6.4 Stresses in Welded Joints in Bending

Figure (6–17*a*) shows a cantilever welded to a support by fillet welds at top and bottom. A free-body diagram of the beam would show a shear-force reaction *V* and a moment reaction *M*. The shear force produces a primary shear in the welds of magnitude

$$\tau' = \frac{V}{A} \qquad \text{a}$$

where A is the total throat area.

The moment M induces a throat shear stress component of 0.707τ in the welds. Treating the two welds of Fig. (6–17b) as lines we find the unit second moment of area to be

$$I_u = \frac{bd^2}{2} \qquad \text{b}$$

The second moment of area I, based on weld throat area, is

$$I = 0.707hI_u = 0.707h(bd^2/2) \qquad \text{c}$$

The nominal throat shear stress is now found to be

$$\tau = \frac{Mc}{I} = \frac{Md/2}{0.707hbd^2/2} = \frac{1.414M}{bdh} \qquad \text{d}$$

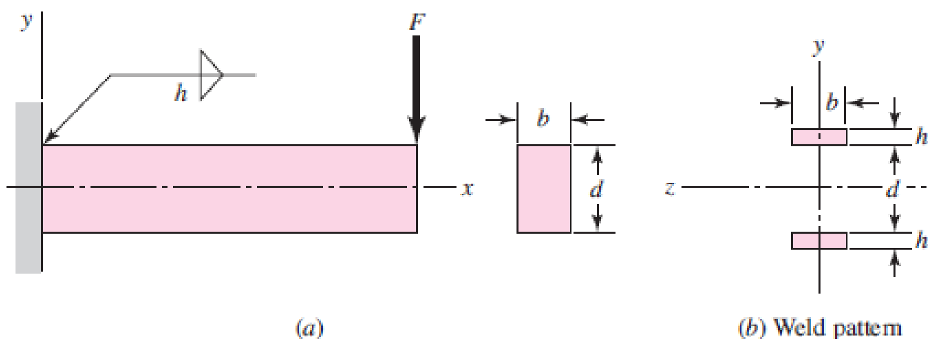


Figure (6–17)

A rectangular cross-section cantilever welded to a support at the top and bottom edges

The second moment of area in Eq. (d) is based on the distance d between the two welds. If this moment is found by treating the two welds as having rectangular footprints, the distance between the weld throat centroids is approximately ($d + h$). This would produce a slightly larger second moment of area, and result in a smaller level of stress. This method of treating welds as a line does not interfere with the conservatism of the model. It also makes Table (6–2) possible with all the conveniences that ensue.

Table (6–2)

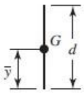
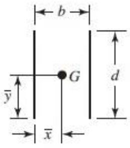
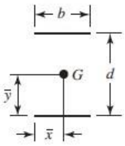
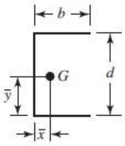
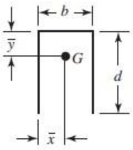
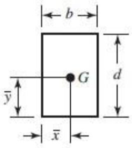
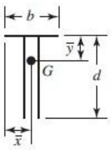
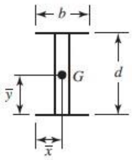
Bending Properties of Fillet Welds

I_u, unit second moment of area, is taken about a horizontal axis through *G*, the centroid of the weld group, *h* is weld size; the plane of the bending

couple is normal to the plane of the paper and parallel to the

y-axis; all welds are of the same size

Weld	Throat Area	Location of G	Unit Second Moment of Area

	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$
	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b+d}$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{12}(6b+d)$
	$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b+2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$
	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{6}(3b+d)$
	$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b+2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$
	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{6}(3b+d)$
	$A = 1.414\pi hr$		$I_u = \pi r^3$

6.5 The Strength of Welded Joints

The properties of electrodes vary considerably, but Table (6–3) lists the minimum properties for some electrode classes.

