CHAPTER **SIX**

# CONTINUOUS CONSTRUCTION DESIGN CONSIDERATIONS



## 6-1 INTRODUCTION

A common form of concrete cast-in-place building construction consists of a continuous one-way slab cast monolithically with supporting continuous beams and girders. In this type of system, all members contribute in carrying the floor load to the supporting columns (see Figure 3-1). The slab steel runs through the beams, the beam steel runs through the girders, and the steel from both the beams and girders runs through the columns. The result is that the whole floor system is tied together, forming a highly indeterminate and complex type of rigid structure. The behavior of the members is affected by their rigid connections. Not only will loads applied directly *on* a member produce moment, shear, and a definite deflected shape, but loads applied to *adjacent* members will produce similar effects because of the rigidity of the connections. The shears and moments transmitted through a joint will depend on the relative stiffnesses of all the members framing into that joint. With this type of condition, a precise evaluation of moments and shears resulting from a floor loading is excessively time-consuming and is outside the scope of this text. Several commercial computer programs are available to facilitate these analysis computations.

In an effort to simplify and expedite the design phase, the ACI Code, Section 8.3.3, permits the use of standard moment and shear equations whenever the span and loading conditions satisfy stipulated requirements. This approach applies to continuous non-prestressed one-way slabs and beams. It is an approximate method and may be used for buildings of the usual type of construction, spans, and story heights. The ACI moment equations result from the product of a coefficient and $w_u\ell_n^2$. Similarly, the ACI shear equations result from the product of a coefficient and $w_u\ell_n$. In these equations, $w_u$ is the factored design uniform load and $\ell_n$ is the *clear span* for positive moment (and shear) and the average of two adjacent clear spans for negative moment. The application of the equations is limited to the following:

1. The equations can be used for two or more approximately equal spans (with the larger of two adjacent spans not exceeding the shorter by more than 20%).
2. Loads must be uniformly distributed (therefore, girders are excluded).
3. The maximum allowable ratio of live load to dead load is 3:1 (based on service loads).
4. Members must be prismatic.

The ACI moment and shear coefficients give the envelopes of the maximum moments and shears, respectively, at critical locations on the flexural member. These maximum moments and shears at the different critical locations along the flexural member do not necessarily occur under the same loading condition. These shear and moment equations generally give reasonably conservative values for the stated conditions. If more precision is required, or desired, for economy, or because the stipulated conditions are not satisfied, a more theoretical and precise analysis must be made. The moment and shear equations are depicted in Figure 6-1. Their use will be demonstrated later in this chapter.

For approximate moments and shears for girders, See Section 14-2.

## 6-2 CONTINUOUS-SPAN BAR CUTOFFS

Using a design approach similar to that for simple spans, the area of main reinforcing steel required at any given point is a function of the design moment. As the moment varies along the span, the steel may be modified or reduced in accordance with the theoretical requirements of the member's strength and the requirements of the ACI Code.

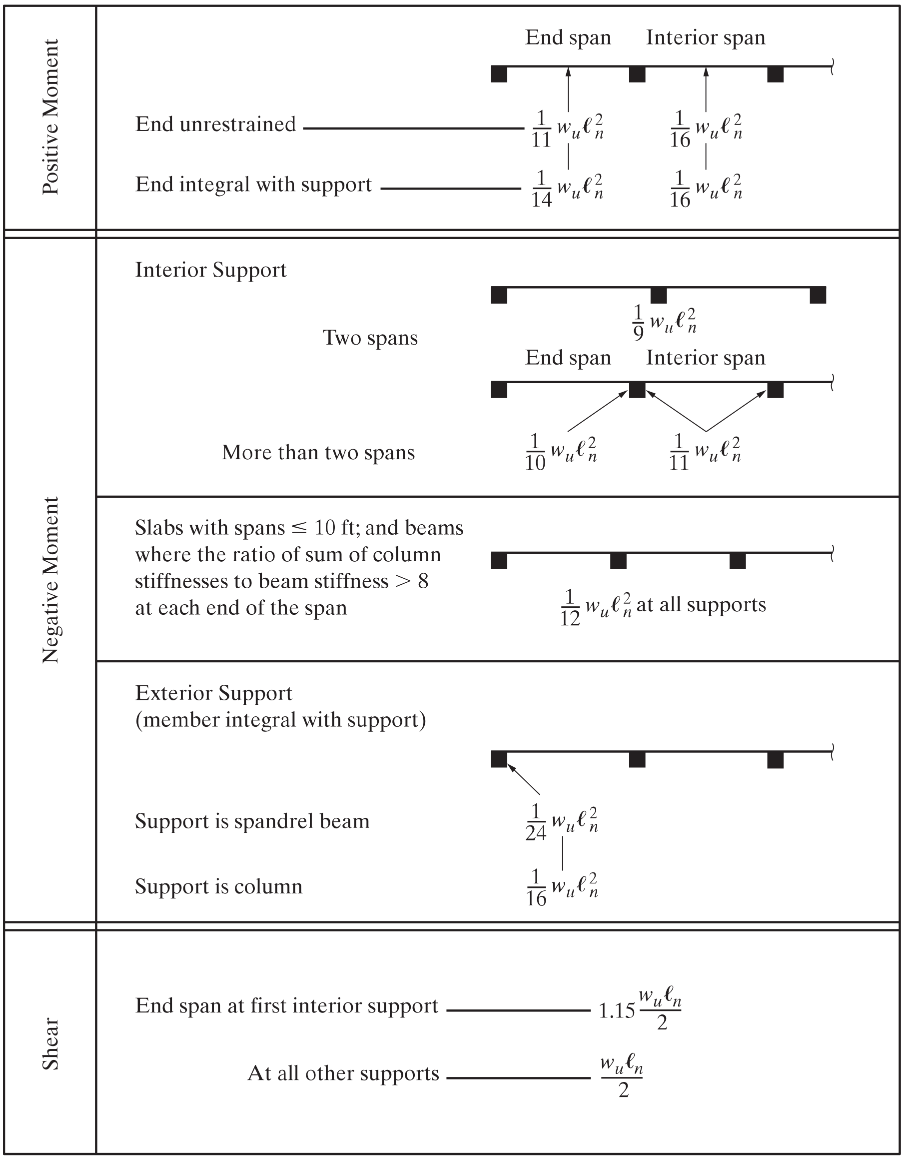



**FIGURE 6-1** ACI Code coefficients and equations for shear and moment for continuous beams and one-way slabs.

Bars can theoretically be stopped or bent in flexural members whenever they are no longer needed to resist moment. A general representation of the bar cutoff requirements for continuous spans (both positive and negative moments) is shown in Figure 6-2.

In continuous members the ACI Code, Section 12.11.1, requires that a minimum of one-fourth of the positive moment steel be extended into the support a distance of at least 6 in. The ACI Code, Section 12.12.3, also requires that at least one-third of the negative moment steel be extended beyond the extreme position of the point of inflection a distance not less than one-sixteenth of the clear span, the effective depth of the member $d$, or 12 bar diameters, whichever is greater. If negative moment bars C (Figure 6-2a) are to be cut off, they must extend at least a full development length $\ell_d$ beyond the face of the support. In addition, they must extend a distance equal to the effective depth of the member or 12 bar diameters, whichever is larger, beyond the theoretical cutoff point defined by the moment diagram. The remaining negative moment bars D (minimum of one-third of total negative steel) must extend at least $\ell_d$ beyond the theoretical point of cutoff of bars C and, in addition, must extend a distance equal to the effective depth of the member, 12 bar diameters, or one-sixteenth of the clear span, whichever is the greater, past the point of inflection. Where negative moment bars are cut off before reaching the point of inflection, the situation is analogous to the simple beam cutoffs where the

