1. Mechanical Springs Mechanical Springs

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. In general, springs may be classified as wire springs, flat springs, or special-shaped springs, and there are variations within these divisions. Wire springs include helical springs of round or square wire, made to resist and deflect under tensile, compressive, or torsional loads. Flat springs include cantilever and elliptical types, wound motor- or clock-type power springs, and flat spring washers, usually called Belleville springs.

7.1 Stresses in Helical Springs

Figure (7–1*a*) shows a round-wire helical compression spring loaded by the axial force ***F***. We designate ***D*** as the *mean coil diameter* and ***d*** as the *wire diameter*. Now imagine that the spring is cut at some point (Fig. 7–1*b*), then, at the *inside* fiber of the spring,

$$\tau_{max} = \frac{Tr}{J} + \frac{F}{A} \qquad \text{7-1}$$

at the inside fiber of the spring. Substitution of $\tau_{max} = \tau$, $T = F\,D/2$, $r = d/2$, $J = \pi d^4/32$, and $A = \pi d^2/4$ gives

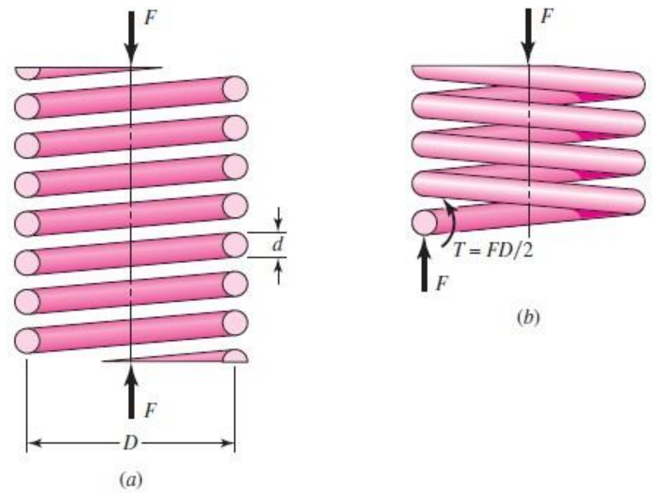


Figure (7–1)
(a) Axially loaded helical spring; (*b*) free-body diagram showing that the wire is

subjected to a direct shear and a torsional shear.

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Now we define the *spring index* 7-2

which is a measure $C = \frac{D}{d}$ of coil curvature. With this relation, Eq. (7–1) can be rearranged to give

$$\tau = K_s \frac{8FD}{\pi d^3} \qquad 7\text{-}3$$

where K_s is a *shear-stress correction factor* and is defined by the equation

$$K_s = \frac{2C + 1}{2C} \qquad 7\text{-}4$$

For most springs, C ranges from about 6 to 12. Equation (7–3) is quite general and applies for both static and dynamic loads.

The use of square or rectangular wire is not recommended for springs unless space limitations make it necessary. Springs of special wire shapes are not made in large quantities, unlike those of round wire; they have not had the benefit of refining development and hence may not be as strong as springs made from round wire. When space is severely limited, the use of nested round-wire springs should always be considered. They may have an economical advantage over the special-section springs, as well as a strength advantage.

7.2 The Curvature Effect

Equation (7–1) is based on the wire being straight. However, the curvature of the wire increases the stress on the inside of the spring but decreases it only slightly on the outside. This curvature stress is primarily important in fatigue because the loads are lower and there is no opportunity for localized yielding. For static loading, these stresses can normally be neglected because of strain-strengthening with the first application of load.

Unfortunately, it is necessary to find the curvature factor in a roundabout way. The reason for this is that the published equations also include the effect of the direct shear stress. Suppose K_s in Eq. (7–3) is replaced by another K factor, which corrects for both curvature and direct shear. Then this factor is given by either of the equations

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad \text{7-5}$$

$$K_B = \frac{4C+2}{4C-3} \quad \text{7-6}$$

The first of these is called the *Wahl factor*, and the second, the *Bergsträsser factor*. Since the results of these two equations differ by less than 1 percent, Eq. (7–6) is preferred. The curvature correction factor can now be obtained by canceling out the effect of the direct shear. Thus, using Eq. (7–6) with Eq. (7–4), the curvature correction factor is found to be

$$K_c = \frac{K_B}{K_s} = \frac{2C(4C+2)}{(4C-3)(2C+1)} \quad \text{1-7}$$

Now, K_S , K_B or K_W , and K_C are simply stress correction factors applied multiplicatively to Tr/J at the critical location to estimate a particular stress. There is *no* stress concentration factor. We will use $\tau = K_B(8FD)/(\pi d^3)$ to predict the largest shear stress.

