CHAPTER 3 DESIGN METHODS AND REQUIREMENTS

3.1 ACI BUILDING CODE

When two different materials, such as steel and concrete, act together, it is understandable that the analysis for strength of a reinforced concrete member has to be partly empirical. These principles and methods are being constantly revised and improved as results of theoretical and experimental research accumulate. The American Concrete Institute (ACI), serving as a clearinghouse for these changes, issues building code requirements, the most recent of which is the Building Code Requirements for Structural Concrete (ACI 318-08), hereafter referred to as the ACI Code.

The ACI Code is a Standard of the American Concrete Institute. In order to achieve legal status, it must be adopted by a governing body as a part of its general building code. The ACI Code is partly a specification-type code, which gives acceptable design and construction methods in detail, and partly a performance code, which states desired results rather than details of how such results are to be obtained. A building code, legally adopted, is intended to prevent people from being harmed; therefore, it specifies minimum requirements to provide adequate safety and serviceability. It is important to realize that a building code is not a recommended practice, nor is it a design handbook, nor is it intended to replace engineering knowledge, judgment, or experience. It does not relieve the designer of the responsibility for having a safe, economical structure.

ACI 318M-08 – Building Code Requirements for Structural Concrete and Commentary.

Two philosophies of design have long been prevalent:

- The working stress method (1900 – 1960).
- The strength design method (1960 till now, with few exceptions).

3.2 WORKING STRESS METHOD

In the working stress method, a structural element is so designed that the stresses resulting from the action of service loads (also called working loads) and computed by the mechanics of elastic members do not exceed some predesignated allowable values.

Service load is the load, such as dead, live, snow, wind, and earthquake, which is assumed actually to occur when the structure is in service.

The working stress method may be expressed by the following:

$$f \le f_{allow}$$

where

$f -$ an elastic stress, such as by using the flexure formula $f = Mc/I$ for a beam, computed under service load.

$f_{allow} -$ a limiting or allowable stress prescribed by a building code as a percentage of the compressive strength f_c' for concrete, or of the yield stress for the steel reinforcing bars.

3.3 STRENGTH DESIGN METHOD

In the strength design method (formerly called ultimate strength method), the service loads are increased by factors to obtain the load at which failure is considered to be "imminent". This load is called the factored load or factored service load. The structure or structural element is then proportioned such that the strength is reached when the factored load is acting. The computation of this strength takes into account the nonlinear stress-strain behavior of concrete.

The strength design method may be expressed by the following,

strength provided ≥ [strength required to carry factored loads]

where the "strength provided" (such as moment strength) is computed in accordance with the provisions of a building code, and the "strength required" is that obtained by performing a structural analysis using factored loads.

3.4 SAFETY PROVISIONS

Structures and structural members must always be designed to carry some reserve load above what is expected under normal use. Such reserve capacity is provided to account for a variety of factors, which may be grouped in two general categories:

- factors relating to overload
- factors relating to understrength (that is, less strength than computed by acceptable calculating procedures).

Overloads may arise from changing the use for which the structure was designed, from underestimation of the effects of loads by oversimplification in calculation procedures, and from effects of construction sequence and methods. Understrength may result from adverse variations in material strength, workmanship, dimensions, control, and degree of supervision, even though individually these items are within required tolerances.

In the strength design method, the member is designed to resist factored loads, which are obtained by multiplying the service loads by load factors. Different factors are used for different loadings. Because dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty. Several load combinations must be considered in the design to compute the maximum and minimum design forces. Reduction factors are used for some combinations of loads to reflect the low probability of their simultaneous occurrences. The ACI Code presents specific values of load factors to be used in the design of concrete structures.

In addition to load factors, the ACI Code specifies another factor to allow an additional reserve in the capacity of the structural member. The nominal strength is generally calculated using accepted analytical procedure based on statistics and equilibrium; however, in order to account for the degree of accuracy within which the nominal strength can be calculated, and for adverse variations in materials and dimensions, a strength reduction factor, ϕ, should be used in the strength design method.

To summarize the above discussion, the ACI Code has separated the safety provision into an overload or load factor and to an undercapacity (or strength reduction) factor, ϕ. A safe design is achieved when the structure's strength, obtained by multiplying the nominal strength by the reduction factor, ϕ, exceeds or equals the strength needed to withstand the factored loadings (service loads times their load factors).

The requirement for strength design may be expressed:

$$\text{Design strength} \geq \text{Factored load (i.e., required strength)}$$

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

where P_n, M_n, and V_n are "nominal" strengths in axial compression, bending moment, and shear, respectively, using the subscript n.

P_u, M_u, and V_u are the factored load effects in axial compression, bending moment, and shear, respectively, using the subscript u.

Given a load factor of 1.2 for dead load and a load factor of 1.6 for live load, the overall safety factor for a structure loaded be a dead load, D, and a live load, L, is

$$\text{Factor of Safety} = \frac{1.2D + 1.6\,L}{D + L}\left(\frac{1}{\phi}\right)$$

3.5 LOAD FACTORS AND STRENGTH REDUCTION FACTORS

Overload Factors *U*

The factors U for overload as given by ACI-9.2 are:

$$U = 1.4(D + F)$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S$$

$$U = 0.9D + 1.6W + 1.6H$$

$$U = 0.9D + 1.0E + 1.6H$$

where

D - dead load; L - live load; L_r- roof live load; S - snow load; R - rain load; W - wind load; E - earthquake load; F - load due to weights and pressures of fluids with well-defined densities and controllable maximum heights; H - load due to weight and pressure of soil, water in soil or other materials; T - the cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage compensating concrete.

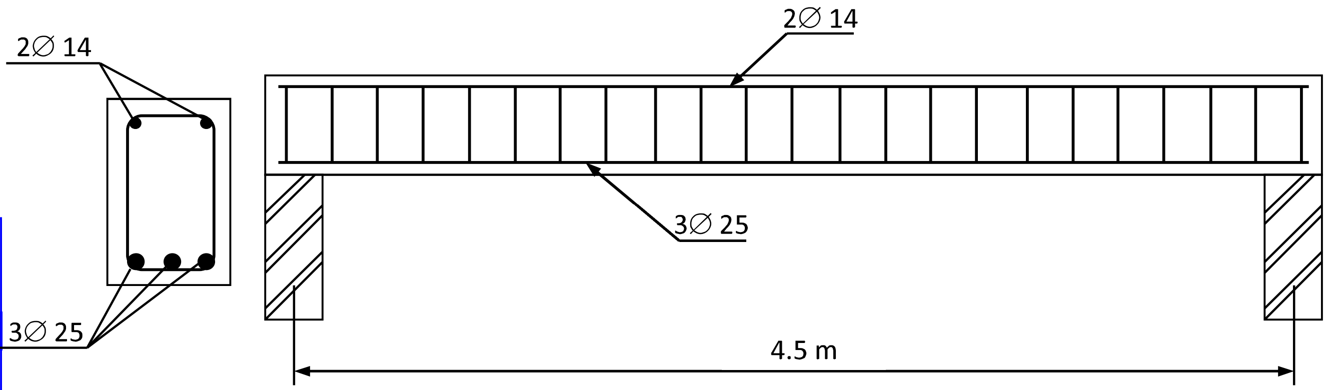
Strength Reduction Factors ϕ

The factors ϕ for understrength are called strength reduction factors according to ACI-9.3. and are as follows:

Strength Condition	ϕ Factors
1. Flexure (with or without axial force)	
Tension-controlled sections	0.90
Compression-controlled sections	
Spirally reinforced	0.75
Others	0.65
2. Shear and torsion	0.75
3. Bearing on concrete	0.65
4. Post-tensioned anchorage zones	0.85
5. Struts, ties, nodal zones, and bearing areas in strut-and-tie models	0.75

Example:

A simple beam is loaded with a dead load of 40 KN/m and a live load of 30 KN/m. Check the strength requirement according to ACI code if the nominal bending moment $M_n = 275\ KN.m$



Solution:

$$M_n = 275\ KN.m \qquad and \qquad \phi = 0.9$$

$$w_u = 1.2D + 1.6L = 1.2 \cdot 40 + 1.6 \cdot 30 = 96\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{96 \cdot 4.5^2}{8} = 243\ KN \cdot m$$

$$\phi M_n \geq M_u$$

$0.9 \cdot 275 = 247.5\ KN \cdot m > 243\ KN \cdot m$ OK Strength requirement is satisfied

$$\text{Factor of Safety} = \frac{1.2D + 1.6\ L}{D + L}\left(\frac{1}{\phi}\right) = \frac{96}{40 + 30}\left(\frac{1}{0.9}\right) = 1.52$$

CHAPTER 4 FLEXURE IN BEAMS

4.1 INTRODUCTION

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of steel, wood, or any other structural material.