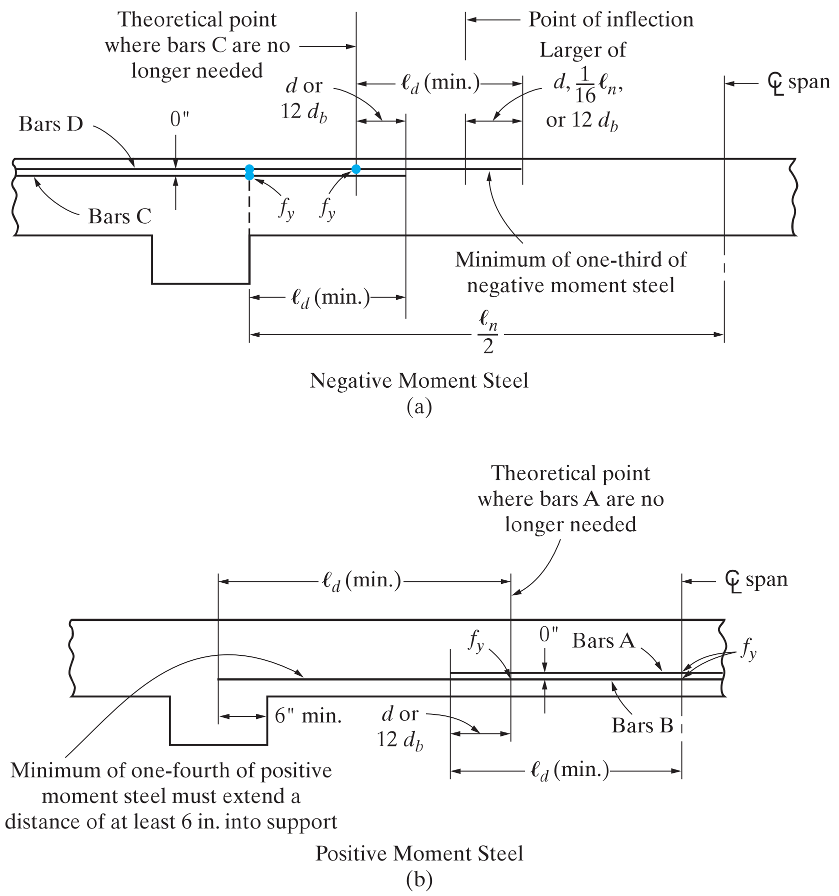



**FIGURE 6-2** Bar cutoff requirements for continuous slabs (ACI Code).

reinforcing bars are being terminated in a tension zone. The reader is referred to Example 5-5.

In Figure 6-2b, if positive moment bars A are to be cut off, they must project $\ell_d$ past the point of maximum positive moment as well as a distance equal to the effective depth of the member or 12 bar diameters, whichever is larger, beyond their theoretical cutoff point. Recall that the location of the theoretical cutoff point depends on the amount of steel to be cut and the shape of the applied moment diagram. The remaining positive moment bars B must extend $\ell_d$ past the theoretical cutoff point of bars A and extend at least 6 in. into the support. Comments on terminating bars in a tension zone again apply. Additionally, the size of the positive moment bars at the point of inflection must meet the requirements of the ACI Code, Section 12.11.3.

Because the determination of cutoff and bend points constitutes a relatively time-consuming chore, it has become customary to use defined cutoff points that experience has indicated are safe. These defined points may be used where the ACI moment coefficients have application but must be applied with judgment where parameters vary. The recommended bar details and cutoffs for continuous spans are shown in Figure 6-3.

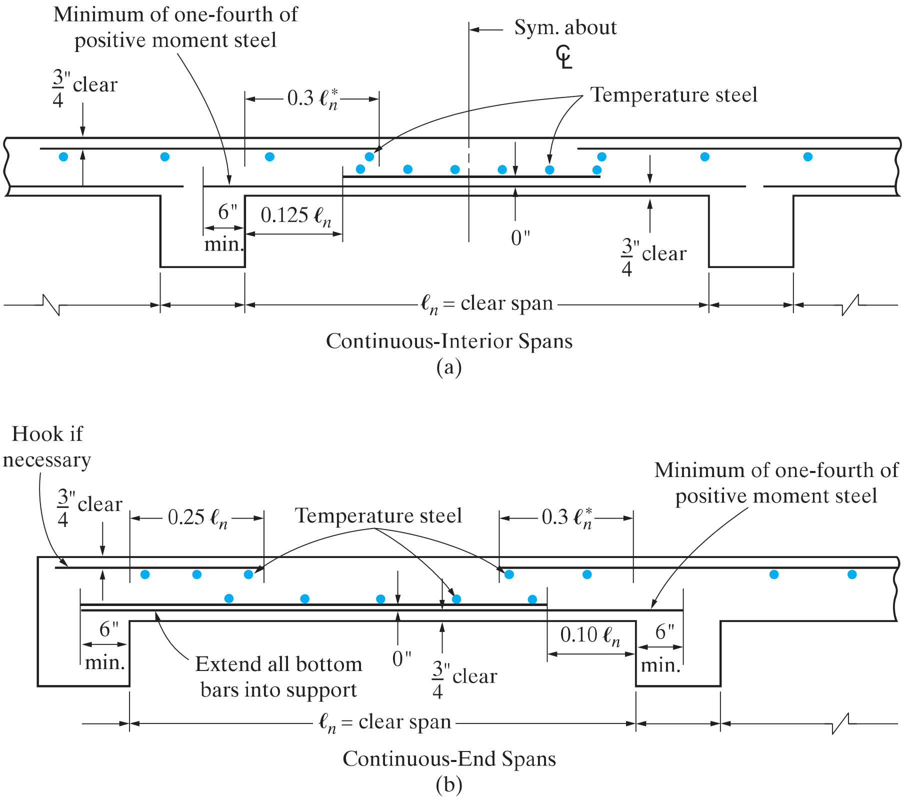
## 6-3 DESIGN OF CONTINUOUS FLOOR SYSTEMS

One common type of floor system consists of a continuous, cast-in-place, one-way reinforced concrete slab supported by monolithic, continuous reinforced concrete beams. Assuming that the floor system parameters and loading conditions satisfy the criteria for application of the ACI Code coefficients, the design of the system may be based on these coefficients. Example 6-1 furnishes a complete design of a typical one-way slab and beam floor system.

### Example 6-1

The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use $f_c' = 3000$ psi (normal-weight concrete) and $f_y = 60{,}000$ psi. The bars are uncoated.

a. Design the continuous one-way floor slab.
b. Design the continuous supporting beam.




*If adjacent spans have different span lengths, use the larger of the two.

**FIGURE 6-3** Recommended bar details and cutoffs, one-way slabs; tensile reinforced beams similar.

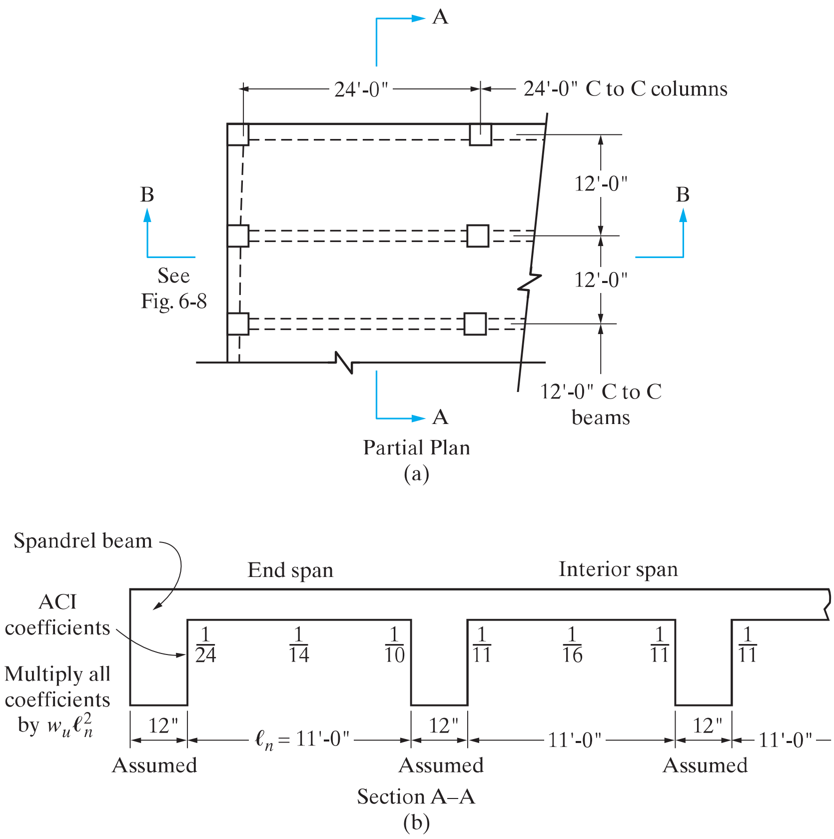



**FIGURE 6-4** Sketches for Example 6-1.

**Solution:**

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.

a. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the Code and this thickness will be used to estimate slab weight.