7.3 Deflection of Helical Springs

The deflection-force relations are quite easily obtained by using Castigliano's theorem. The total strain energy for a helical spring is composed of a torsional component and a shear component. The strain energy is

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Substituting T = F.D/2, l = π DN, J = πd^4 /32, and A = πd^2 /4 results in

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

where N = N_a = number of active coils. Then using Castigliano's theorem, to find total deflection y gives:

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N}{d^4 G} + \frac{4FDN}{d^2 G}$$

Since C = D/d, the previous Equation can be rearranged to yield

$$y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) \doteq \frac{8FD^3N}{d^4G}$$

The spring rate, also called the scale of the spring, is k = F/y, and so

$$k \doteq \frac{d^4G}{8D^3N}$$

7.4 Compression Springs

The four types of ends generally used for compression springs are illustrated in Fig. (7–2). A spring with *plain ends* has a noninterrupted helicoid; the ends are the same as if a long spring had been cut into sections. A spring with plain ends that are *squared* or *closed* is obtained by deforming the ends to a zero-degree helix angle. Springs should always be both squared and ground for important applications, because a better transfer of the load is obtained.

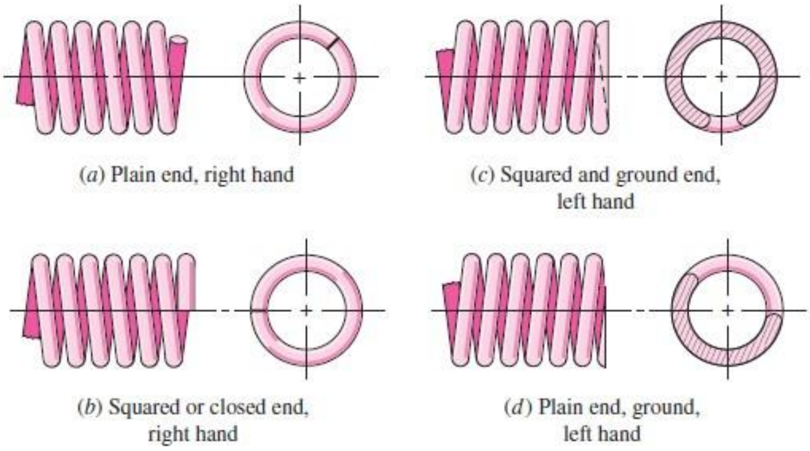


Figure (7–2)

Types of ends for compression springs: (*a*) both ends plain;

(*b*) both ends squared; (*c*) both ends squared and ground;
(*d*) both ends plain and ground.

Table (7–1) shows how the type of end used affects the number of coils and the spring length. Note that the digits 0, 1, 2, and 3 appearing in Table (7–1) are often used without question. *Some of these need closer scrutiny as they may not be integers.*This depends on how a springmaker forms the ends. Forys pointed out that squared and ground ends give a solid length L_s of

$$L_s = (N_t - a)\, d$$

where a varies, with an average of 0.75, so the entry dN_t in Table (7–1) may be overstated. The way to check these variations is to take springs from a particular springmaker, close them solid, and measure the solid height. Another way is to look at the spring and count the wire diameters in the solid stack.

Set removal or *presetting* is a process used in the manufacture of compression springs to induce useful residual stresses. It is done by making the spring longer than needed and then compressing it to its solid height. This operation *sets* the spring to the required final free length and, since the torsional yield strength has been exceeded, induces residual stresses opposite in direction to those induced in service. Springs to be preset should be designed so that 10 to 30 percent of the initial free length is removed during the operation. If the stress at the solid height is greater than 1.3 times the torsional yield strength, distortion may occur. If this stress is much less than 1.1 times, it is difficult to control the resulting free length.