It is preferable, in designing welded components, to select a steel that will result in a fast, economical weld even though this may require a sacrifice of other qualities such as machinability. Under the proper conditions, all steels can be welded, but best results will be obtained if steels having a UNS specification between G10140 and G10230 are chosen. All these steels have a tensile strength in the hot-rolled condition in the range of 60 to 70 kpsi.

Table (6–3)

Minimum Weld-Metal Properties

AWS Electrode Number	Tensile Strength kpsi (MPa)	Yield Strength kpsi (MPa)	Percent Elongation
E60xx	62 (427)	50 (345)	17–25
E70xx	70 (482)	57 (393)	22
E80xx	80 (551)	67 (462)	19
E90xx	90 (620)	77 (531)	14–17
E100xx	100 (689)	87 (600)	13–16
E120xx	120 (827)	107 (737)	14

The designer can choose factors of safety or permissible working stresses with more confidence if he or she is aware of the values of those used by others. One of the best standards to use is the American Institute of Steel Construction (AISC) code for building construction. Table (6–4) lists the formulas specified by the code for calculating these permissible stresses for various loading conditions. The factors of safety implied by this code are easily calculated. For tension, $n = 1/0.6 = 1.67$. For shear, $n = 0.577/0.4 = 1.44$, using the distortion-energy theory as the criterion of failure.

It is important to observe that the electrode material is often the strongest material present. If a bar of AISI 1010 steel is welded to one of 1018 steel, the weld metal is actually a mixture of the electrode material and the 1010 and 1018 steels. Furthermore, a welded cold-drawn bar has its cold-drawn properties replaced with the hot-rolled properties in the vicinity of the weld. Finally, remembering that the weld metal is usually the strongest, do check the stresses in the parent metals.

Table (6–4)

Stresses Permitted by the AISC Code for Weld Metal

Type of Loading	Type of Weld	Permissible Stress	n
Tension	Butt	$0.60S_y$	1.67
Bearing	Butt	$0.90S_y$	1.11
Bending	Butt	$0.60–0.66S_y$	1.52–1.67
Simple compression	Butt	$0.60S_y$	1.67
Shear	Butt or fillet	$0.30S_{ut}^{*}$	

* Shear stress on base metal should not exceed $0.4S_y$ of base metal.

The fatigue stress-concentration factors listed in Table (6–5) are suggested for use. These factors should be used for the parent metal as well as for the weld metal.

Table (6–5)

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

6.6 Static Loading

Some examples of statically loaded joints are useful in comparing and contrasting the conventional method of analysis and the welding code methodology.

EXAMPLE 6–2

A 1/2-in by 2-in rectangular-cross-section 1015 bar carries a static load of 16.5 kip. It is welded to a gusset plate with a 3/8-in fillet weld 2 in long on both sides with an E70XX electrode (the allowable force per unit length is 5.57 kip/in of weldment) as depicted in Fig. (6–18). Use the welding code method.

(a) Is the weld metal strength satisfactory?

(b) Is the attachment strength satisfactory?

Solution

(*a*)

$F = 5.57l = 5.57(4) = 22.28$ kip

Since 22.28 > 16.5 kip, weld metal strength is satisfactory.

(*b*) Check shear in attachment adjacent to the welds. From Table (6–4) and Table (3–4), from which $S_y = 27.5$ kpsi, the allowable attachment shear stress is

$$\tau_{all} = 0.4S_y = 0.4(27.5) = 11 \text{ kpsi}$$

The shear stress τ on the base metal adjacent to the weld is

$$\tau = \frac{F}{2hl} = \frac{16.5}{2(0.375)2} = 11 \text{ kpsi}$$

Since $\tau_{all} \geq \tau$, the attachment is satisfactory near the weld beads. The tensile stress in the shank of the attachment σ is

$$\sigma = \frac{F}{tl} = \frac{16.5}{(1/2)2} = 16.5 \text{ kpsi}$$

The allowable tensile stress σ_{all}, from Table (6–4), is $0.6S_y$ and, with welding code safety level preserved,

$$\sigma_{all} = 0.6S_y = 0.6(27.5) = 16.5 \text{ kpsi}$$

Since $\sigma_{all} \geq \sigma$, the shank tensile stress is satisfactory.