With both ends continuous,

$$\text{minimum } h = \frac{1}{28}\ell_n = \frac{1}{28}(11)(12) = 4.71 \text{ in.}$$

With one end continuous,

$$\text{minimum } h = \frac{1}{24}\ell_n = \frac{1}{24}(11)(12) = 5.5 \text{ in.}$$

Try a $5\frac{1}{2}$-in.-thick slab. Design a 12-in.-wide segment ($b = 12$ in.).

2. Determine the load:

$$\text{slab dead load} = \frac{5.5}{12}(150) = 68.8 \text{ psf}$$

$$\begin{aligned}\text{total dead load} &= 25.0 + 68.8 = 93.8 \text{ psf}\\ w_u &= 1.2w_{\text{DL}} + 1.6w_{\text{LL}}\\ &= 1.2(93.8) + 1.6(250)\\ &= 112.6 + 400\\ &= 512.6 \text{ psf} \quad \text{(design load)}\end{aligned}$$

Because we are designing a slab segment that is 12-in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

$$+M_u = \frac{1}{14}w_u\ell_n^2 = \frac{1}{14}(0.513)(11)^2 = 4.43 \text{ ft.-kips}$$

$$+M_u = \frac{1}{16}w_u\ell_n^2 = \frac{1}{16}(0.513)(11)^2 = 3.88 \text{ ft.-kips}$$

$$-M_u = \frac{1}{10}w_u\ell_n^2 = \frac{1}{10}(0.513)(11)^2 = 6.21 \text{ ft.-kips}$$

$$-M_u = \frac{1}{11}w_u\ell_n^2 = \frac{1}{11}(0.513)(11)^2 = 5.64 \text{ ft.-kips}$$

$$-M_u = \frac{1}{24}w_u\ell_n^2 = \frac{1}{24}(0.513)(11)^2 = 2.59 \text{ ft.-kips}$$

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

$$V_u = 1.15\frac{w_u\ell_n}{2} = 1.15(0.513)\left(\frac{11}{2}\right) = 3.24 \text{ kips}$$

whereas at all other supports,

$$V_u = \frac{w_u\ell_n}{2} = 0.513\left(\frac{11}{2}\right) = 2.82 \text{ kips}$$

4. Design the slab. Using the assumed slab thickness of $5\frac{1}{2}$ in., find the approximate $d$. Assume No. 5 bars for main steel and $\frac{3}{4}$-in. cover for the bars in the slab. Thus

$$d = 5.5 - 0.75 - 0.31 = 4.44 \text{ in.}$$

5. Design the steel reinforcing. Assume a tension-controlled section ($\epsilon_t \geq 0.005$) and $\phi = 0.90$. Select the point of maximum moment. This is a negative moment and occurs in the end span at the first interior support, and

$$M_u = \frac{w_u\ell_n^2}{10} = 6.21 \text{ ft.-kips}$$

$$\phi M_n = \phi bd^2\bar{k}$$

Because for design purposes $M_u = \phi M_n$ as a limit, then

$$\text{required } \bar{k} = \frac{M_u}{\phi bd^2} = \frac{6.21(12)}{0.90(12)(4.44)^2} = 0.3500 \text{ ksi}$$

From Table A-8,

$$\rho = 0.0063 < \rho_{\max} = 0.01355 \qquad \text{(O.K.)}$$

$$\begin{aligned}\text{required } A_s &= \rho bd = 0.0063(12)(4.44)\\ A_s &= 0.34 \text{ in.}^2\end{aligned}$$

As the steel area required at all other points will be less, the preceding process will be repeated for the other points. The expression

$$\text{required } \bar{k} = \frac{M_u}{\phi bd^2}$$

can be simplified because all values are constant except $M_u$:

$$\text{required } \bar{k} = \frac{M_u(12)}{0.9(12)(4.44)^2} = \frac{M_u}{17.74}$$

where $M_u$ must be in ft-kips. In the usual manner, the required steel ratio $\rho$ and the required steel area $A_s$ may then be determined. The results of these calculations are listed in Table 6-1.

Minimum reinforcement for slabs of constant thickness is that required for shrinkage and temperature reinforcement:

$$\begin{aligned}\text{minimum required } A_s &= 0.0018bh\\ &= 0.0018(12)(5.5) = 0.12 \text{ in.}^2\end{aligned}$$

Also, maximum $\rho = 0.01355$, corresponding to a net tensile strain $\epsilon_t$ of 0.005 (see Table A-8). Therefore the slab steel requirements for flexure as shown in Table 6-1 are within acceptable

**TABLE 6-1** Slab Steel Area Requirements

| Location | Moment equation | $\bar{k}$ (ksi) | Required $\rho$ | $A_s$ (in.$^2$/ft) |
|---|---|---|---|---|
| End span | | | | |
| At spandrel | $-\frac{1}{24}w_u\ell_n^2$ | 0.1460 | 0.0025 | 0.13 |
| Midspan | $+\frac{1}{14}w_u\ell_n^2$ | 0.2497 | 0.0044 | 0.24 |
| Interior spans | | | | |
| Interior support | $-\frac{1}{11}w_u\ell_n^2$ | 0.3179 | 0.0057 | 0.30 |
| Midspan | $+\frac{1}{16}w_u\ell_n^2$ | 0.2187 | 0.0038 | 0.20 |

limits. The shrinkage and temperature steel may be selected based on the preceding calculation:

use No. 3 bars at 11 in. o.c. ($A_s = 0.12$ in.$^2$)

Recall that the maximum spacing allowed is the smaller of $5h$ or 18 in. Because $5h = 5(5.5) = 27.5$ in., the 18 in. would control and the spacing is acceptable.

6. Check the shear strength. From step 3, maximum $V_u = 3.24$ kips at the face of the support. A check of shear at the face of the support, rather than at the critical section that is at a distance equal to the effective depth of the member from the face of the support, is conservative. Slabs are not normally reinforced for shear; therefore

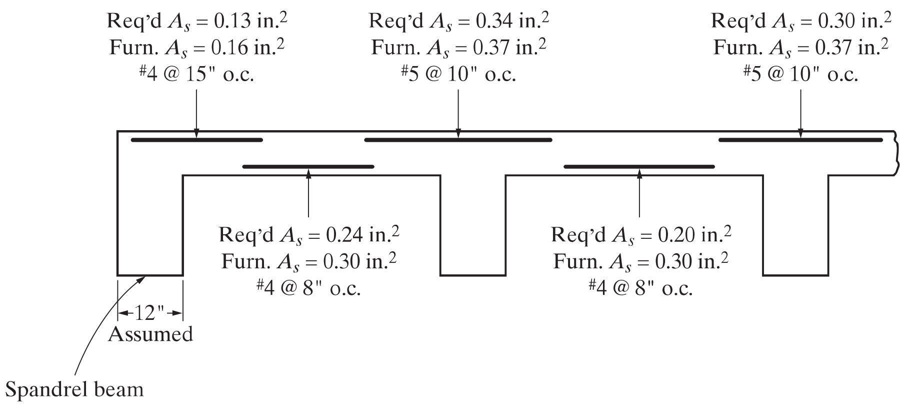
$$\phi V_n = \phi V_c = \phi 2\sqrt{f'_c}\, b_w d$$
$$= \frac{0.75(2\sqrt{3000})(12)(4.44)}{1000} = 4.38 \text{ kips}$$
$$\phi V_n > V_u$$

Therefore the thickness is O.K.

7. Select the main steel. Using Table A-4, establish a pattern in which the number of bar sizes and the number of different spacings are kept to a minimum. The maximum spacing for the main steel is 16.5 in. (the smaller of $3h$ or 18 in.). A work sketch (see Figure 6-5) is recommended to establish steel pattern and cutoff points. With regard to the steel selection, note that in the positive moment areas of both the end span and interior span, No. 4 bars at 9 in. could be used. If alternate bars were terminated, however, the spacing of the bars remaining would exceed the maximum spacing of 16.5 in. The use of No. 4 bars at 8 in. avoids this problem. The steel selected is conservative.