Set removal increases the strength of the spring and so is especially useful when the spring is used for energy-storage purposes. However, set removal should not be used when springs are subject to fatigue.

Table (7–1)

Formulas for the Dimensional Characteristics of Compression-Springs.

(N_a = Number of Active Coils)

Term	*Type of Spring Ends*			
	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

7.5 Spring Materials

Springs are manufactured either by hot- or cold-working processes, depending upon the size of the material, the spring index, and the properties desired. In general, prehardened wire should not be used if $D/d < 4$ or if $d > 1/4$ in. Winding of the spring induces residual stresses through bending, but these are normal to the direction of the torsional working stresses in a coil spring. Quite frequently in spring manufacture, they are relieved, after winding, by a mild thermal treatment.

A great variety of spring materials are available to the designer, including plain carbon steels, alloy steels, and corrosion-resisting steels, as well as nonferrous materials such as phosphor bronze, spring brass, beryllium copper, and various nickel alloys.

Spring materials may be compared by an examination of their tensile strengths; these vary so much with wire size that they cannot be specified until the wire size is known. The material and its processing also, of course, have an effect on tensile strength. It turns out that the graph of tensile strength versus wire diameter is almost a straight line for some materials when plotted on log-log paper. Writing the equation of this line as

$$S_{ut} = \frac{A}{d^m}$$

furnishes a good means of estimating minimum tensile strengths when the intercept A and the slope m of the line are known. Values of these constants have been worked out from recent data and are given for strengths in units of kpsi and MPa in Table (7–3). In Eq. (7–10) when d is measured in millimeters, then A is in MPa · mm^m and when d is measured in inches, then A is in kpsi · in^m.

A very rough estimate of the torsional yield strength can be obtained by assuming that the tensile yield strength is between 60 and 90 percent of the tensile strength. Then the distortion-energy theory can be employed to obtain the torsional yield strength ($S_{ys} = 0.577S_y$). This approach results in the range

$$0.35S_{ut} \leq S_{sy} \leq 0.52S_{ut} \qquad \textit{for steels} \qquad 7\text{-}11$$

For wires listed in Table (7–4), the maximum allowable shear stress in a spring can be seen in column 3. Music wire and hard-drawn steel spring wire have a low end of range $S_{sy} = 0.45S_{ut}$. Valve spring wire, Cr-Va, Cr-Si, and other (not shown) hardened and tempered carbon and low-alloy steel wires as a group have $S_{sy} \geq 0.50S_{ut}$. Many nonferrous materials (not shown) as a group have $S_{sy} \geq 0.35S_{ut}$. In view of this, *Joerres* uses the maximum allowable torsional stress for static application shown in Table (7–5). For specific materials for which you have torsional yield information use this table as a guide. *Joerres* provides set-removal information in Table (7–5), that $S_{sy} \geq 0.65S_{ut}$ increases strength through cold work, but at the cost of an additional operation by the springmaker. Sometimes the additional operation can be done by the manufacturer during assembly. Some correlations with carbon steel springs show that the tensile yield strength of spring wire in torsion can be estimated from $0.75S_{ut}$. The corresponding estimate of the yield strength in shear based on distortion energy theory is

$S_{sy} = 0.577(0.75)S_{ut} = 0.433S_{ut} = 0.45S_{ut}$. *Samónov* discusses the problem of allowable stress and shows that

$$S_{sy} = \tau_{all} = 0.56S_{ut} \qquad 7\text{-}12$$

for high-tensile spring steels, which is close to the value given by Joerres for hardened alloy steels. He points out that this value of allowable stress is specified by Draft Standard 2089 of the German Federal Republic when Eq. (7–3) is used without stress-correction factor.

Table (7–2)

High-Carbon and Alloy Spring Steels

Name of Material	Similar Specifications	Description
Music wire, 0.80–0.95*C*	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures.
Oil-tempered wire, 0.60–0.70*C*	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures.
Hard-drawn wire, 0.60–0.70*C*	UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures.
Chrome-vanadium	UNS G61500 AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pretempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter.

Chrome-silicon	UNS G92540 AISI 9254	This alloy is an excellent material for highly stressed springs that require long life and are subjected to shock loading. Rockwell hardnesses of C50 to C53 are quite common, and the material may be used up to 250°C (475°F). Available from 0.8 to 12 mm (0.031 to 0.500 in) in diameter.