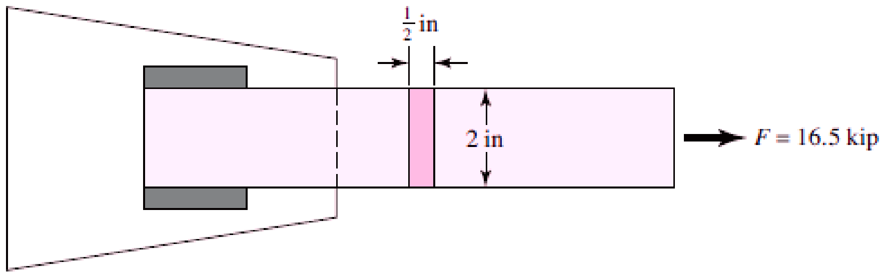


Figure (6–18)

EXAMPLE 6–3

A specially rolled A36 structural steel section for the attachment has a cross section as shown in Fig. (6–19) and has yield and ultimate tensile strengths of 36 and 58 kpsi, respectively. It is statically loaded through the attachment centroid by a load of F = 24 kip. Unsymmetrical weld tracks can compensate for eccentricity such that there is no moment to be resisted by the welds. Specify the weld track lengths l_1 and l_2 for a 5/16-in fillet weld using an E70XX electrode. This is part of a design problem in which the design variables include weld lengths and the fillet leg size.

Solution

The y coordinate of the section centroid of the attachment is

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{1(0.75)2 + 3(0.375)2}{0.75(2) + 0.375(2)} = 1.67 \text{ in}$$

Summing moments about point B to zero gives

$$\sum M_B = 0 = -F_1 b + F\bar{y} = -F_1(4) + 24(1.67)$$

from which
F_1 = 10 kip

It follows that
$F_2 = 24 - 10 = 14$ kip

The weld throat areas have to be in the ratio 14/10 = 1.4, that is, $l_2 = 1.4 l_1$. The weld length design variables are coupled by this relation, so l_1 is the weld length design variable. The other design variable is the fillet weld leg size h, which has been decided by the problem statement. From Table (6–4), the allowable shear stress on the throat τ_{all} is
$\tau_{all} = 0.3(70) = 21$ kpsi

The shear stress τ on the 45° throat is

$$\tau = \frac{F}{(0.707)h(l_1 + l_2)} = \frac{F}{(0.707)h(l_1 + 1.4l_1)}$$
$$= \frac{F}{(0.707)h(2.4l_1)} = \tau_{all} = 21 \text{ kpsi}$$

from which the weld length l_1 is

$$l_1 = \frac{24}{21(0.707)0.3125(2.4)} = 2.16 \text{ in}$$

and

$$l_2 = 1.4l_1 = 1.4(2.16) = 3.02 \text{ in}$$

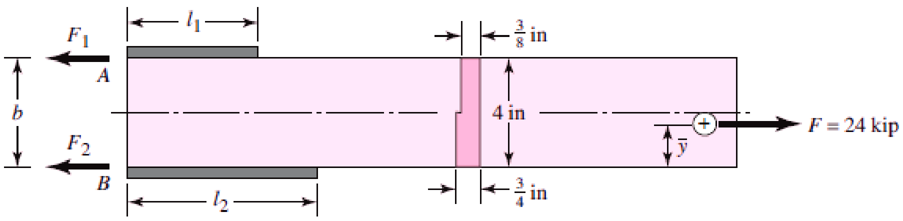


Figure (6–19)