8. Check anchorage into the spandrel beam. The steel is No. 4 bars at 15 in. o.c. Refer to the procedure for development length calculation in Section 5-2.
   a. From Table 5-1, $K_D = 82.2$.
   b. Establish values for the factors $\psi_t$, $\psi_e$, $\psi_s$, and $\lambda$.
      1. $\psi_t = 1.3$ (the bars are top bars).
      2. The bars are uncoated; $\psi_e = 1.0$.
      3. The bars are No. 4; $\psi_s = 0.8$.
      4. Normal-weight concrete is used; $\lambda = 1.0$.
   c. The product $\psi_t \times \psi_e = 1.3 < 1.7$. (O.K.)
   d. Determine $c_b$. Based on cover (center of bar to nearest concrete surface),

$$c_b = \frac{3}{4} + \frac{0.5}{2} = 1 \text{ in.}$$




Note: Bar cutoffs and temperature steel may be observed in the design sketch, Fig. 6-7.

**FIGURE 6-5** Work sketch for Example 6-1.

Based on bar spacing (one-half the center-to-center distance),

$$c_b = \tfrac{1}{2}(15) = 7.5 \text{ in.}$$

Therefore use $c_b = 1.0$ in.

e. $K_{tr}$ is taken as zero. There is no transverse steel that crosses the potential plane of splitting.

f. Check $(c_b + K_{tr})/d_b \le 2.5$:

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.0 + 0}{0.5} = 2.0 < 2.5 \qquad \text{(O.K.)}$$

g. Calculate the excess reinforcement factor:

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}} = \frac{0.130}{0.160} = 0.813$$

h. Calculate $\ell_d$:

$$\ell_d = \frac{K_D}{\lambda}\left[\frac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right]K_{ER}d_b$$

$$= \frac{82.2}{1.0}\left[\frac{1.3(1.0)(0.8)}{2.0}\right](0.813)(0.5)$$

$$= 17.4 \text{ in.} > 12 \text{ in.} \qquad \text{(O.K.)}$$

Use $\ell_d$ = 18 in. (minimum).

9. Because the 18-in. length cannot be furnished, a hook will be provided. Determine if a 180° standard hook will be adequate.

a. Calculate $\ell_{dh}$.

$$\ell_{dh} = \left(\frac{0.02\psi_e f_y}{\lambda\sqrt{f_c'}}\right)d$$

b. The bars are uncoated and the concrete is normal weight. Therefore $\psi_e$ and $\lambda$ are both 1.0.

c. Modification factors are as follows:

1. Assume the concrete side cover is $2\frac{1}{2}$ in. normal to the plane of the hook; use 0.7.

2. For excess steel, use

$$\frac{A_s \text{ required}}{A_s \text{ provided}} = \frac{0.13}{0.16} = 0.813$$

d. Therefore, the required development length is

$$\ell_{dh} = \left(\frac{0.02(1.0)(60{,}000)}{(1.0)\sqrt{3000}}\right)(0.50)(0.7)(0.813)$$

$$= 6.23 \text{ in.}$$

Minimum $\ell_{dh}$ is 6 in. or $8d_b$, whichever is greater:

$$8d_b = 8\left(\tfrac{1}{2}\right) = 4 \text{ in.}$$

Therefore the minimum is 6 in.:

$$6.23 \text{ in.} > 6 \text{ in.} \qquad \text{(O.K.)}$$

e. Check the total width of beam required at the discontinuous end (see Figure 6-6):

$$6.23 + 2 = 8.23 \text{ in.} < 12 \text{ in.} \qquad \text{(O.K.)}$$

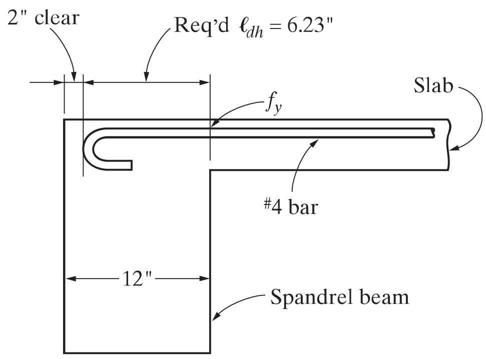



**FIGURE 6-6** Hook detail for Example 6-1.

10. Determine the bar cutoff points. For the normal type of construction for which the typical bar cutoff points shown in Figure 6-3 are used, the cutoff points are located so that all bars terminate in compression zones. Thus, the requirements of the ACI Code, Section 12.10.5, need not be checked, and the recommended bar cutoff points, as shown in Figure 6-3, are used.

11. Prepare the design sketches. The final design sketch for the slab is shown in Figure 6-7. For clarity, the interior and end spans are shown separately.

b. Continuous supporting beam: The second part of Example 6-1 involves the design of the continuous supporting beam. From Figure 6-4a, it is seen that these beams span between columns. The ACI coefficients to be used for moment determination are shown in Figure 6-8.

1. Determine the loading:

$$\text{service live load} = 250 \text{ psf} \times 12 = 3000 \text{ lb/ft}$$

$$\text{service dead load} = 25 \text{ psf} \times 12 = 300 \text{ lb/ft}$$

$$\text{weight of slab} = \left(\frac{5.5}{12}\right)(150)(12) = 825 \text{ lb/ft}$$

Assuming a beam width of 12 in. and an overall depth of 30 in. for purposes of member weight estimate (see Figure 6-9),

$$\text{weight of beam} = \frac{12(30 - 5.5)}{144}(150)$$

$$= 306.3 \text{ lb/ft}$$

$$\text{total service live load} = 3000 \text{ lb/ft}$$

$$\text{total service dead load} = 1431.3 \text{ lb/ft} \quad \text{say, } 1431 \text{ lb/ft}$$

2. Calculate the design load:

$$w_u = 1.2w_{DL} + 1.6w_{LL}$$

$$= 1.2(1431) + 1.6(3000)$$

$$= 1717 + 4800$$

$$= 6517 \text{ lb/ft} \quad \text{say, } 6.5 \text{ kips/ft}$$

The loaded beam is depicted in Figure 6-10.