Two different types of problems arise in the study of reinforced concrete:

1. Analysis. Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.
2. Design. Given a factored design moment, normally designated as M_u. select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

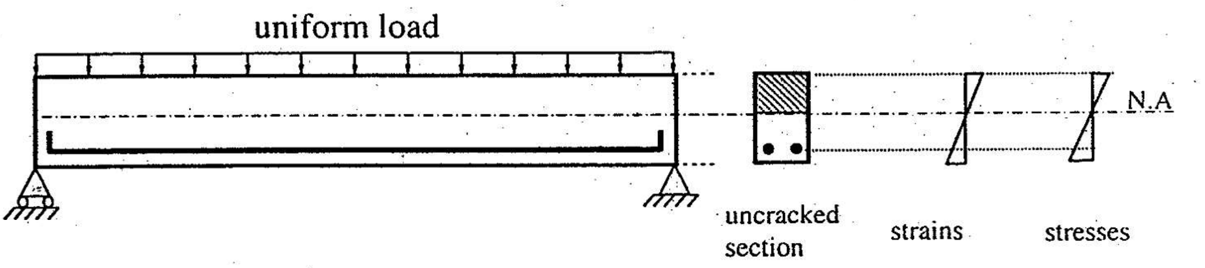
The Strength Design Method requires the conditions of static equilibrium and strain compatibility across the depth of the section to be satisfied.

The following are the assumptions for Strength Design Method:

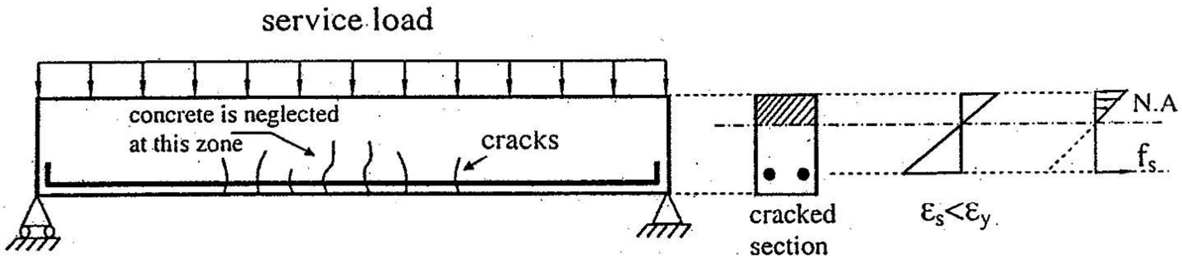
1. Strains in reinforcement and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
2. Concrete sections are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
3. Stress in reinforcement varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
4. Tensile strength of concrete is neglected.
5. Compressive stress distribution of concrete can be represented by the corresponding stress-strain relationship of concrete. This stress distribution may be simplified by a rectangular stress distribution as described later.

4.2 REINFORCED CONCRETE BEAM BEHAVIOR

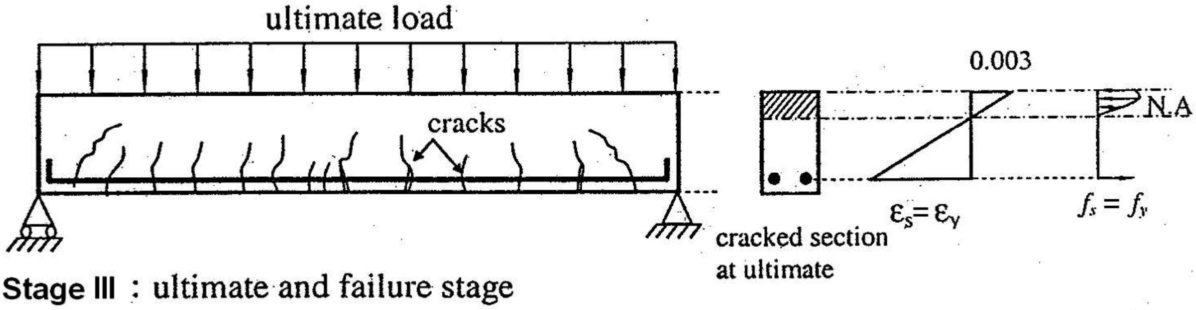
Consider a simply supported and reinforced concrete beam with uniformly distributed load on top. Under such loading and support conditions, flexure-induced stresses will cause compression at the top and tension at the bottom of the beam. Concrete, which is strong in compression, but weak in tension, resists the force in the compression zone, while steel reinforcing bars are placed in the bottom of the beam to resist the tension force. As the applied load is gradually increased from zero to failure of the beam (ultimate condition), the beam may be expected to behave in the following manner:



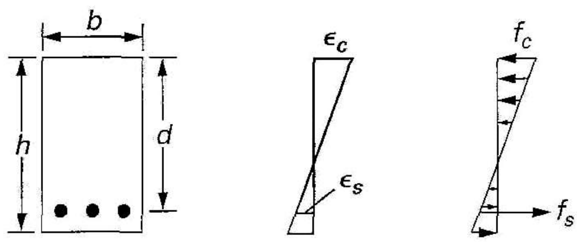
Stage I : before cracking



Stage II : cracking stage, before yield, working load

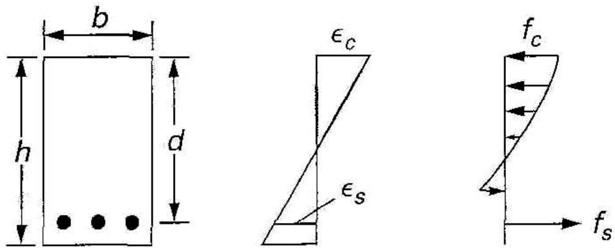


Stage III : ultimate and failure stage



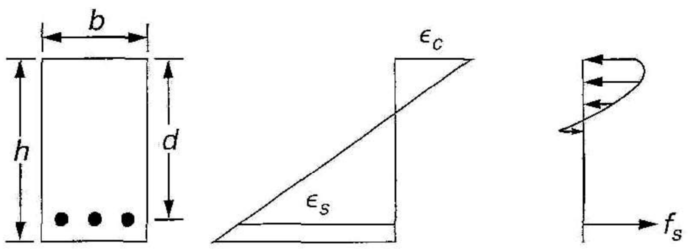
Stage I: when the applied load is low, the stress distribution is essentially linear over the depth of the section. The tensile stresses in the concrete are low enough so that the entire cross-section remains uncracked and the stress distribution is as shown in (a). In the compression zone, the concrete stresses are low enough (less than about $0.5\ f_c'$) so that their distribution is approximately linear.