These are the weld-bead lengths required by weld metal strength. The attachment shear stress allowable in the base metal, from Table (6–4), is

$$\tau_{all} = 0.4\, S_y = 0.4\,(36) = 14.4 \text{ kpsi}$$

The shear stress τ in the base metal adjacent to the weld is

$$\tau = \frac{F}{h(l_1 + l_2)} = \frac{F}{h(l_1 + 1.4l_1)} = \frac{F}{h(2.4l_1)} = \tau_{all} = 14.4 \text{ kpsi}$$

from which

$$l_1 = \frac{F}{14.4h(2.4)} = \frac{24}{14.4(0.3125)2.4} = 2.22 \text{ in}$$

$$l_2 = 1.4l_1 = 1.4(2.22) = 3.11 \text{ in}$$

$l_2 = 1.4\, l_1 = 1.4\ (2.22) = 3.11$ in

These are the weld-bead lengths required by base metal (attachment) strength. The base metal controls the weld lengths. For the allowable tensile stress σ_{all} in the shank of the attachment, the AISC allowable for tension members is $0.6Sy$; therefore,

$$\sigma_{all} = 0.6\, S_y = 0.6\ (36) = 21.6 \text{ kpsi}$$

The nominal tensile stress σ is *uniform* across the attachment cross section because of the load application at the centroid. The stress σ is

$$\sigma = \frac{F}{A} = \frac{24}{0.75(2) + 2(0.375)} = 10.7 \text{ kpsi}$$

Since $\sigma_{all} \geq \sigma$, the shank section is satisfactory. With l_1 set to a nominal 2.25 in, l_2 should be 1.4(2.25) = 3.15 in.

Decision

Set l_1 = 2.25 in, l_2 = 3.25 in. The small magnitude of the departure from l_2/l_1 = 1.4 is not serious. The joint is essentially moment-free.

EXAMPLE 6–4

Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. (6–20). The cantilever is made of AISI 1018 HR steel and welded with a 3/8-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.
(a) Use the conventional method for the weld metal.
(b) Use the conventional method for the attachment (cantilever) metal.

Solution

(a) From Table (6–3), S_y = 50 kpsi, S_{ut} = 62 kpsi. From Table (6–2), second pattern, b = 0.375 in, d = 2 in, so

$A = 1.414hd = 1.414\ (0.375)\ (2) = 1.06\ \text{in}^2$

$I_u = d^3/6 = 2^3/6 = 1.33\ \text{in}^3$

$I = 0.707hI_u = 0.707\ (0.375)\ (1.33) = 0.353\ \text{in}^4$

$$\tau' = \frac{F}{A} = \frac{500(10^{-3})}{1.06} = 0.472\ \text{kpsi}$$

Primary shear:

Secondary shear:

$$\tau'' = \frac{Mr}{I} = \frac{500(10^{-3})(6)(1)}{0.353} = 8.50 \text{ kpsi}$$

The shear magnitude τ is the Pythagorean combination:

$$\tau = (\tau'^2 + \tau''^2)^{1/2} = (0.472^2 + 8.50^2)^{1/2} = 8.51 \text{ kpsi}$$

The factor of safety based on a minimum strength and the distortion- energy criterion is:

$$n = \frac{S_{sy}}{\tau} = \frac{0.577(50)}{8.51} = 3.39$$

Since $n > n_d$, that is, 3.39 > 3.0, the weld metal has satisfactory strength.

(b) From Table (3–4), minimum strengths are S_{ut} = 58 kpsi and S_y = 32 kpsi. Then

$$\sigma = \frac{M}{I/c} = \frac{M}{bd^2/6} = \frac{500(10^{-3})6}{0.375(2^2)/6} = 12 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma} = \frac{32}{12} = 2.67$$

Since $n < n_d$, that is, 2.67 < 3.0, the joint is unsatisfactory as to the attachment strength.

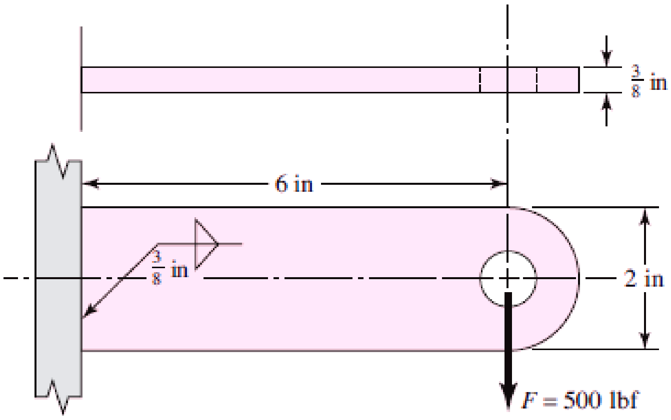


Figure (6–20)

6.7 Fatigue Loading

In fatigue, the Gerber criterion is best; however, you will find that the Goodman criterion is in common use. Recall, that the fatigue stress concentration factors are given in Table (6–5).