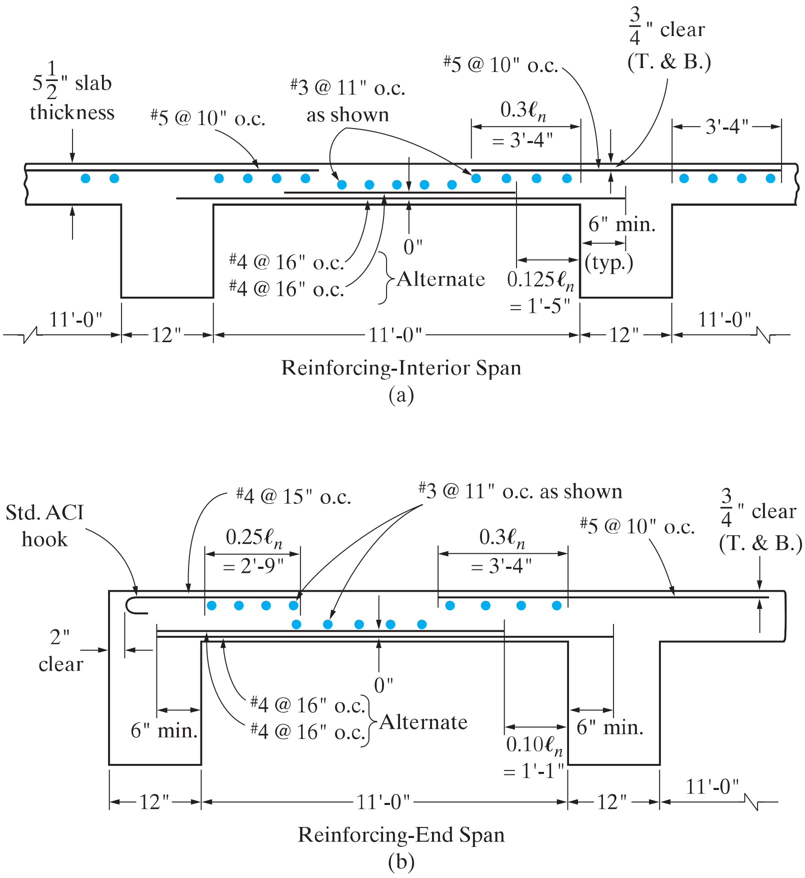



**FIGURE 6-7** Design sketches for Example 6-1.

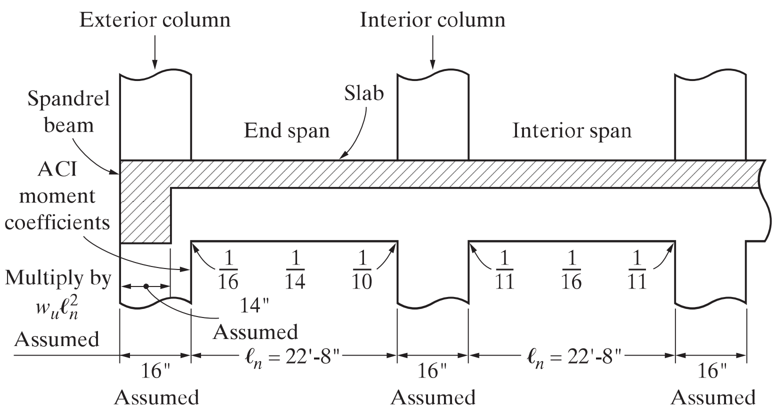



**FIGURE 6-8** Section B–B from Figure 6-4a.

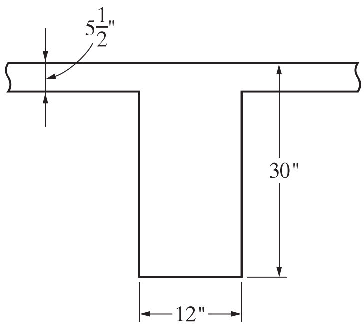



**FIGURE 6-9** Sketch for Example 6-1.

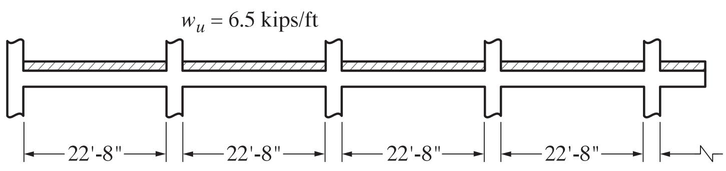



**FIGURE 6-10** Beam design load and spans.

3. Calculate the design moments and shears. The design moments and shears are calculated by using the ACI equations:

$$+M_u = \frac{1}{14} w_u \ell_n^2 = \frac{1}{14}(6.5)(22.67)^2 = 238.6 \text{ ft.-kips}$$

$$+M_u = \frac{1}{16} w_u \ell_n^2 = \frac{1}{16}(6.5)(22.67)^2 = 208.8 \text{ ft.-kips}$$

$$-M_u = \frac{1}{10} w_u \ell_n^2 = \frac{1}{10}(6.5)(22.67)^2 = 334.1 \text{ ft.-kips}$$

$$-M_u = \frac{1}{11} w_u \ell_n^2 = \frac{1}{11}(6.5)(22.67)^2 = 303.7 \text{ ft.-kips}$$

$$V_u = \frac{w_u \ell_n}{2} = \frac{6.5(22.67)}{2} = 73.7 \text{ kips}$$

$$V_u = 1.15\frac{w_u \ell_n}{2} = 1.15(6.5)\left(\frac{22.67}{2}\right) = 84.7 \text{ kips}$$

4. Design the beam. Establish concrete dimensions based on the maximum bending moment. This occurs in the end span at the first interior support where the negative moment $M_u = w_u \ell_n^2/10$. Since the top of the beam is in tension, the design will be that of a rectangular beam.
   a. Maximum moment (negative) = 334.1 ft.-kips.
   b. From Table A-5, assume that $\rho = 0.0090$ (which is less than $\rho_{max}$ of 0.01355 from Table A-8). A check of minimum steel required will be made shortly.
   c. From Table A-8, $\bar{k} = 0.4828$ ksi.
   d. Assume that $b = 12$ in.:

$$\text{required } d = \sqrt{\frac{M_u}{\phi b \bar{k}}} = \sqrt{\frac{334.1(12)}{0.9(12)(0.4828)}} = 28.0 \text{ in.}$$

$$d/b \text{ ratio} = \frac{28.0}{12} = 2.34,$$ which is within the acceptable range

   e. Check the estimated beam weight assuming one layer of No. 11 bars and No. 3 stirrups:

$$\text{required } h = 28.0 + \frac{1.41}{2} + 0.38 + 1.5 = 30.6 \text{ in.}$$

   Use $h = 31$ in. with an assumed $d$ of 28 in. Also, check the minimum $h$ from the ACI Code, Table 9.5(a):

$$\text{minimum } h = \frac{1}{18.5}(22.67)(12) = 14.7 \text{ in.} < 31 \text{ in.} \quad \text{(O.K.)}$$

   Note that the estimated beam weight based on $b = 12$ in. and $h = 30$ in. is slightly on the low side but may be considered acceptable.
   f. Design the steel reinforcing for points of negative moment as follows:

1. At the first interior support, based on an assumed $\rho$ of 0.0090 (see the preceding step 4),

$$\text{required } A_s = \rho bd = 0.0090(12)(28) = 3.0 \text{ in.}^2$$

From Table A-5,

$$A_{s,min} = 0.0033(12)(28) = 1.11 \text{ in.}^2$$

$$3.0 \text{ in.}^2 > 1.11 \text{ in.}^2 \quad \text{(O.K.)}$$

2. At the other interior supports, $-M_u = 303.7$ ft.-kips:

$$\text{required } \bar{k} = \frac{M_u}{\phi b d^2} = \frac{303.7(12)}{0.9(12)(28)^2} = 0.4304 \text{ ksi}$$

From Table A-8, $\rho = 0.0079$. Check $\rho$ with $\rho_{max}$:

$$0.0079 < 0.01355 \quad \text{(O.K.)}$$

Therefore

$$\text{required } A_s = \rho bd = 0.0079(12)(28) = 2.65 \text{ in.}^2$$

Check the required $A_s$ with $A_{s,min}$. From Table A-5,

$$A_{s,min} = 0.0033(12)(28) = 1.11 \text{ in.}^2$$

$$2.65 \text{ in.}^2 > 1.11 \text{ in.}^2 \quad \text{(O.K.)}$$

3. At the exterior support (exterior column), $-M_u = 208.8$ ft.-kips:

$$\text{required } \bar{k} = \frac{M_u}{\phi b d^2} = \frac{208.8(12)}{0.9(12)(28)^2} = 0.2959 \text{ ksi}$$

From Table A-8, $\rho = 0.0053$. Check $\rho$ with $\rho_{max}$:

$$0.0053 < 0.01355 \quad \text{(O.K.)}$$

Therefore

$$\text{required } A_s = \rho bd = 0.0053(12)(28) = 1.78 \text{ in.}^2$$

Check the required $A_s$ with $A_{s,min}$. From Table A-5,

$$A_{s,min} = 0.0033(12)(28) = 1.11 \text{ in.}^2$$

$$1.78 \text{ in.}^2 > 1.11 \text{ in.}^2 \quad \text{(O.K.)}$$

   g. Design steel reinforcing for points of positive moment as follows: At points of positive moment, the top of the beam is in compression; therefore, the design will be that of a T-beam.

1. End-span positive moment:
   a. Design moment = 238.6 ft.-kips = $M_u$.
   b. Effective depth $d = 28$ in. (see negative moment design).