Table (7–3)

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

*Surface is smooth, free of defects, and has a bright, lustrous finish. †Has a slight heat-treating scale which must be removed before plating. ‡Surface is smooth and bright with no visible marks. §Aircraft-quality tempered wire, can also be obtained annealed. "Tempered to Rockwell C49, but may be obtained untempered. #Type 302 stainless steel. * *Temper CA510.

Material	ASTM No.	Exponent m	Diameter, in	A, Kpsi.in^m	Diameter, mm	A, MPa.mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

Table (7–4)

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

*Also includes 302, 304, and 316.

Table (7–5)
Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Material	*Maximum Percent of Tensile Strength*	
	Before Set Removed (includes K_W or K_B)	After Set Removed (includes K_S)
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

EXAMPLE 7–1

A helical compression spring is made of no.16 music wire of diameter (d = 0.037 in). The outside diameter of the spring is 7/16 in. The ends are squared and there are 12.5 total turns.

(a) Estimate the torsional yield strength of the wire.
(b) Estimate the static load corresponding to the yield strength.
(c) Estimate the scale of the spring.
(d) Estimate the deflection that would be caused by the load in part (b).
(e) Estimate the solid length of the spring.

Solution

(a) From Table (7–3), we find A = 201 kpsi·in^m and m = 0.145. Therefore, from Eq. (7–10)

$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.037^{0.145}} = 324 \text{ kpsi}$$

Then, from Table (7–5),

$$S_{sy} = 0.45S_{ut} = 0.45(324) = 146 \text{ kpsi}$$

(b) The mean spring coil diameter is D = 7/16 − 0.037 = 0.4 in, and so the spring index is C = 0.4/0.037 = 10.8. Then, from Eq. (7–6),

$$K_B = \frac{4C+2}{4C-3} = \frac{4\,(10.8)+2}{4\,(10.8)-3} = 1.124$$

Now rearrange Eq. (7–3) replacing K_S and τ with K_B and S_{ys}, respectively, and solve for F:

$$F = \frac{\pi d^3 S_{sy}}{8 K_B D} = \frac{\pi (0.037^3) 146(10^3)}{8(1.124)\,0.400} = 6.46 \text{ lbf}$$

(c) From Table (7–1), N_a = 12.5 − 2 = 10.5 turns. In Table (7–4), G = 11.85 Mpsi, and the scale of the spring is found to be, from Eq. (7–9),

$$k = \frac{d^4 G}{8 D^3 N_a} = \frac{0.037^4\,(11.85)10^6}{8(0.400^3)10.5} = 4.13 \text{ lbf/in}$$

(d)

$$y = \frac{F}{k} = \frac{6.46}{4.13} = 1.56 \text{ in}$$

(*e*) From Table (7–1),

$$L_S = (N_t + 1)\, d = (12.5 + 1)\, 0.037 = 0.5 \text{ in}$$

7.6 Helical Compression Spring Design for Static Service

The preferred range of spring index is $4 \leq C \leq 12$, with the lower indexes being more difficult to form (because of the danger of surface cracking) and springs with higher indexes tending to tangle often enough to require individual packing. This can be the first item of the design assessment. The recommended range of active turns is $3 \leq Na \leq 15$. To maintain linearity when a spring is about to close, it is necessary to avoid the gradual touching of coils (due to non-perfect pitch). A helical coil spring force-deflection characteristic is ideally linear. Practically, it is nearly so, but not at each end of the force-deflection curve. The spring force is not reproducible for very small deflections, and near closure, nonlinear behavior begins as the number of active turns diminishes as coils begin to touch. The designer confines the spring's operating point to the central 75 percent of the curve between no load, $F = 0$, and closure, $F = F_S$. Thus, the maximum operating force should be limited to $F_{max} \leq 7/8\ F_S$.