Some examples of fatigue loading of welded joints follow.

EXAMPLE 6–5

The 1018 steel strap of Fig. (6–21) has a 1000-lbf, completely reversed load applied. Determine the factor of safety of the weldment for infinite life.

Solution

From Table (3–4) for the 1018 attachment metal the strengths are S_{ut} = 58 kpsi and S_y = 32 kpsi. For the E6010 electrode, S_{ut} = 62 kpsi and S_y = 50 kpsi. The fatigue stress-concentration factor, from Table (6–5), is K_{fs} = 2.7. From Table (3–1), p. 49,

$$k_a = 39.9(58)^{-0.995} = 0.702$$

The shear area is:

$$A = 2(0.707)(0.375)(2) = 1.061 \text{ in}^2$$

For a uniform shear stress on the throat, $k_b = 1$.

From Eq. (3–8), p. 51, for torsion (shear), $k_c = 0.59$

$k_d = k_e = k_f = 1$

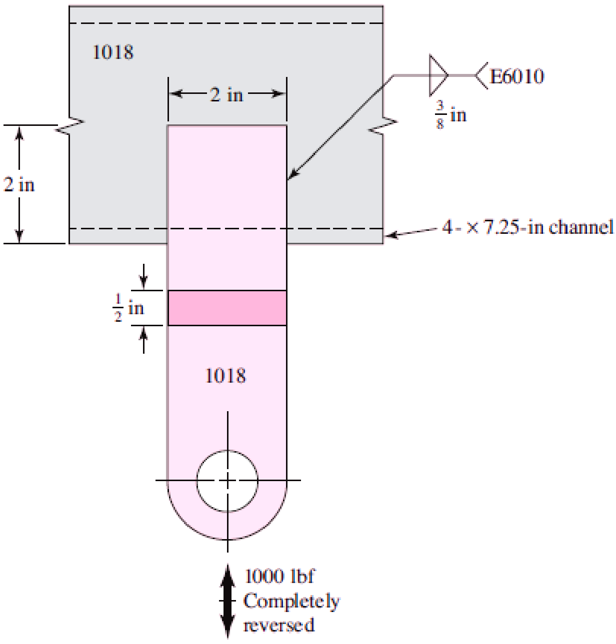


Figure (6–21)

From Eqs. (3–1), p. 48, and (3–2), p. 49,

$S_{se} = 0.702(1)(0.59)(1)(1)(1)(0.5)(58) = 12$ kpsi

$K_{fs} = 2.7$ $\quad F_a = 1000$ lbf $\quad F_m = 0$

Only primary shear is present:

$$\tau'_a = \frac{K_{fs} F_a}{A} = \frac{2.7(1000)}{1.061} = 2545 \text{ psi} \qquad \tau'_m = 0 \text{ psi}$$

In the absence of a midrange component, the fatigue factor of safety n_f is given by

$$n_f = \frac{S_{se}}{\tau'_a} = \frac{12\,000}{2545} = 4.72$$

EXAMPLE 6–6

The 1018 steel strap of Fig. (6–22) has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.