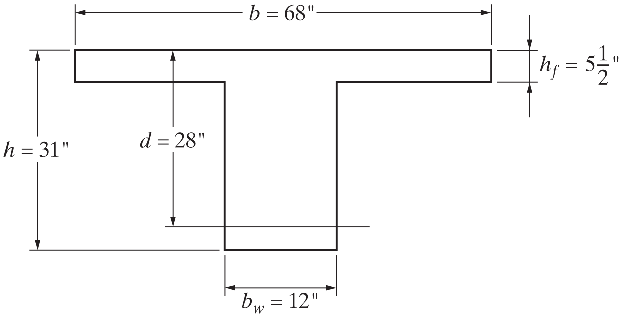



**FIGURE 6-11** Beam cross section.

c. Effective flange width:

$$\tfrac{1}{4}\text{ span length} = 0.25(22.67)(12) = 68\text{ in.}$$
$$b_w + 16h_f = 12 + 16(5.5) = 100\text{ in.}$$
$$\text{beam spacing} = 144\text{ in.}$$

Use an effective flange width $b = 68$ in. (see Figure 6-11).

d. Assuming total flange in compression, $\epsilon_t = 0.005$, and $\phi = 0.90$:

$$\phi M_{nf} = \phi(0.85f'_c)bh_f\left(d - \frac{h_f}{2}\right)$$
$$= 0.9(0.85)(3)(68)(5.5)\left(\frac{28 - 5.5/2}{12}\right)$$
$$= 1806\text{ ft.-kips}$$

e. Because $1806 > 238.6$, the member behaves as a wide rectangular T-beam with $b = 68$ in. and $d = 28$ in.

f. $$\text{required } \bar{k} = \frac{M_u}{\phi bd^2}$$
$$= \frac{238.6(12)}{0.9(68)(28)^2} = 0.0597\text{ ksi}$$

g. From Table A-8,

$$\text{required } \rho = 0.0010$$

h. The required steel area is

$$\text{required } A_s = \rho bd$$
$$= 0.0010(68)(28) = 1.90\text{ in.}^2$$

i. Use three No. 8 bars ($A_s = 2.37$ in.$^2$):

$$\text{required } b = 9.0\text{ in.} \qquad \text{(O.K.)}$$

j. Check $d$. With a No. 3 stirrup and $1\tfrac{1}{2}$-in. cover,

$$d = 31 - 1.5 - 0.38 - 1.00/2$$
$$= 28.6\text{ in.} > 28\text{ in.} \qquad \text{(O.K.)}$$

k. Check the required $A_s$ with $A_{s,\min}$. From Table A-5,

$$A_{s,\min} = 0.0033(12)(28) = 1.11\text{ in.}^2$$
$$2.37\text{ in.}^2 > 1.11\text{ in.}^2 \qquad \text{(O.K.)}$$

l. Check $\epsilon_t$ and $\phi$. From Table A-8, with $\rho = 0.001$, $\epsilon_t \geq 0.005$. Therefore, the assumed $\phi$ of 0.90 is O.K.

2. Interior span positive moment:

a. Design moment = 208.8 ft.-kips = $M_u$.

b. through (e) See the end-span positive moment computations. Use an effective flange width $b = 68$ in. and an effective depth $d = 28$ in. Also, for total flange in compression, $\phi M_n = 1806$ ft.-kips $> M_u$. Therefore, this member also behaves as a rectangular T-beam.

c. $$\text{required } \bar{k} = \frac{M_u}{\phi bd^2}$$
$$= \frac{208.8(12)}{0.9(68)(28)^2} = 0.0522\text{ ksi}$$

d. From Table A-8,

$$\text{required } \rho = 0.0010$$

e. The required steel area is

$$\text{required } A_s = \rho bd$$
$$= 0.0010(68)(28) = 1.90\text{ in.}^2$$

f. Use three No. 8 bars ($A_s = 2.37$ in.$^2$):

$$\text{required } b = 9.0\text{ in.} \qquad \text{(O.K.)}$$

g. through l. are identical to those for the end-span positive moment.

5. Check the distribution of negative moment steel. The ACI Code, Section 10.6.6, requires that where flanges are in tension, a part of the main tension reinforcement be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. The use of smaller bars spread out into part of the flange will also be advantageous where a beam is supported by a spandrel girder or exterior column and the embedment length for the negative moment steel is limited. Thus

$$\frac{\text{span}}{10} = \frac{22.67(12)}{10} = 27\text{ in.}$$
$$\text{effective flange width} = b = 68\text{ in.}$$

Therefore distribute the negative moment bars over a width of 27 in. Figure 6-12 shows suitable bars and patterns to use to satisfy the foregoing code requirement and furnish the cross-sectional area of steel required for flexure.

The ACI Code also stipulates that if the effective flange width exceeds one-tenth of the span, some longitudinal reinforcement shall be provided in the outer portions of the flange. In this design no additional steel will be furnished. In the authorsí opinion, this requirement is satisfied by the slab temperature and shrinkage steel (see Figures 6-7 and 6-18).

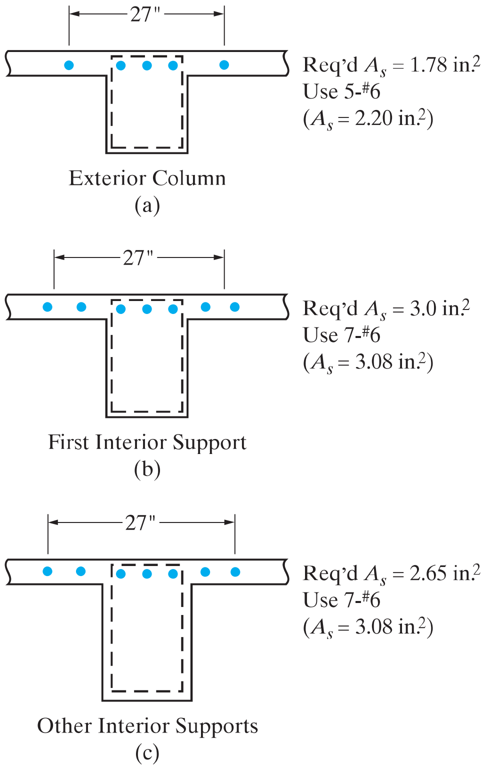



**FIGURE 6-12** Negative moment steel for beam of Example 6-1.

6. Prepare the work sketch. A work sketch is developed in Figure 6-13, which includes the bars previously chosen.
7. Check the anchorage into the exterior column.
   a. From Table 5-1, $K_D = 82.2$.
   b. Establish values for the factors $\psi_t$, $\psi_e$, $\psi_s$, and $\lambda$.
      1. $\psi_t = 1.3$ (the bars are top bars).
      2. The bars are uncoated; $\psi_e = 1.0$.
      3. The bars are No. 6; $\psi_s = 0.8$.
      4. Normal-weight concrete is used; $\lambda = 1.0$.
   c. The product $\psi_t \times \psi_e = 1.3 < 1.7$. (O.K.)
   d. Determine $c_b$. Based on cover (center of bar to nearest concrete surface), assume $1\frac{1}{2}$-in. clear cover and a No. 3 stirrup:

$$c_b = 1.5 + 0.375 + \frac{0.75}{2} = 2.25 \text{ in.}$$

   Based on bar spacing (one-half the center-to-center distance), refer to Figures 6-12. Consider the three No. 6 bars that are located within the No. 3 loop stirrup. Here

$$c_b = \left[\frac{12 - 2(1.5) - 2(0.375) - 0.75}{2}\right]\left(\frac{1}{2}\right)$$
$$= 1.875 \text{ in.}$$

   Therefore use $c_b = 1.875$ in.
   e. $K_{tr}$ may be conservatively taken as zero.
   f. Check $(c_b + K_{tr})/d_b \le 2.5$:

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.875 + 0}{0.75} = 2.5(\le 2.5) \qquad \text{(O.K.)}$$

   g. Determine the excess reinforcement factor:

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}} = \frac{1.78}{2.20} = 0.809$$

   h. Calculate $\ell_d$:

$$\ell_d = \frac{K_D}{\lambda}\left[\frac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right]d_b K_{ER}$$
$$= \frac{82.2}{1.0}\left[\frac{1.3(1.0)(0.8)}{2.5}\right](0.75)(0.809)$$
$$= 20.7 \text{ in.} > 12 \text{ in.} \qquad \text{(O.K.)}$$

   Use $\ell_d = 21$ in. (minimum).

   With 2.0 in. clear at the end of the bar, the embedment length available in the column is $16.0 - 2.0 = 14.0$ in.
8. Because 21 in. > 14.0 in., a hook is required. Determine if a 90° standard hook will be adequate.