Stage II: On increasing the applied load, the tensile stresses at the bottom of the beam become high enough to exceed the tensile strength at which the concrete cracks. After cracking, the tensile force is resisted mainly by the steel reinforcement. Immediately below the neutral axis, a small portion of the



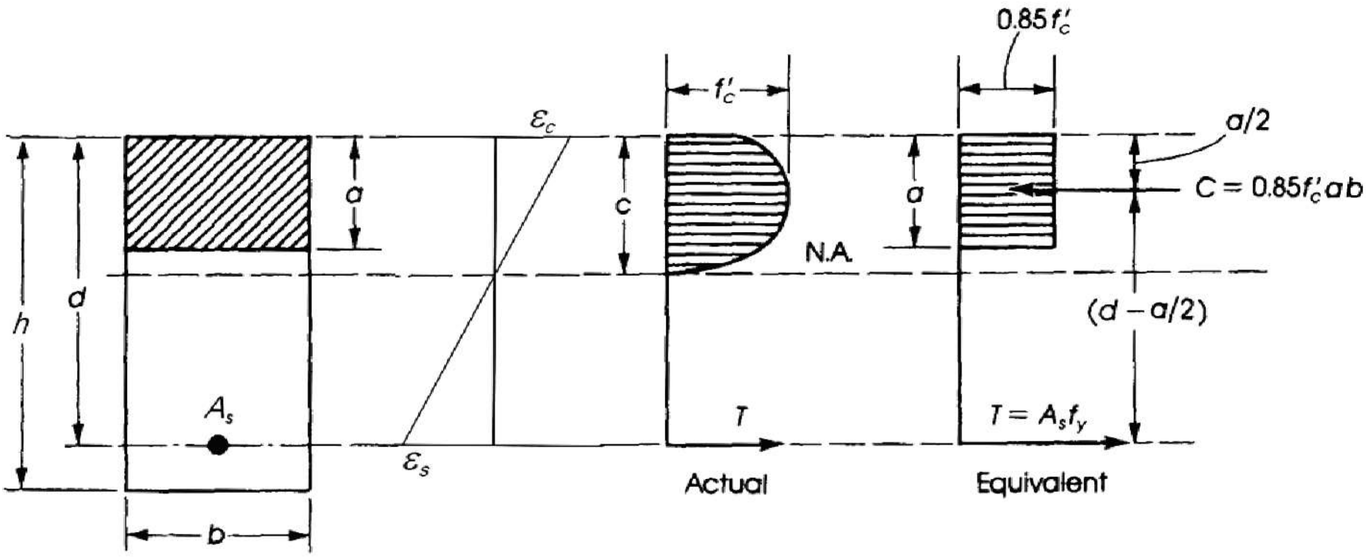
beam remains uncracked. These tensile stresses in the concrete offer, however, only a small contribution to the flexural strength. The concrete stress distribution in the compression zone becomes nonlinear.

Stage III: at nominal (so,-called ultimate) strength, the neutral axis has moved farther up-ward as flexural cracks penetrate more and more toward the compression face. The steel reinforcement has yielded and the concrete stress distribution in the compression zone becomes more nonlinear. Below the neutral axis, the concrete is cracked except for a very small zone.



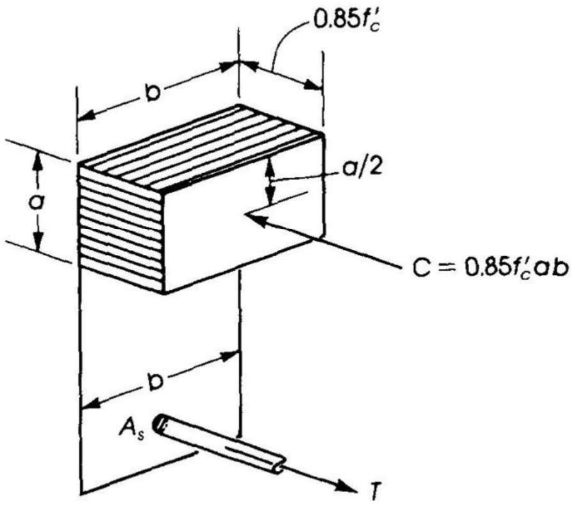
At the ultimate stage, two types of failure can be noticed. If the beam is reinforced with a small amount of steel, ductile failure will occur. In this type of failure, the steel yields and the concrete crushes after experiencing large deflections and lots of cracks. On the other hand, if the beam is reinforced with a large amount of steel, brittle failure will occur. The failure in this case is sudden and occurs due to the crushing of concrete in the compression zone without yielding of the steel and under relatively small deflections and cracks. This is not a preferred mode of failure because it does not give enough warning before final collapse.

4.3 THE EQUIVALENT RECTANGULAR COMPRESSIVE STRESS DISTRIBUTION (COMPRESSIVE STRESS BLOCK)



The actual distribution of the compressive stress in a section has the form of rising parabola. It is time consuming to evaluate the volume of compressive stress block. An equivalent rectangular stress block can be used without loss of accuracy.

The flexural strength M_n, using the equivalent rectangular, is obtained as follows:



$$C = 0.85\, f'_c ab$$
$$T = A_s f_y$$

$\sum F_x = 0$ gives $T = C$

$$A_s f_y = 0.85\, f'_c ab$$

or

$$a = \frac{A_s f_y}{0.85\, f'_c b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f'_c ab\left(d - \frac{a}{2}\right)$$

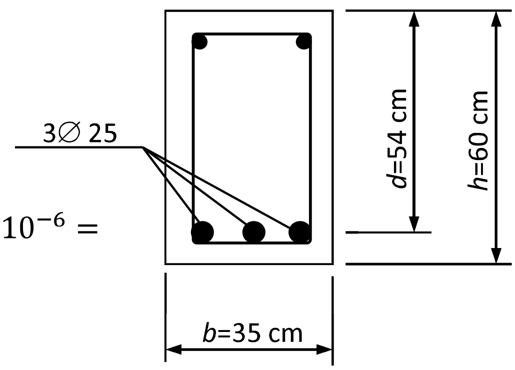
Notation:

a – depth of rectangular compressive stress block,
b – width of the beam at the compression side,
c – depth of the neutral axis measured from the extreme compression fibers,
d – effective depth of the beam, measured from the extreme compression fibers to the centroid of the steel area,
h – total depth of the beam,
ε_c – strain in extreme compression fibers,
ε_s – strain at tension steel,
f'_c – compressive strength of concrete,
f_y – yield stress of steel,
A_s – area of the tension steel,
C – resultant compression force in concrete,
T – resultant tension force in steel,
M_n – nominal moment strength of the section.

Example:

Determine the nominal moment strength of the beam section. Take $f'_c = 20\, MPa$, $f_y = 400\, MPa$.

Solution:



$A_s(3\varnothing\, 25) = 14.72\, cm^2$

$$a = \frac{A_s f_y}{0.85\, f'_c b} = \frac{14.72 \cdot 100 \cdot 400}{0.85 \cdot 20 \cdot 350} = 98.96\, mm$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 14.72 \cdot 100 \cdot 400\left(540 - \frac{98.96}{2}\right) \cdot 10^{-6} =$$