Solution

From Table (3–1), p. 49,

$k_a = 39.9(58)^{-0.995} = 0.702$

$A = 2(0.707)(0.375)(2) = 1.061\ \text{in}^2$

For uniform shear stress on the throat $k_b = 1$

From Eq. (3–8), p. 51, $k_c = 0.59$

From Eqs. (3–1), p. 48, and (3–2), p. 49,

$$S_{se} = 0.702(1)(0.59)(1)(1)(1)(0.5)(58) = 12\ \text{kpsi}$$

From Table (6–5), $K_{fs} = 2$

Only primary shear is present:

$$\tau'_a = \tau'_m = \frac{K_{fs} F_a}{A} = \frac{2(1000)}{1.061} = 1885\ \text{psi}$$

$S_{su} = 0.67S_{ut}$, This, together with the Gerber fatigue failure criterion for shear stresses from Table (3–6), p. 66, gives

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\,\right]$$

$$n_f = \frac{1}{2}\left[\frac{0.67(58)}{1.885}\right]^2 \frac{1.885}{12.0}\left\{-1+\sqrt{1+\left[\frac{2(1.885)12.0}{0.67(58)1.885}\right]^2}\,\right\} = 5.85$$

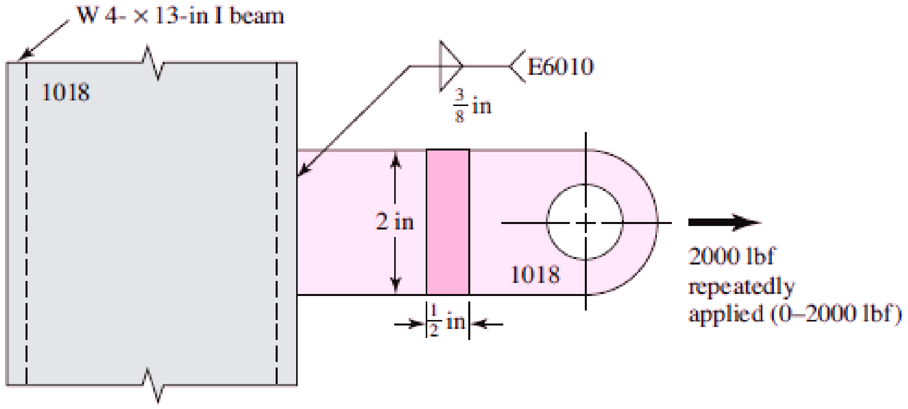


Figure (6–22)

6.8 Resistance Welding

The heating and consequent welding that occur when an electric current is passed through several parts that are pressed together is called *resistance welding*. *Spot welding* and *seam welding* are forms of resistance welding most often used. The advantages of resistance welding over other forms are the speed, the accurate regulation of time and heat, the uniformity of the weld, and the mechanical properties that result. In addition the process is easy to automate, and filler metal and fluxes are not needed.

The spot- and seam-welding processes are illustrated schematically in Fig. (6–23). Seam welding is actually a series of overlapping spot welds, since the current is applied in pulses as the work moves between the rotating electrodes.

Failure of a resistance weld occurs either by shearing of the weld or by tearing of the metal around the weld. Because of the possibility of tearing, it is good practice to avoid loading a resistance-welded joint in tension. Thus, for the most part, design so that the spot or seam is loaded in pure shear. The shear stress is then simply the load divided by the area of the spot. Because the thinner sheet of the pair being welded may tear, the strength of spot welds is often specified by stating the load per spot based on the thickness of the thinnest sheet. Such strengths are best obtained by experiment.

Somewhat larger factors of safety should be used when parts are fastened by spot welding rather than by bolts or rivets, to account for the metallurgical changes in the materials due to the welding.

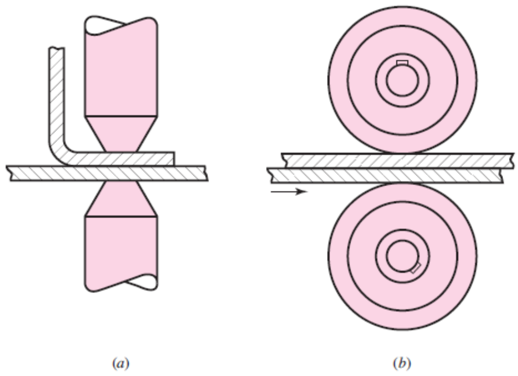


Figure (6–23)