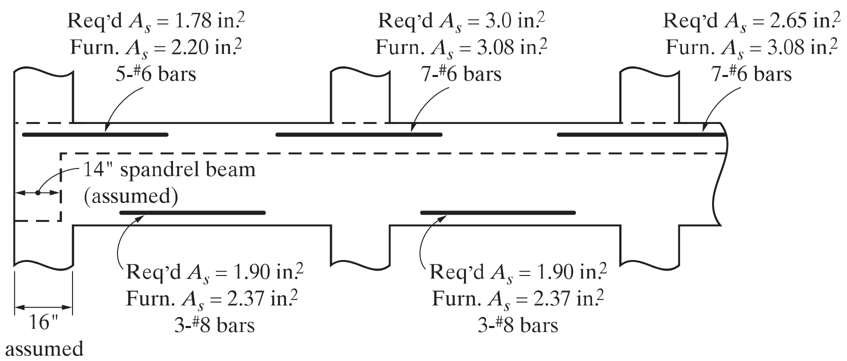



**FIGURE 6-13** Work sketch for beam design of Example 6-1.

a. From Table A-13, $\ell_{dh} = 16.4$ in.

b. Modification factors (MF) to be used are

1. Assume concrete cover $\geq 2\frac{1}{2}$ in. and cover on the bar extension beyond the hook = 2 in.; use 0.7.
2. For excess steel, use

$$\frac{\text{required } A_s}{\text{provided } A_s} = \frac{1.78}{2.20} = 0.809$$

c. The required development length for the hook is

$$\ell_{dh} = 16.4(0.7)(0.809) = 9.29 \text{ in.}$$

Minimum $\ell_{dh}$ is 6 in. or $8d_b$, whichever is greater:

$$8d_b = 8(\tfrac{3}{4}) = 6 \text{ in.}$$

$$9.29 \text{ in.} > 6 \text{ in.} \qquad \text{(O.K.)}$$

d. Check the total width of column required (see Figure 6-14):

$$9.29 + 2 = 11.3 \text{ in.} < 16 \text{ in.} \qquad \text{(O.K.)}$$

For other points along the continuous beam, use bar cutoff points recommended in Figure 6-3 and as shown in Figure 6-18.

9. Prepare the stirrup design. Established values are $b_w = 12$ in., effective depth $d = 28$ in., $f'_c = 3000$ psi, and $f_y = 60{,}000$ psi.

a. The shear force $V_u$ diagram may be observed in Figure 6-15. Note that the shear diagram is unsymmetrical with respect to the centerline of the span, and the point of zero shear does not occur at the midspan of the beam. Furthermore, the point of zero shear for the right-hand section of the end span of the beam occurs at 13.03 ft. (that is, 84.7 kips/$w_u$ = 84.7/6.5) from the face of the first interior support. It should be noted that the point of zero shear for the left-hand section of the beam occurs at 11.34 ft. (that is, 73.7 kips/6.5) from the interior face of the end support. The points of zero shears for both sections of the beam occur at different locations because of the approximate nature of the ACI shear coefficients as pointed out earlier. The stirrup design will be based on shear in the interior portion of the end span where the maximum values occur. The resulting stirrup pattern will be used throughout the continuous beam. Only the applicable portion of the $V_u$ diagram is shown.

b. Determine if stirrups are required:

$$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d = \frac{0.75(2\sqrt{3000})(12)(28)}{1000}$$

$$= 27.6 \text{ kips}$$

$$\tfrac{1}{2}\phi V_c = \tfrac{1}{2}(27.6) = 13.8 \text{ kips}$$

At the critical section $d$ distance (28 in.) from the face of the support,

$$V_u^* = 84.7 - \frac{28}{12}(6.5) = 69.5 \text{ kips}$$

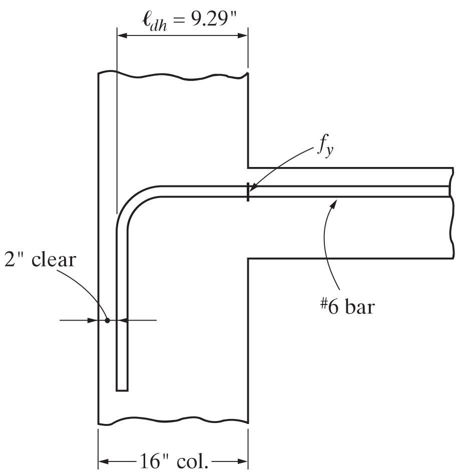



**FIGURE 6-14** Anchorage at column.

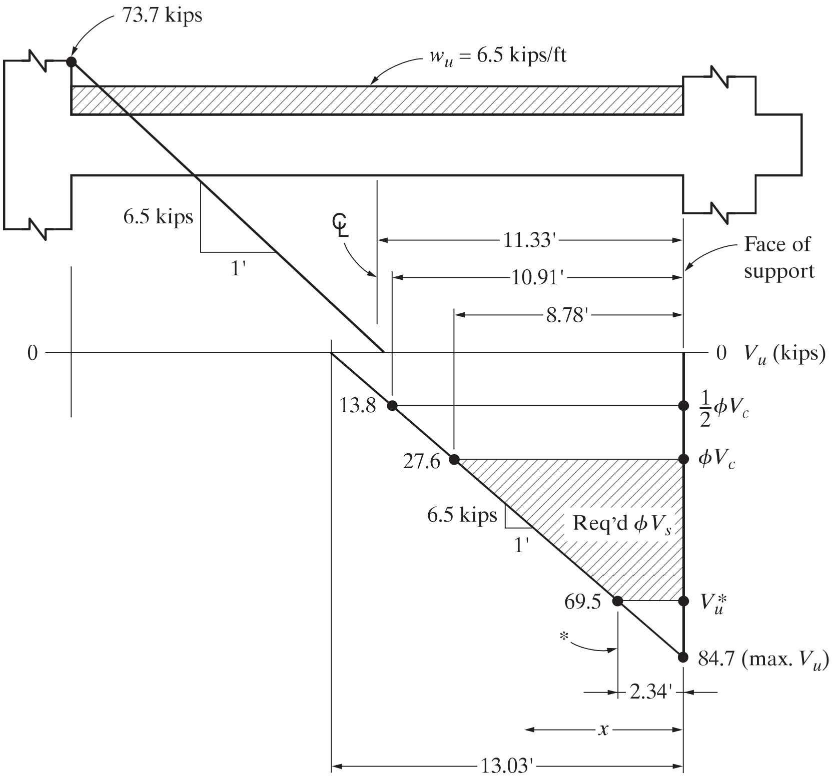



**FIGURE 6-15** $V_u$ diagram for stirrup design (Example 6-1).

(Quantities at the critical section are designated with an asterisk.) Stirrups are required because

$$V_u^* > \tfrac{1}{2}\phi V_c (69.4 \text{ kips} > 13.8 \text{ kips})$$

c. Find the length of span over which stirrups are required. Stirrups are required to the point where

$$V_u = \tfrac{1}{2}\phi V_c = 13.8 \text{ kips}$$

From Figure 6-15 and referencing from the face of the support, $V_u = 13.8$ kips at

$$\frac{84.7 - 13.8}{6.5} = 10.91 \text{ ft}$$

The distance from the face of the support to where $V_u = \phi V_c = 27.6$ kips is

$$\frac{84.7 - 27.6}{6.5} = 8.78 \text{ ft}$$

d. On the $V_u$ diagram, designate the area between the $\phi V_c$ line, the $V_u^*$ line, and the sloping $V_u$ line as "Req'd $\phi V_s$." At locations between 2.34 ft and 8.78 ft from the face of the support, the required $\phi V_s$ varies. Designating the slope of the $V_u$ diagram as $m$ (kips/ft) and taking $x$ (ft) from the face of the support ($2.34 \le x \le 8.78$) yields

$$\begin{aligned} \text{required } \phi V_s &= \text{maximum } V_u - \phi V_c - mx \\ &= 84.7 - 27.6 - 6.5x \\ &= 57.1 - 6.5x \end{aligned}$$

e. Assume a No. 3 vertical stirrup ($A_v = 0.22$ in.$^2$):

$$\begin{aligned} \text{required } s^* &= \frac{A_v f_{yt} d}{V_s} \\ &= \frac{\phi A_v f_{yt} d}{\text{required } \phi V_s^*} \\ &= \frac{\phi A_v f_{yt} d}{V_u^* - \phi V_c} \\ &= \frac{0.75(0.22)(60)(28)}{69.5 - 27.6} = 7.57 \text{ in.} \end{aligned}$$

