for bolts and cap screws is

$$l_T = \begin{cases} 2d + 6 \text{ mm}, & L \leq 125, d \leq 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \leq 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

where L is the bolt length. The thread length for structural bolts is slightly shorter than given above.

*Minimum strengths are strength exceeded by 99 percent of fasteners.

Homework

(1) A power screw is 25 mm in diameter and has a thread pitch of 5 mm. (a) Find the thread depth, the thread width, the mean and root diameters, and the lead, provided square threads are used. (b) Repeat part (a) for Acme threads.

(2) Show that for zero collar friction the efficiency of a square-thread screw is given by the equation

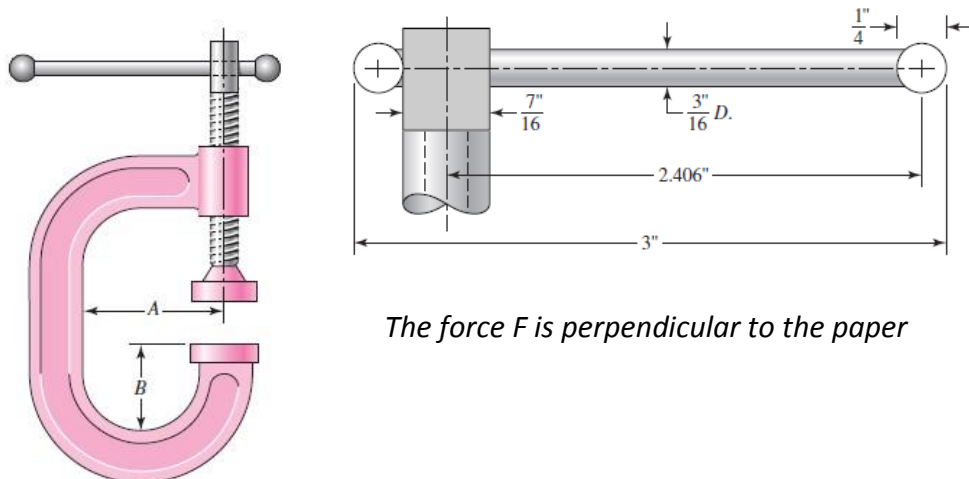
$$e = \tan \lambda \frac{1 - f \tan \lambda}{\tan \lambda + f}$$

(3) A single-threaded power screw is 25 mm in diameter with a pitch of 5 mm. A vertical load on the screw reaches a maximum of 6 kN. The coefficients of friction are 0.05 for the collar and 0.08 for the threads. The frictional diameter of the collar is 40 mm. Find the overall efficiency and the torque to “raise” and “lower” the load.

(Ans./ 0.294, 16.23 N.m, 6.622 N.m)

(4) A screw clamp similar to the one shown in the figure has a handle with diameter $\frac{3}{16}$ in made of cold-drawn AISI 1006 steel. The overall length is 3 in. The screw is $\frac{7}{16}$ in-14 UNC and is $5\frac{3}{4}$ in long, overall. Distance A is 2 in. The clamp will accommodate parts up to $4\frac{3}{16}$ in high.

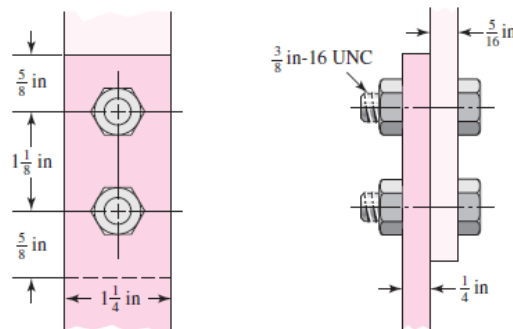
(a) What screw torque will cause the handle to bend permanently?
 (b) What clamping force will the answer to part (a) cause if the collar friction is neglected and if the thread friction is 0.075?



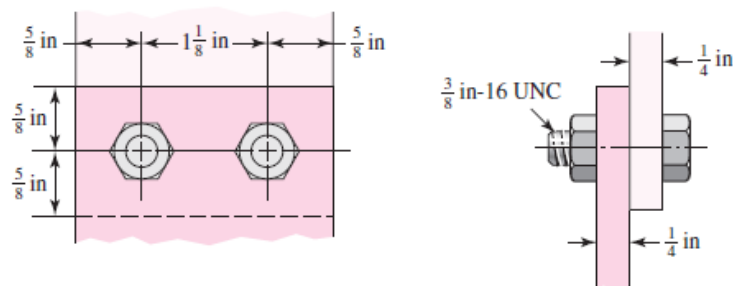
(5) Find the power required to drive a 40-mm power screw having double square threads with a pitch of 6 mm. The nut is to move at a velocity of 48 mm/s and move a load of $F = 10$ kN. The frictional coefficients are 0.1 for the threads and 0.15 for the collar. The frictional diameter of the collar is 60 mm. (Ans./ 2.086 kW)