$$F_s = (1 + \xi)F_{\max}$$

it follows that

$$F_s = (1 + \xi)F_{\max} = (1 + \xi)\left(\frac{7}{8}\right)F_s$$

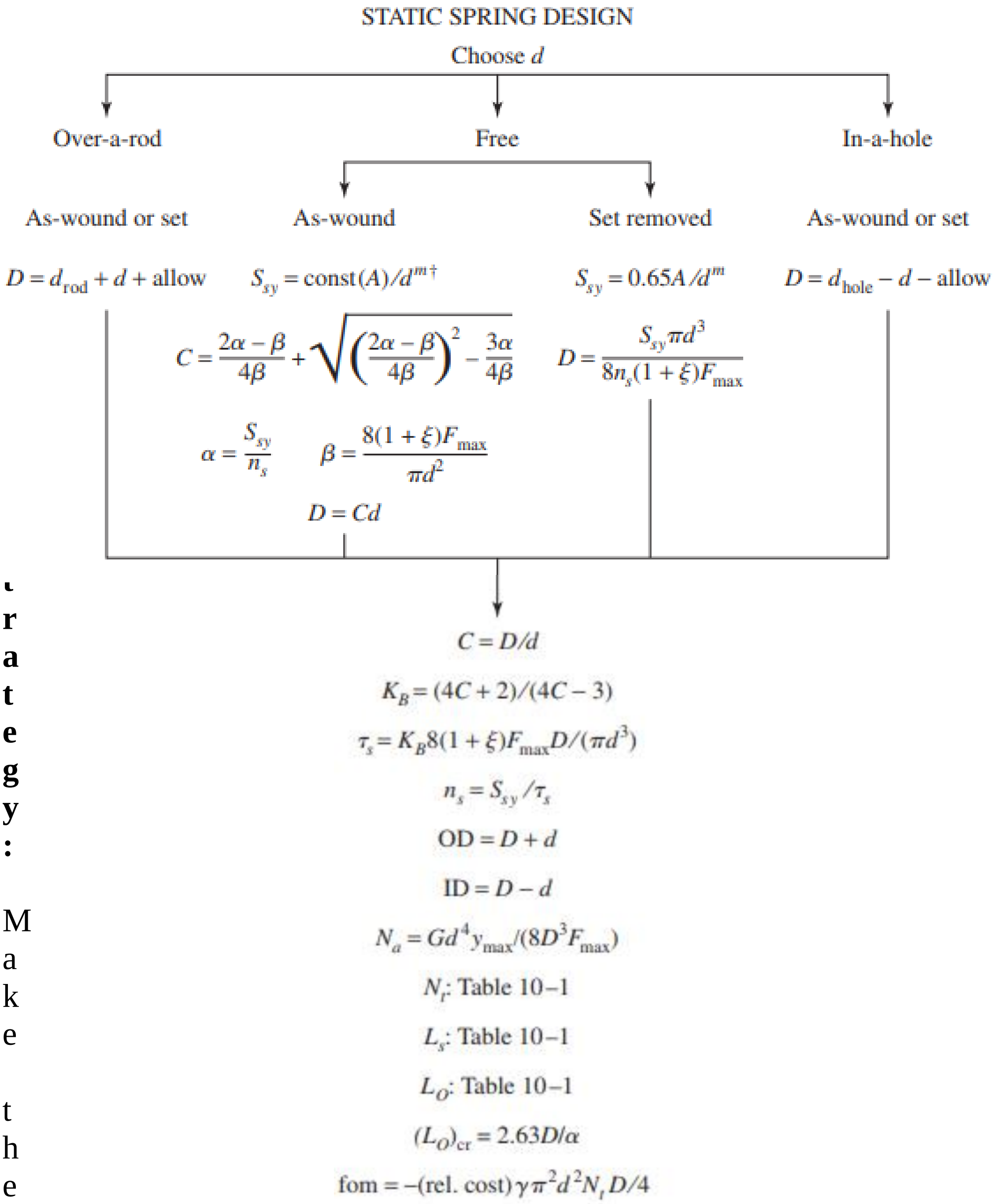
From the outer equality $\xi = 1/7 = 0.143 = 0.15$. Thus, it is recommended that $\xi \geq 0.15$. In addition to the relationships and material properties for springs, we now have some recommended design conditions to follow, namely:

$$4 \leq C \leq 12 \qquad \xi \geq 0.15$$

$$3 \leq N_a \leq 15 \qquad n_s \geq 1.2$$

where ns is the factor of safety at closure (solid height). When considering designing a spring for high volume production, the figure of merit can be the cost of the wire from which the spring is wound. The fom would be proportional to the relative material cost, weight density, and volume:

$$\text{fom} = -(\text{relative material cost})\frac{\gamma\pi^2 d^2 N_t D}{4}$$



trategy: Make the a priori decisions, with hard-drawn steel wire the first choice (relative material cost is 1.0). Choose a wire size d. With all decisions made, generate a column of parameters: d, D, C, OD or ID, Na, Ls, L_0, $(L_0)_{cr}$, n_s , and fom. By incrementing wire sizes available, we can scan the table of parameters and apply the design recommendations by inspection. After wire sizes are eliminated, choose the spring design with the highest figure of merit. This will give the optimal design despite the presence of a discrete design variable d and aggregation of equality and inequality constraints. The column vector of information can be generated by using the flowchart displayed in Fig. 10–3. It is general enough to accommodate to the situations of as-wound and set-removed springs, operating over a rod, or in a hole free of rod or hole. In as-wound springs the controlling equation must be solved for the spring index as follows. $\tau = S_{sy}/ns$, C = D/d, KB from Eq. (10–6), and Eq. (10–17),

$$\frac{S_{sy}}{n_s} = K_B \frac{8F_s D}{\pi d^3} = \frac{4C+2}{4C-3}\left[\frac{8(1+\xi)\, F_{\max} C}{\pi d^2}\right]$$

Let:

$$\alpha = \frac{S_{sy}}{n_s}$$

$$\beta = \frac{8\,(1+\xi)\,F_{\max}}{\pi d^2}$$

Substituting previous Equations and simplifying yields a quadratic equation in C. The larger of the two solutions will yield the spring index

$$C = \frac{2\alpha-\beta}{4\beta} + \sqrt{\left(\frac{2\alpha-\beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

Example: A music wire helical compression spring is needed to support a 20-lbf load after being compressed 2 in. Because of assembly considerations the solid height cannot exceed 1 in and the free length cannot be more than 4 in. Design the spring.

Solution: The a priori decisions are • Music wire, A228; from Table 10–4, A = 201 000 psi-in^m ; m = 0.145; from Table 10–5, E = 28.5 Mpsi, G = 11.75 Mpsi (expecting d > 0.064 in)

• Ends squared and ground
• Function: F_{max} = 20 lbf, y_{max} = 2 in
• Safety: use design factor at solid height of $(n_s)_d$ = 1.2
• Robust linearity: ξ = 0.15
• Use as-wound spring (cheaper), $S_{sy} = 0.45S_{ut}$ from Table 10–6
• Decision variable: d = 0.080 in, music wire gage #30, Table A–28. From Fig. 10–3 and Table 10–6,

$$S_{sy} = 0.45\frac{201\,000}{0.080^{0.145}} = 130\,455 \text{ psi}$$

$$\alpha = \frac{S_{sy}}{} = \frac{130\ 455}{} = 108\ 713\,\text{psi}$$

$$\text{OD} = 0.843 + 0.080 = 0.923\text{ in}$$

$$N_a = \frac{0.080^4(11.75)10^6(2)}{8(0.843)^3 20} = 10.05\text{ turns}$$

$$\frac{(108\ 713)}{4(9151.4)} = 10.53$$

$$N_t = 10.05 + 2 = 12.05\text{ total turns}$$

$$L_s = 0.080(12.05) = 0.964\text{ in}$$

(

$$L_0 = 0.964 + (1 + 0.15)2 = 3.264\text{ in}$$

$$(L)_{cr} = 2.63(0.843/0.5) = 4.43\text{ in}$$

$$\text{fom} = -2.6\pi^2(0.080)^2 12.05(0.843)/4 = -0.417$$

$$\tau_s = 1.128\frac{8(1 + 0.15)20(0.8424)}{\pi(0.080)^3} = 108\ 700\text{ psi}$$

$$n_s = \frac{130\ 445}{108\ 700} = 1.2$$

e

: Indexing is used in machine operations when a circular part being manufactured must be divided into a certain number of segments. Figure 10–4 shows a portion of an indexing fixture used to successively position a part for the operation. When the knob is momentarily pulled up, part 6, which holds the workpiece, is rotated about a vertical axis to the next position and locked in place by releasing the index pin. In this example we wish to design the spring to exert a force of about 3 lbf and to fit in the space defined in the figure caption.