Use $6\tfrac{1}{2}$-in. spacing between the critical section and the face of the support.

f. Establish ACI Code maximum spacing requirements:

$$4\sqrt{f_c'}\, b_w d = \frac{4\sqrt{3000}\,(12)(28)}{1000} = 73.6 \text{ kips}$$

Calculating $V_s^*$ at the critical section yields

$$\begin{aligned} \phi V_s^* &= V_u^* - \phi V_c \\ &= 69.5 - 27.6 = 41.9 \text{ kips} \end{aligned}$$

$$V_s^* = \frac{\phi V_s^*}{\phi} = \frac{41.9}{0.75} = 55.9 \text{ kips}$$

Because 55.9 kips < 73.6 kips, the maximum spacing should be the smaller of $d/2$ or 24 in.:

$$\frac{d}{2} = \frac{28}{2} = 14 \text{ in.}$$

Also check:

$$s_{\max} = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} \le \frac{A_v f_{yt}}{50 b_w}$$

$$\frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} = \frac{0.22(60{,}000)}{0.75\sqrt{3000}(12)} = 26.7 \text{ in.}$$

and

$$\frac{A_v f_{yt}}{50 b_w} = \frac{0.22(60{,}000)}{50(12)} = 22.0 \text{ in.}$$

Therefore, use a maximum spacing of 14 in.

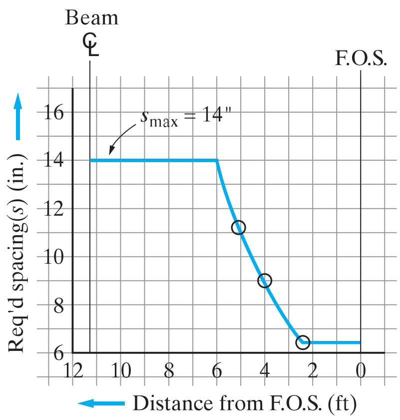
g. Determine the spacing requirements based on shear strength to be furnished. The denominator of the following formula for required spacing uses the expression for required $\phi V_s$ from step d:

$$\begin{aligned} \text{required } s &= \frac{\phi A_v f_{yt} d}{\text{required } \phi V_s} \\ &= \frac{0.75(0.22)(60)(28)}{57.1 - 6.5x} = \frac{277.2}{57.1 - 6.5x} \end{aligned}$$

The results for several arbitrary values of $x$ are shown tabulated and plotted in Figure 6-16.

h. Using Figure 6-16, the stirrup pattern shown in Figure 6-17 is developed. Despite the lack of symmetry in the shear diagram, the stirrup pattern is symmetrical with respect to the centerline of the span. This is conservative and will be used for all spans.

The design sketches are shown in Figure 6-18. As with the slab design, the typical bar cutoff points of Figure 6-3 are used for this beam. All bars therefore terminate in compression zones, and the requirements of the ACI Code, Section 12.10.5, need not be checked.




| $x$ (ft) | Req'd $s$ (in.) |
|---|---|
| 2.34 | 6.62 |
| 4 | 8.91 |
| 5 | 11.27 |
| 6 | 15.3 |

**FIGURE 6-16** Stirrup spacing requirements for Example 6-1.

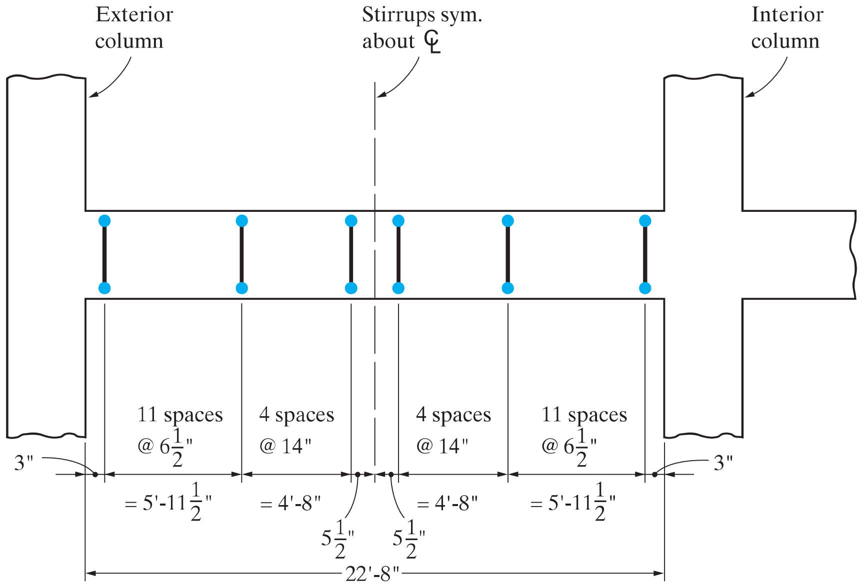



**FIGURE 6-17** Stirrup spacing for Example 6-1 end span (interior spans similar).

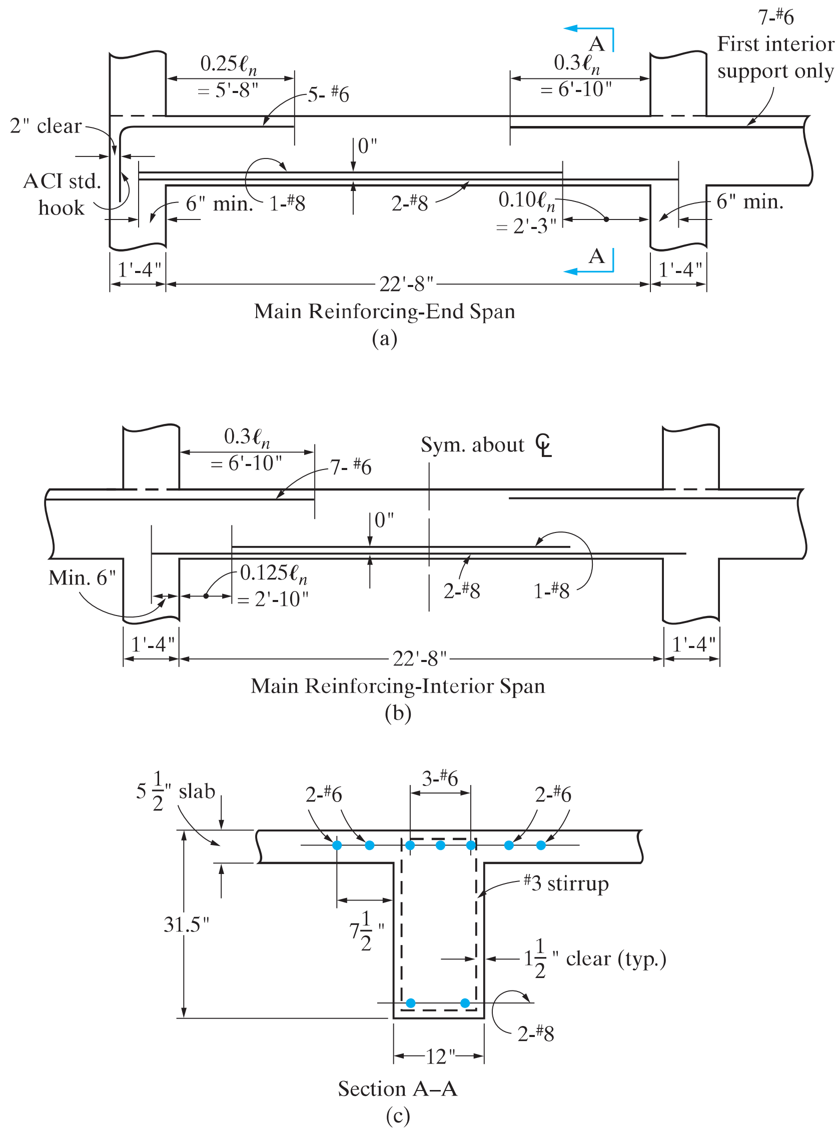



**FIGURE 6-18** Design sketches for Example 6-1 (stirrup spacings not shown).