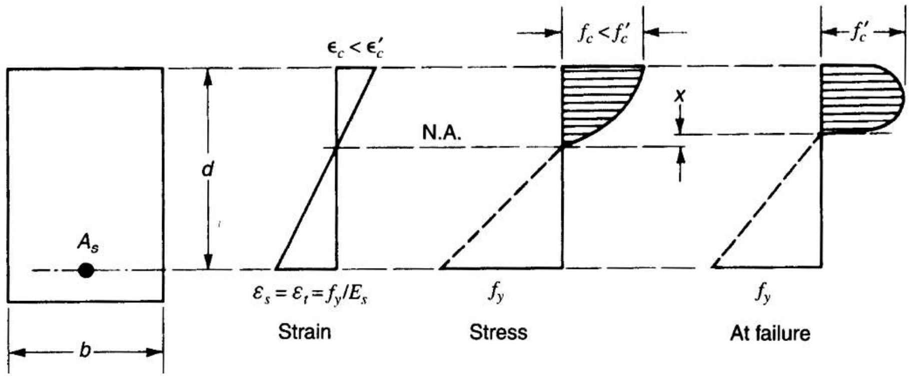
$= 288.82\, KN \cdot m$

4.4 TYPES OF FAILURE AND STRAIN LIMITS

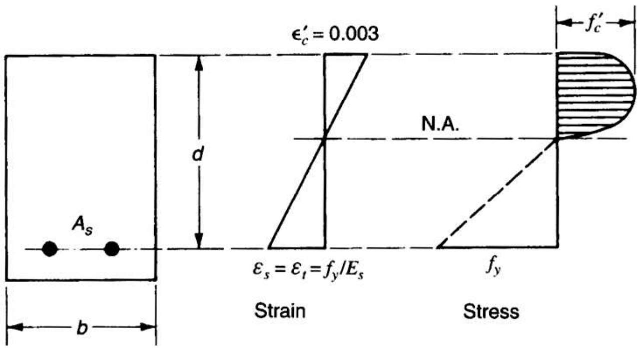
Types of failure

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

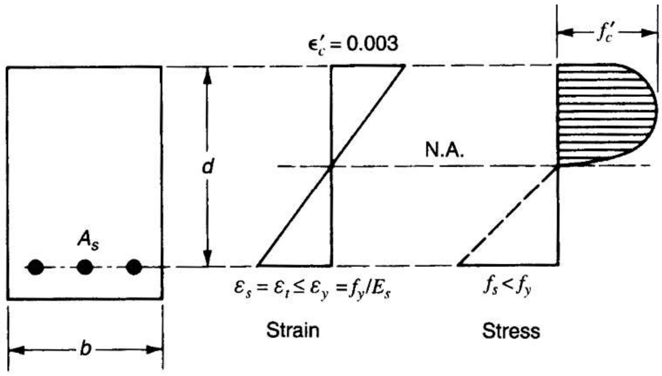
1. Steel may reach its yield strength before the concrete reaches its maximum strength, In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and is called a **tension-controlled section**.



2. Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a **balanced section**.



3. Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is, f_s is less than f_y. The strain in the steel is equal to or less than 0.002. This section is called a **compression-controlled section**.



The ACI Code assumes that concrete fails in compression when the concrete strain reaches 0.003.

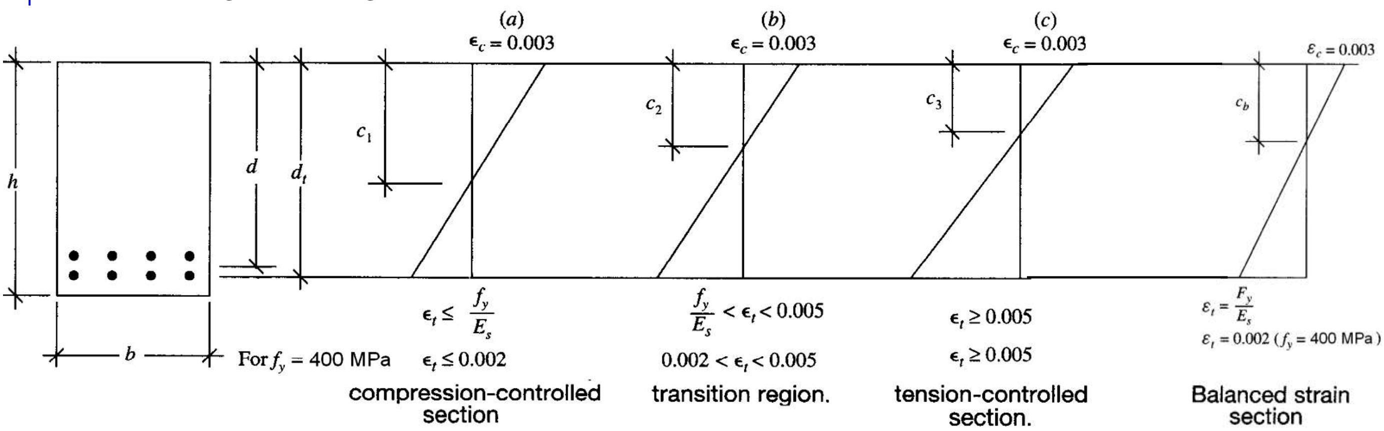
In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code **does not allow** this type of design.

Strain Limits for Tension and Tension-Controlled Sections

The ACI Code, Section 10.3. defines the concept of tension or compression-controlled sections in terms of net tensile strain ε_t (net tensile strain in the reinforcement closest to the tension face). Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition.
These four conditions are defined as follows:

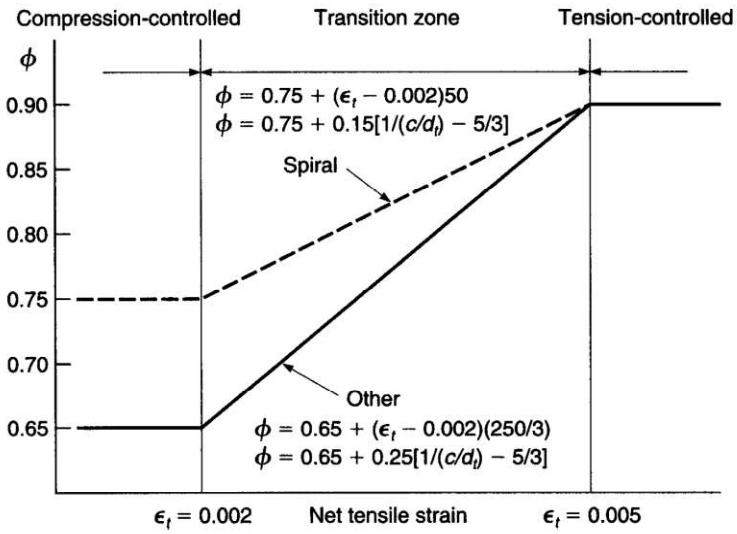
1. Compression-controlled sections are those sections in which ε_t at nominal strength is equal to or less than the compression-controlled strain limit (the compression-controlled strain limit may be taken as a net strain of $\varepsilon_y = 0.002 -$ for $f_y = 400\ MPa$) at the time when concrete in compression reaches its assumed strain limit of 0.003, ($\varepsilon_c = 0.003$). This case occurs mainly in columns subjected to axial forces and moments.
2. Tension-controlled sections are those sections in which the ε_t is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003
3. Sections in which the ε_t lies between the compression-controlled strain limit of 0.002 (for $f_y = 400\ MPa$) and the tension-controlled strain limit of 0.005 constitute the transition region.
4. The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, f_y or $\varepsilon_s = \frac{f_y}{E_s}$, just as the maximum strain in concrete at the extreme compression fibers reaches 0.003.

In addition to the above four conditions, Section 10.3.5 of the ACI Code indicates that the net tensile strain, ε_t, at nominal strength, within the transition region, shall not be less than 0.004 for reinforced concrete flexural members without or with an axial load less than $0.10\ f_c' A_g$, where $A_g =$ gross area of the concrete section.



Strain limit distribution

Note that in cases where strain is less than 0.005 namely, the section is in the transition zone, a value of the reduction ϕ lower than 0.9 for flexural has to be used for final design moment, with a strain not less than 0.004 as a limit.



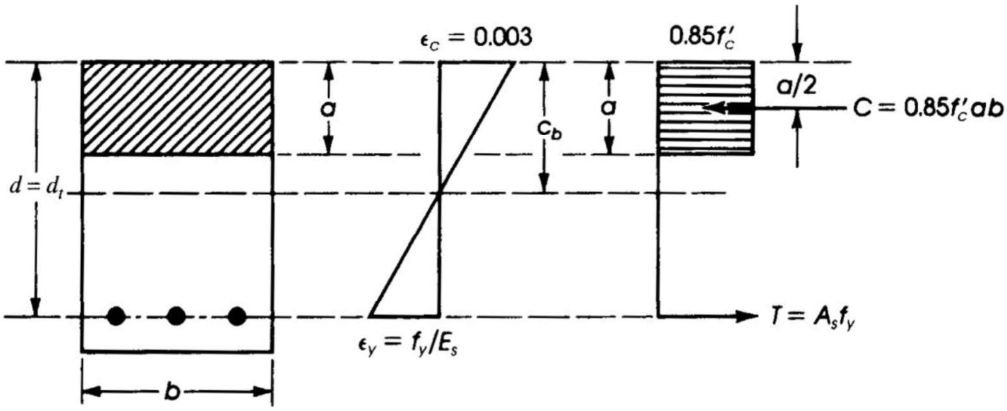
For transition region ϕ may be determined by linear interpolation:

$$\phi = 0.75 + (\varepsilon_t - 0.002)50 - \; for\ spiral\ members$$

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) - \; for\ other\ members$$