(6) A single square-thread power screw has an input power of 3 kW at a speed of 1 rev/s. The screw has a diameter of 36 mm and a pitch of 6 mm. The frictional coefficients are 0.14 for the threads and 0.09 for the collar, with a collar friction radius of 45 mm. Find the axial resisting load F and the combined efficiency of the screw and collar. (Ans./ 65 kN, 0.13)

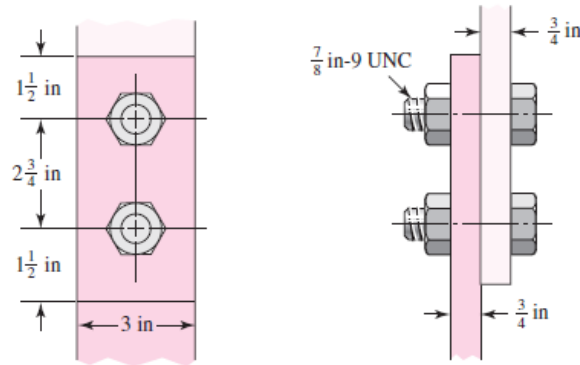
(7) The figure shows a bolted lap joint that uses SAE grade 8 bolts. The members are made of cold-drawn AISI 1040 steel. Find the safe tensile shear load F that can be applied to this connection if the following factors of safety are specified: shear of bolts 3, bearing on bolts 2, bearing on members 2.5, and tension of members 3. (Ans./ 5.18 kip)



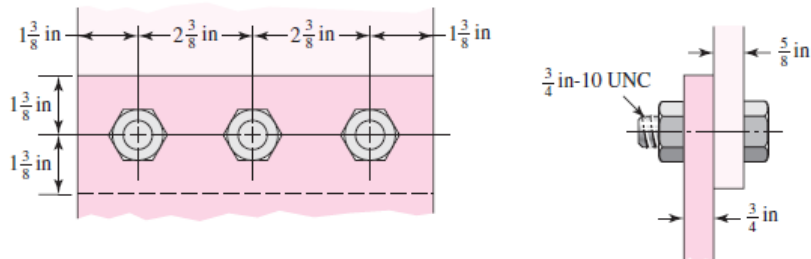
(8) The bolted connection shown in the figure uses SAE grade 5 bolts. The members are hot-rolled AISI 1018 steel. A tensile shear load $F = 4000$ lbf is applied to the connection. Find the factor of safety for all possible modes of failure. (Ans./ 2.93, 4.32, 1.5, 3.25)



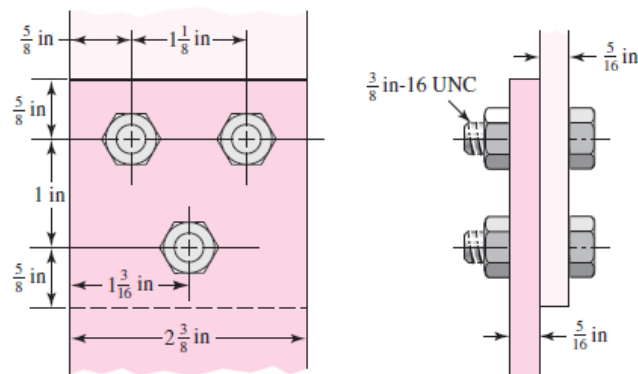
(9) A bolted lap joint using SAE grade 5 bolts and members made of cold-drawn SAE 1040 steel is shown in the figure. Find the tensile shear load F that can be applied to this connection if the following factors of safety are specified: shear of bolts 1.8, bearing on bolts 2.2, bearing on members 2.4, and tension of members 2.6. (Ans./ 35.46)



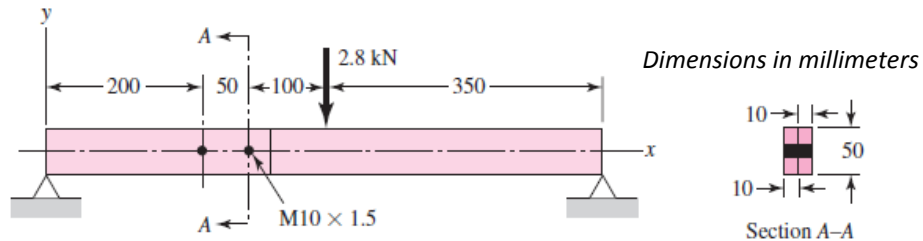
(10) The bolted connection shown in the figure is subjected to a tensile shear load of 20 kip. The bolts are SAE grade 5 and the material is cold-drawn AISI 1015 steel. Find the factor of safety of the connection for all possible modes of failure. (Ans./ 3.52, 6.47, 3.31, 7.71)