Solution: Since the fixture is not a high-production item, a stock spring will be selected. These are available in music wire. In one catalog there are 76 stock springs available having an outside diameter of 0.480 in and designed to work in a 1 2 -in hole. These are made in seven different wire sizes, ranging from 0.038 up to 0.063 in, and in free lengths from 1 2 to 2 1 2 in, depending upon the wire size.

Since the pull knob must be raised 3 4 in for indexing and the space for the spring is 1 3 8 in long when the pin is down, the solid length cannot be more than 5 8 in. Let us begin by selecting a spring having an outside diameter of 0.480 in, a wire size of 0.051 in, a free length of 1 3 4 in, 111 2 total turns, and plain ends. Then m = 0.145 and A = 201 kpsi · in^m for music wire. Then

$$S_{sy} = 0.45\frac{A}{d^m} = 0.45\frac{201}{0.051^{0.145}} = 139.3 \text{ kpsi}$$

With plain ends, from Table 10–1, the number of active turns is

$$N_a = N_t = 11.5 \text{ turns}$$

The mean coil diameter is $D = \text{OD} - d = 0.480 - 0.051 = 0.429$ in. From Eq. (10–9) the spring rate is, for $G = 11.85(10^6)$ psi from Table 10–5,

$$k = \frac{d^4G}{8D^3N_a} = \frac{0.051^4(11.85)10^6}{8(0.429)^3 11.5} = 11.0 \text{ lbf/in}$$

From Table 10–1, the solid height L_s is

$$L_s = d(N_t + 1) = 0.051(11.5 + 1) = 0.638 \text{ in}$$

The spring force when the pin is down, $F_{\min}$, is

$$F_{\min} = ky_{\min} = 11.0(1.75 - 1.375) = 4.13 \text{ lbf}$$

When the spring is compressed solid, the spring force F_s is

$$F_s = ky_s = k(L_0 - L_s) = 11.0(1.75 - 0.638) = 12.2 \text{ lbf}$$

Since the spring index is $C = D/d = 0.429/0.051 = 8.41$,

$$K_B = \frac{4C+2}{4C-3} = \frac{4(8.41)+2}{4(8.41)-3} = 1.163$$

and for the as-wound spring, the shear stress when compressed solid is

$$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.163\frac{8(12.2)0.429}{\pi(0.051)^3} = 116\,850 \text{ psi}$$

The factor of safety when the spring is compressed solid is

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{139.3}{116.9} = 1.19$$

ate and Ls is larger than 5 8 in, we must investigate other springs with a smaller wire size. After several investigations another spring has possibilities. It is as-wound music wire, d = 0.045 in, 20 gauge (see Table A–25) OD = 0.480 in, N_t = 11.5 turns, L_0 = 1.75 in. S_{sy} is still 139.3 kpsi, and

$$D = \text{OD} - d = 0.480 - 0.045 = 0.435 \text{ in}$$

$$N_a = N_t = 11.5 \text{ turns}$$

$$k = \frac{0.045^4(11.85)10^6}{8(0.435)^3 11.5} = 6.42 \text{ lbf/in}$$

$$L_s = d(N_t + 1) = 0.045(11.5 + 1) = 0.563 \text{ in}$$

$$F_{\text{min}} = ky_{\text{min}} = 6.42(1.75 - 1.375) = 2.41 \text{ lbf}$$

$$F_s = 6.42(1.75 - 0.563) = 7.62 \text{ lbf}$$

$$C = \frac{D}{d} = \frac{0.435}{0.045} = 9.67$$

$$K_B = \frac{4(9.67) + 2}{4(9.67) - 3} = 1.140$$

$$\tau_s = 1.140 \frac{8(7.62)0.435}{\pi(0.045)^3} = 105\,600 \text{ psi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{139.3}{105.6} = 1.32$$

Now ns > 1.2, buckling is not possible as the coils are guarded by the hole surface, and the solid length is less than 5 8 in, so this spring is selected. By using a stock spring, we take advantage of economy of scale.