4.5 THE BALANCED CONDITION

Let us consider the case of balanced section, which implies that at ultimate load the strain in concrete equals 0.003 and that of steel equals $\varepsilon_t = \frac{f_y}{E_s}$ (at distance d_t).



$$\frac{c_b}{0.003} = \frac{d}{0.003 + \frac{f_y}{E_s}}, \qquad or \qquad c_b = \frac{d}{0.003 + \frac{f_y}{E_s}} 0.003$$

Substituting $E_s = 200\ 000\ MPa$

$$c_b = \frac{600}{600 + f_y} d$$

From equation of equilibrium $\sum F_x = 0$

$$T = C \qquad \Longrightarrow \qquad A_s f_y = 0.85\, f_c' ab$$

$a -$ the depth of compressive block and equal $a = \beta_1 c$.
For balanced condition, $a_b = \beta_1 c_b$.
where β_1 as defined in ACI 10.2.7.3 equal:

$$\beta_1 = 0.85 - 0.007(f_c' - 28) \qquad 0.65 \le \beta_1 \le 0.85$$

The reinforcement ratio for tension steel

$$\rho = \frac{A_s}{bd} \qquad and\ balanced\ reinforcement\ ratio\ \rho_b = \frac{(A_s)_b}{bd}$$

$$\frac{(A_s)_b}{bd} = 0.85 \frac{f_c'}{f_y} \beta_1 \frac{600}{600 + f_y}$$

$$\rho_b = 0.85 \frac{f_c'}{f_y} \beta_1 \left(\frac{600}{600 + f_y} \right)$$

4.6 UPPER AND LOWER (MINIMUM) STEEL PERCENTAGES.

The maximum reinforcement ratio ρ_{max} that ensures a minimum net tensile steel strain of 0.004.

$$\rho\ (\varepsilon_t = 0.004) = \frac{0.003 + \varepsilon_y}{0.003 + 0.004} \rho_b = \frac{0.003 + \varepsilon_y}{0.007} \rho_b = \rho_{max}$$

For Grade 420 reinforcing bars $\varepsilon_y = 0.002$, then

$$\rho_{max} = \frac{0.003 + 0.002}{0.007} \rho_b = \frac{0.005}{0.007} \rho_b = 0.724 \rho_b$$

If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment, the calculation may show that very small or no steel reinforcement is required. The ACI Code, 10.5, specifies a minimum steel area, $A_{s,min}$

$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d$$

and not less than

$$A_{s,min} = \frac{1.4}{f_y} b_w d$$

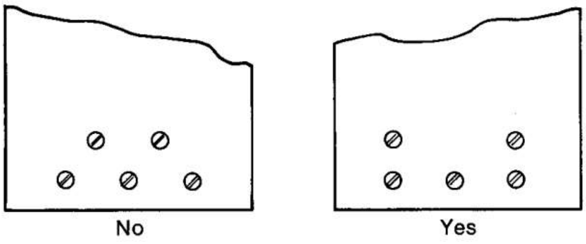
The above requirements of $A_{s,min}$ need not be applied if, at every section, A_s provided is at least one-third greater than that required by analysis ($A_{s,provided} \ge 1.33 A_{s,required}$). This exception provides sufficient additional reinforcement in large members where the amount required by the above equations would be excessive.

$b_w -$ width of section, width of web for T-section, mm.

4.7 SPACING LIMITS AND CONCRETE PROTECTION FOR REINFORCEMENT.

The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking.

According to ACI 7.6. The minimum clear spacing between parallel bars in a layer shall be d_b, but not less than $25\ mm$. Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than $25\ mm$.



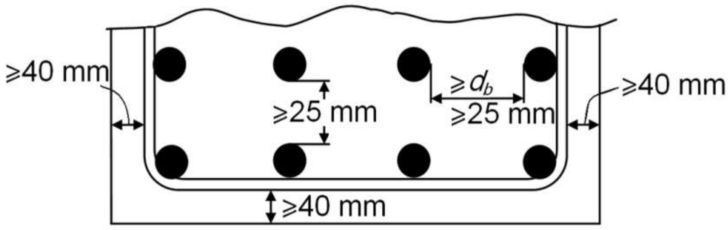
Arrangement of bars in two layers (ACI Section 7.6.2).

In addition, the nominal maximum size of coarse aggregate shall be not larger than:

(a) $1/5$ the narrowest dimension between sides of forms, nor

(b) $1/3$ the depth of slabs, nor

(c) $3/4$ the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, individual tendons, bundled tendons, or ducts.



Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where concrete cover is prescribed for a class of structural members, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars. According to ACI, 7.7, minimum clear cover in cast-in-place concrete beams and columns should not be less than $40\ mm$.

To limit the widths of flexural cracks in beams and slabs, ACI Code Section 10.6.4 defines upper limit on the center-to-center spacing between bars in the layer of reinforcement closest to the tension face of a member. In some cases, this requirement could force a designer to select a larger number of smaller bars in the extreme layer of tension reinforcement. The spacing limit is:

$$s = 380\left(\frac{280}{f_s}\right) - 2.5C_c \qquad \text{but} \qquad s \le 300\left(\frac{280}{f_s}\right)$$

where C_c is the least distance from surface of reinforcement to the tension face. It shall be permitted to take f_s as $\frac{2}{3}f_y$.

4.8 ANALYSIS OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

Given: section dimensions b, h; reinforcement A_s; material strength f_c', f_y.

Required: M_n – Nominal moment strength.

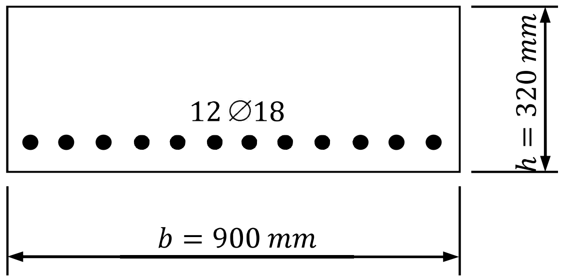
$$T = C \qquad \Longrightarrow \qquad A_s f_y = 0.85\, f_c' ab \qquad \Longrightarrow \qquad a = \frac{A_s f_y}{0.85\, f_c' b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' ab \left(d - \frac{a}{2}\right)$$

Example:

Determine the nominal moment strength of the beam section. Take $f_c' = 30\ MPa$, $f_y = 420\ MPa$.



Solution:

$A_s(12\varnothing\, 18) = 30.536\ cm^2 = 3053.6\ mm^2$

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{3053.6\ \cdot 420}{0.85 \cdot 30 \cdot 900} = 55.88\ mm$$

$$d = 320 - 40 - 10 - \frac{18}{2} = 261\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3053.6\ \cdot 420 \left(261 - \frac{55.88}{2}\right) \cdot 10^{-6} = 298.9\ KN \cdot m$$

Check for strain:

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right)$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{55.88}{0.836} = 66.84\ mm$$

$$\varepsilon_s = 0.003 \left(\frac{261 - 66.84}{66.84}\right) = 0.00871 > 0.005$$

Take $\phi = 0.9\ for\ flexure$

$$\phi M_n = 0.9 \cdot 298.9 = 269.01\ KN \cdot m$$

4.9 DESIGN OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

Given: M_u – factored moment ($M_u \le \phi M_n$); material strength f_c', f_y.

Required: section dimensions b, h; reinforcement A_s.