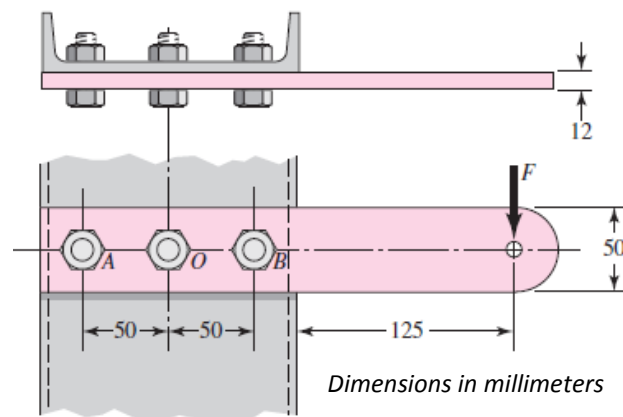
(11) The figure shows a connection that employs three SAE grade 5 bolts. The tensile shear load on the joint is 5400 lbf. The members are cold-drawn bars of AISI 1020 steel. Find the factor of safety for each possible mode of failure. (Ans./ 3.26, 5.99, 3.71, 5.36)



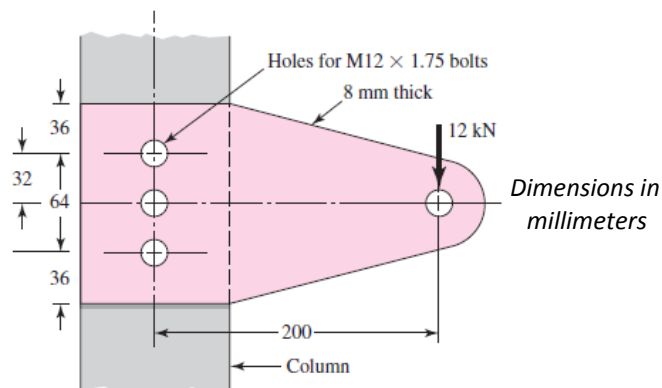
(12) A beam is made up by bolting together two cold-drawn bars of AISI 1018 steel as a lap joint, as shown in the figure. The bolts used are ISO 5.8. Ignoring any twisting, determine the factor of safety of the connection. (*Ans./ n = the minimum of (2.72, 5.29, 3.15) = 2.72*)



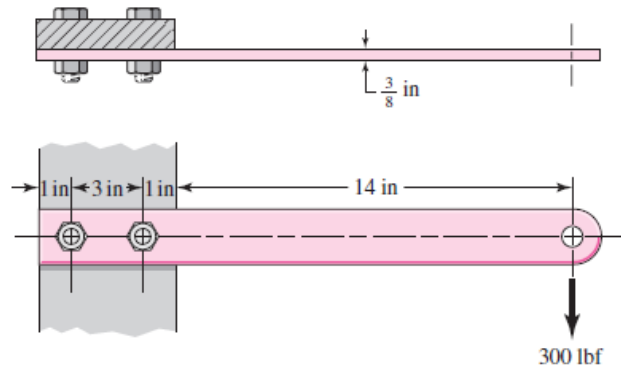
(13) A vertical channel 152×76 ($t = 6.4$ mm) has a cantilever beam bolted to it as shown. The channel is hot-rolled AISI 1006 steel. The bar is of hot-rolled AISI 1015 steel. The shoulder bolts are $M12 \times 1.75$ ISO 5.8. For a design factor of 2.8, find the safe force F that can be applied to the cantilever. (*Ans./ F = 1.99 kN based on bearing on channel*)



(14) Find the total shear load on each of the three bolts for the connection shown in the figure and compute the significant bolt shear stress and bearing stress. Find the second moment of area of the 8-mm plate on a section through the three bolt holes, and find the maximum bending stress in the plate. (*Ans./ $1.48(10)^6$ mm⁴, 110 MPa*)

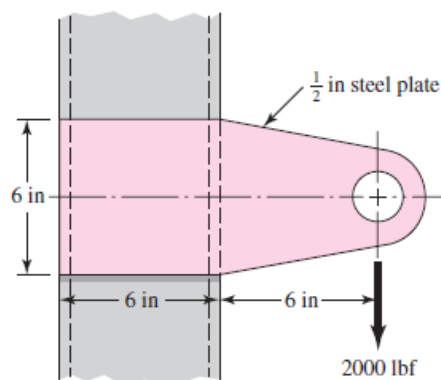


(15) A $\frac{3}{8}$ - \times 2-in AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 300 lbf as illustrated. The bar is secured to the support using two $\frac{1}{2}$ in-13 UNC SAE 5 bolts. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member. (Ans./ 5.79, 9.58, 5.63, 2.95)



(16) A cantilever is to be attached to the flat side of a channel used as a column. The cantilever is to carry a load as shown in the figure. To a designer the choice of a bolt array is usually an a priori decision. Such decisions are made from a background of knowledge of the effectiveness of various patterns.

- (a) If two fasteners are used, should the array be arranged vertically, horizontally, or diagonally? How would you decide?
- (b) If three fasteners are used, should a linear or triangular array be used? For a triangular array, what should be the orientation of the triangle? How would you decide?



(17) Using your experience with Problem (15), specify a bolt pattern for this Problem and size the bolts.