The two conditions of equilibrium are

$$T = C \qquad (1)$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right) \quad (2)$$

Reinforcement ratio

$$\rho = \frac{A_s}{bd} \qquad or \qquad A_s = \rho bd$$

Substituting into (1)

$$\rho bdf_y = 0.85\, f_c' ab$$

$$a = \rho\left(\frac{f_y}{0.85 f_c'}\right) d \quad (3)$$

Substituting (3) into (2)

$$M_n = \rho bdf_y\left[d - \frac{\rho}{2}\left(\frac{f_y}{0.85 f_c'}\right) d\right] \quad (4)$$

A strength coefficient of resistance R_n is obtained by dividing (4) by (bd^2) and letting

$$m = \left(\frac{f_y}{0.85 f_c'}\right)$$

Thus

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{\rho m}{2}\right) \quad (5)$$

From which ρ may be determined

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \quad (6)$$

Design Procedure:

1. Set $M_u = \phi M_n = \phi R_n\, bd^2$
2. For ductile behavior such that beam is well into the tension controlled zone, a reinforcement percentage ρ should be chosen in the range of $(40 - 60)\%$ of ρ_b. Assume $\rho = (0.4 - 0.6)\rho_b$.

$$\rho_b = 0.85\, \frac{f_c'}{f_y} \beta_1 \left(\frac{600}{600 + f_y}\right).$$

3. Find the flexural resistance factor R_n

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right), \qquad m = \left(\frac{f_y}{0.85 f_c'}\right)$$

4. Determine the required dimensions b, d

$$bd^2 = \frac{M_n}{R_n} = \frac{M_u}{\phi R_n}$$

5. Determine the required steel area for the chosen b, d

$$A_s = \rho b d$$

Where

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right), \qquad R_n = \frac{M_n}{bd^2}, \qquad m = \left(\frac{f_y}{0.85 f_c'}\right)$$

6. Check for minimum steel reinforcement area

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

Or

$$\rho_{min} = 0.25\frac{\sqrt{f_c'}}{f_y} \geq \frac{1.4}{f_y}$$

$$If \quad A_{s,provided} \geq \frac{4}{3} A_{s,required} \quad - \quad NO\ need\ to\ use\ A_{s,min}$$

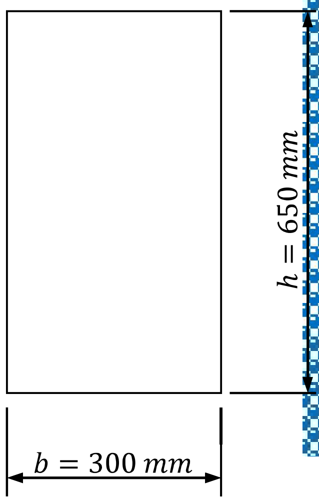
7. Check for strain ($\varepsilon_s \geq 0.005$) – tension-controlled section.
8. Check for steel bars arrangement in section.

Example:

Calculate the area of steel reinforcement required for the beam. $M_u = 360\ KN \cdot m$

Take $f_c' = 30\ MPa$, $f_y = 400\ MPa.$

Assume ∅ 25 with one layer arrangement.



<u>Solution:</u>

$$d = h - cover - \varnothing stirrups - \frac{\varnothing bar}{2} = 650 - 40 - 10 - \frac{25}{2} = 587.5\ mm$$

Take $\phi = 0.9\ for\ flexure$

$$R_n = \frac{M_n}{bd^2} = \frac{M_u}{\phi bd^2} = \frac{360 \cdot 10^6}{0.9 \cdot 300 \cdot 587.5^2} = 3.86\ MPa$$

$$m = \left(\frac{f_y}{0.85 f_c'}\right) = \frac{400}{0.85 \cdot 30} = 15.69$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{15.69}\left(1 - \sqrt{1 - \frac{2 \cdot 15.69 \cdot 3.86}{400}}\right) = 0.0105$$

$$A_s = \rho b d = 0.0105 \cdot 300 \cdot 587.5 = 1850.625\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{30}}{400} 300 \cdot 587.5 = 603.35\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 300 \cdot 587.5 = 617\ mm^2 \quad - control$$

$$A_s = 1850.625\ mm^2 > A_{s,min} = 617\ mm^2 \quad - OK$$

Use $4 \varnothing 25$ with $A_s(4 \varnothing 25) = 19.634\ cm^2 > A_{s,req} = 18.5\ cm^2 \quad - OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{1963.4 \cdot 400}{0.85 \cdot 30 \cdot 300} = 102.66\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{102.66}{0.836} = 122.8\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{587.5 - 122.8}{122.8}\right) = 0.01135 > 0.005 \qquad OK$$

Check for bar placement:

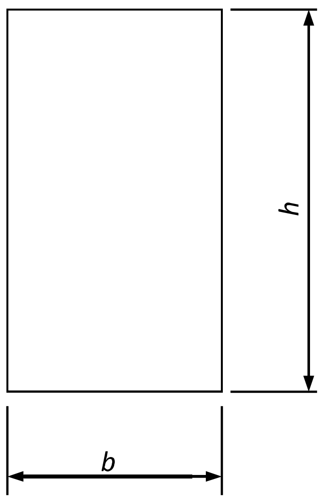
$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 4 \times 25}{3} = 33.33\ mm > d_b = 25\ mm, \quad > 25\ mm \qquad OK$$

Example:

Select an economical rectangular beam sizes and select bars using ACI strength method. The beam is a simply supported span of a 12 m and it is to carry a live load of 20 KN/m and a dead load of 25 KN/m including beam weight.

Take $f_c' = 28\ MPa,\ f_y = 400\ MPa.$

Assume $d \approx 2b$



Solution:

$$w_u = 1.2DL + 1.6LL = 1.2 \cdot 25 + 1.6 \cdot 20 = 62\ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{62 \cdot 12^2}{8} = 1116\ KN \cdot m$$

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85\ \ (f_c' = 28\ MPa)$

$$\rho_b = 0.85\frac{f_c'}{f_y}\beta_1\left(\frac{600}{600 + f_y}\right) = 0.85\ \frac{28}{400}0.85\left(\frac{600}{600 + 400}\right) = 0.030345$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.030345 = 0.012138$$

$$m = \left(\frac{f_y}{0.85 f_c'}\right) = \left(\frac{400}{0.85 \cdot 28}\right) = 16.807$$

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right) = 0.012138 \cdot 400\left(1 - \frac{0.012138 \cdot 16.807}{2}\right) = 4.36\ MPa$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{1116 \cdot 10^6}{0.9 \cdot 4.36} = 4b^3 \qquad \rightarrow \quad b = \sqrt[3]{\frac{1116 \cdot 10^6}{4 \cdot 0.9 \cdot 4.36}} = 414.28\ mm$$

Take $b = 400\ mm$ and $d = 2b = 2 \cdot 400 = 800\ mm$

$$R_n = \frac{M_u}{\phi bd^2} = \frac{1116 \cdot 10^6}{0.9 \cdot 400 \cdot 800^2} = 4.84\ MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{16.807}\left(1 - \sqrt{1 - \frac{2 \cdot 16.807 \cdot 4.84}{400}}\right) = 0.01367$$

$$A_s = \rho bd = 0.01367 \cdot 400 \cdot 800 = 4374.54\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

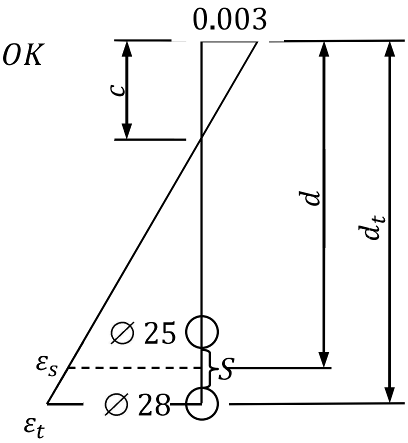
$$A_{s,min} = 0.25\frac{\sqrt{28}}{400} 400 \cdot 800 = 1058.3\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 400 \cdot 800 = 1120\ mm^2 \quad - control$$

$$A_s = 4374.54\ mm^2 > A_{s,min} = 1120\ mm^2 \quad - OK$$

Take 4 ∅ 28 + 4 ∅ 25 in two layers with

$$A_s = 24.63 + 19.635 = 44.265\ cm^2 > A_{s,req} = 43.74\ cm^2 \quad - OK$$



Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{4426.5 \cdot 400}{0.85 \cdot 28 \cdot 400} = 185.99\ mm$$

$$c = \frac{a}{\beta_1} = \frac{185.99}{0.85} = 218.81\ mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 800 + \frac{25}{2} + \frac{28}{2} = 826.5\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{826.5 - 218.81}{218.81}\right) = 0.00833 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{400 - 40 \times 2 - 10 \times 2 - 4 \times 28}{3} = 62.67\ mm > d_b = 28\ mm, \quad > 25\ mm \qquad OK$$

$$h = d_t + \frac{d_b}{2} + \varnothing\text{stirrups} + cover = 826.5 + \frac{28}{2} + 10 + 40 = 890.5\ mm$$

Take $b = 400\ mm$ and $h = 900\ mm.$

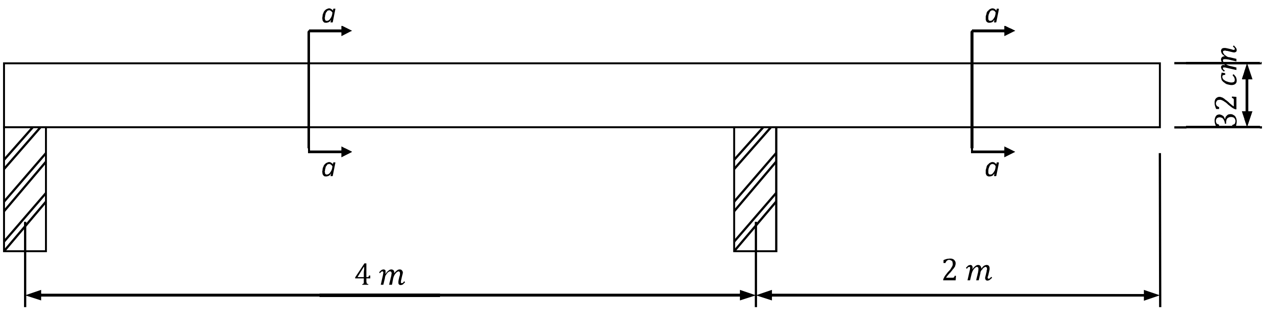
Example:

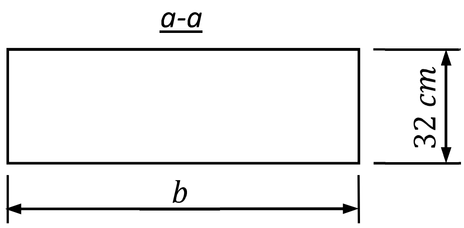
The beam shown below is loaded by service (unfactored) dead load of 45 KN/m and service live load of 25 KN/m. Design the beam for flexure given the following information:

$f_c' = 24\ MPa, \qquad f_y = 420\ MPa.$

Assume the depth of the beam $h = 32\ cm$

Use bars $\varnothing 16$





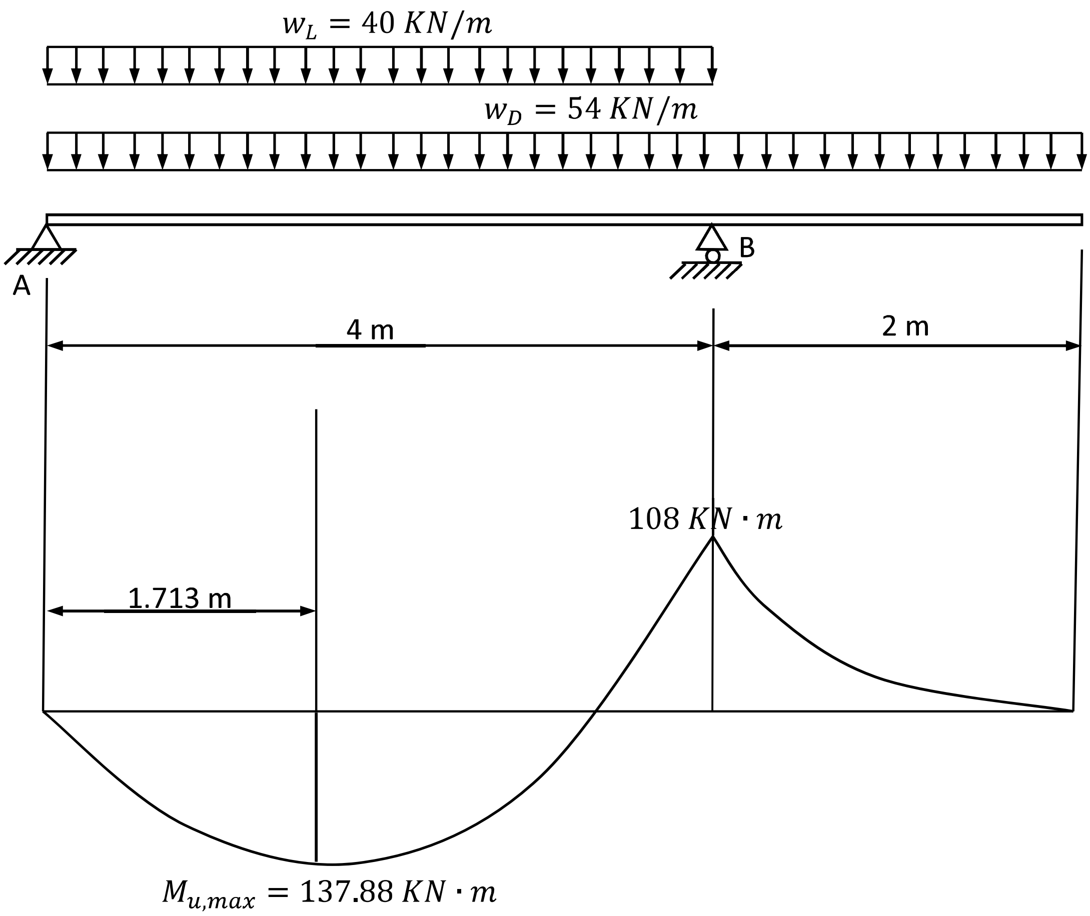
Solution:

$w_D = 1.2 \cdot 45 = 54\ KN/m$

$w_L = 1.6 \cdot 25 = 40\ KN/m$

Determination the maximum positive and negative bending moments for the beam:

- Maximum positive bending moment.



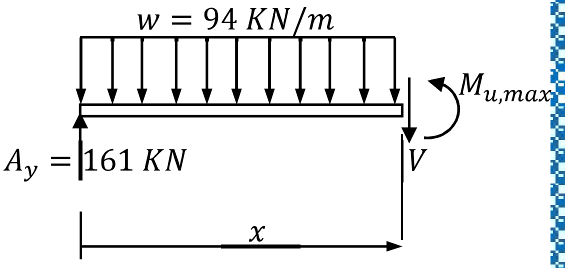
$\curvearrowright + \sum M_B = 0, \qquad A_y \cdot 4 - 94 \cdot 4 \cdot 2 + 54 \cdot 2 \cdot 1 = 0 \qquad\qquad A_y = 161\ KN$

Location of Maximum positive moment at distance x from support A from condition of zero shear force.

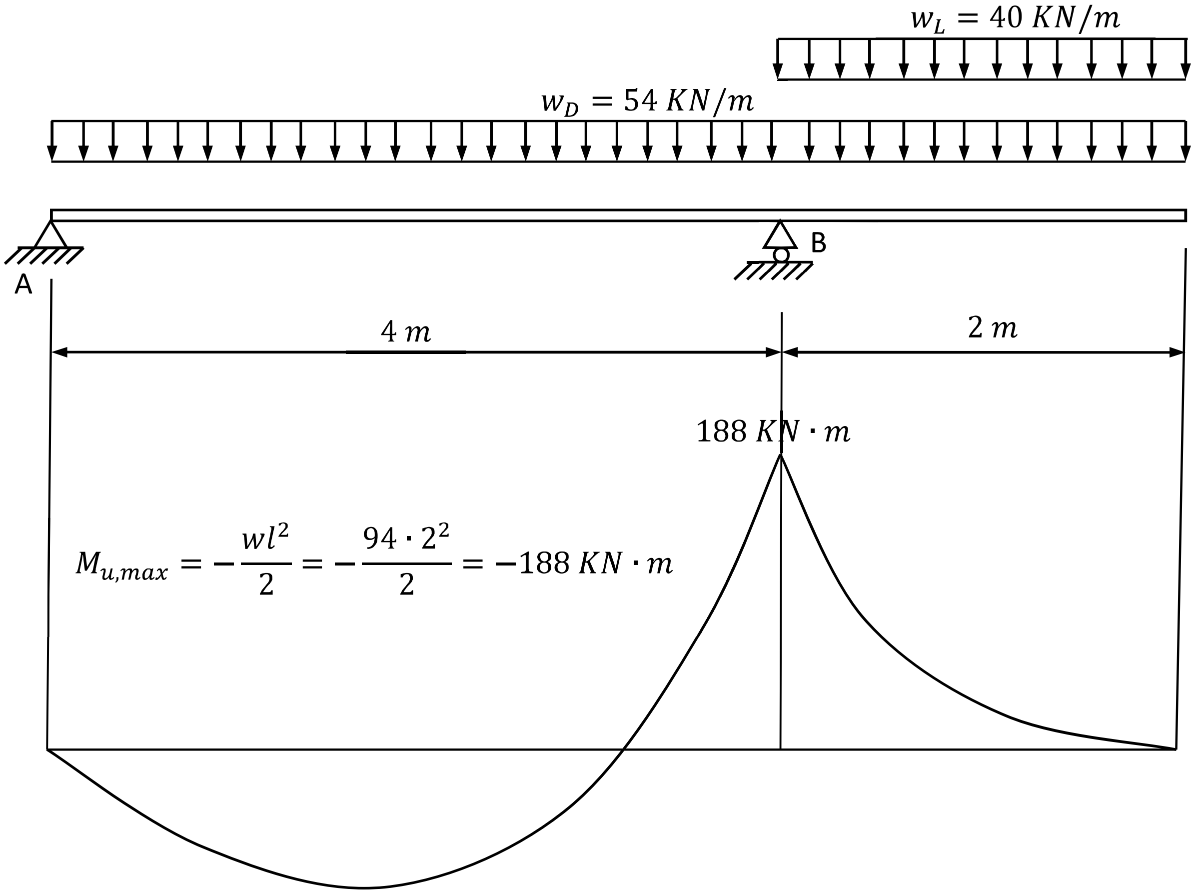
$V(x) = 0, \quad 161 - 94 \cdot x = 0 \quad x = 1.713\ m$

$M_{u,max} = 161 \cdot 1.713 - 94 \cdot \frac{1.713^2}{2} = 137.88\ KN \cdot m$

$M_B = -54 \cdot \frac{2^2}{2} = -108\ KN \cdot m$



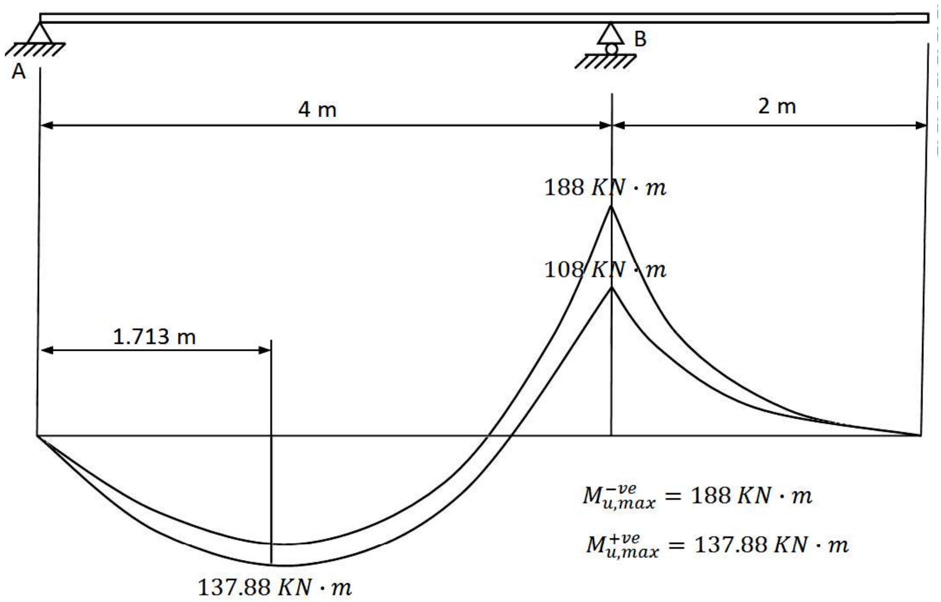
- Maximum negative bending moment.



The maximum moments from all cases (envelope):

$M^{+ve}_{u,max} = 137.88\ KN \cdot m$

$M^{-ve}_{u,max} = 188\ KN \cdot m$



Determination of the beam width b and Design for negative moment $M_u = 188\ KN \cdot m$

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85$ $(f_c' = 24\ MPa)$

$$\rho_b = 0.85\frac{f_c'}{f_y}\beta_1\left(\frac{600}{600+f_y}\right) = 0.85\frac{24}{420}0.85\left(\frac{600}{600+420}\right) = 0.02429$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.02429 = 0.01$$

$$m = \left(\frac{f_y}{0.85f_c'}\right) = \left(\frac{420}{0.85 \cdot 24}\right) = 20.6$$

$$R_n = \rho f_y\left(1-\frac{\rho m}{2}\right) = 0.01 \cdot 420\left(1-\frac{0.01 \cdot 20.6}{2}\right) = 3.767\ MPa$$

$$d = h - cover - \varnothing\text{stirrups} - \frac{\varnothing\text{bar}}{2} = 320 - 40 - 10 - \frac{16}{2} = 262\ mm$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{188 \cdot 10^6}{0.9 \cdot 3.767} = b \cdot 262^2 \qquad \rightarrow \qquad b = \frac{188 \cdot 10^6}{0.9 \cdot 3.767 \cdot 262^2} = 807.8\ mm$$

Take $b = 900\ mm$

$$R_n = \frac{M_u}{\phi bd^2} = \frac{188 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 3.38\ MPa$$

$$\rho = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1-\sqrt{1-\frac{2 \cdot 20.6 \cdot 3.38}{420}}\right) = 0.0089$$

$$A_s = \rho bd = 0.0089 \cdot 900 \cdot 262 = 2099\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{24}}{420}900 \cdot 262 = 688\ mm^2$$

$$A_{s,min} = \frac{1.4}{420}900 \cdot 262 = 786\ mm^2 \quad - control$$

$$A_s = 2099\ mm^2 > A_{s,min} = 786\ mm^2 \quad - OK$$

Take 11 $\varnothing$ 16 in one layer with $A_s = 22.11\ cm^2 > A_{s,req} = 20.99\ cm^2$ $\quad - OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{2211 \cdot 420}{0.85 \cdot 24 \cdot 900} = 50.6\ mm$$

$$c = \frac{a}{\beta_1} = \frac{50.6}{0.85} = 59.5\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{262-59.5}{59.5}\right) = 0.01 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{900 - 40 \times 2 - 10 \times 2 - 11 \times 16}{10} = 62.4\,mm > 25\,mm \qquad OK$$

Design for positive moment $M_u = 137.88\ KN \cdot m$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{137.88 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 2.48\ MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1 - \sqrt{1 - \frac{2 \cdot 20.6 \cdot 2.48}{420}}\right) = 0.0063$$

$$A_s = \rho b d = 0.0063 \cdot 900 \cdot 262 = 1486\ mm^2$$

$$A_{s,min} = 786\ mm^2$$

$$A_s = 1486\ mm^2 > A_{s,min} = 786\ mm^2 \quad - OK$$

Take 8 ∅ 16 in one layer with $A_s = 16.08\ cm^2 > A_{s,req} = 14.86\ cm^2 \quad - OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{1608 \cdot 420}{0.85 \cdot 24 \cdot 900} = 36.78\ mm$$

$$c = \frac{a}{\beta_1} = \frac{37}{0.85} = 43.28\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d - c}{c}\right) = 0.003\left(\frac{262 - 43.28}{43.28}\right) = 0.0152 > 0.005 \qquad OK$$

Check for bar placement: $S_b > 25\ mm \qquad OK$