The ***price elasticity of supply*** is defined in an analogous way to that of demand. We define

$$E = \frac{\text{percentage change in supply}}{\text{percentage change in price}}$$

This time, however, there is no need to fiddle the sign. An increase in price leads to an increase in supply, so E is automatically positive. In symbols,

$$E = \frac{P}{Q} \times \frac{\Delta Q}{\Delta P}$$

If (Q_1, P_1) and (Q_2, P_2) denote two points on the supply curve then arc elasticity is obtained, as before, by setting

$$\Delta P = P_2 - P_1$$

$$\Delta Q = Q_2 - Q_1$$

$$P = \tfrac{1}{2}(P_1 + P_2)$$

$$Q = \tfrac{1}{2}(Q_1 + Q_2)$$

The corresponding formula for point elasticity is

$$E = \frac{P}{Q} \times \frac{\mathrm{d}Q}{\mathrm{d}P}$$

Example

Given the supply function

$$P = 10 + \sqrt{Q}$$

find the price elasticity of supply

(a) averaged along an arc between $Q = 100$ and $Q = 105$

(b) at the point $Q = 100$

Solution

(a) We are given that

$$Q_1 = 100, \; Q_2 = 105$$

so that

$$P_1 = 10 + \sqrt{100} = 20 \quad \text{and} \quad P_2 = 10 + \sqrt{105} = 20.247$$

Hence

$$\Delta P = 20.247 - 20 = 0.247, \qquad \Delta Q = 105 - 100 = 5$$

$$P = \frac{1}{2}(20 + 20.247) = 20.123, \qquad Q = \frac{1}{2}(100 + 105) = 102.5$$

The formula for arc elasticity gives

$$E = \frac{P}{Q} \times \frac{\Delta Q}{\Delta P} = \frac{20.123}{102.5} \times \frac{5}{0.247} = 3.97$$

(b) To evaluate the elasticity at the point $Q = 100$, we need to find the derivative, $\frac{dQ}{dP}$. The supply equation

$$P = 10 + Q^{1/2}$$

differentiates to give

$$\frac{dP}{dQ} = \frac{1}{2}Q^{-1/2} = \frac{1}{2\sqrt{Q}}$$

so that

$$\frac{dQ}{dP} = 2\sqrt{Q}$$

At the point $Q = 100$, we get

$$\frac{dQ}{dP} = 2\sqrt{100} = 20$$

The formula for point elasticity gives

$$E = \frac{P}{Q} \times \frac{dQ}{dP} = \frac{20}{100} \times 20 = 4$$

Notice that, as expected, the answers to parts (a) and (b) are nearly the same.

Practice Problem

4 If the supply equation is

$$Q = 150 + 5P + 0.1P^2$$

calculate the price elasticity of supply

(a) averaged along an arc between $P = 9$ and $P = 11$

(b) at the point $P = 10$

Advice

The concept of elasticity can be applied to more general functions and we consider some of these in the next chapter. For the moment we investigate the theoretical properties of demand elasticity. The following material is more difficult to understand than the foregoing, so you may prefer just to concentrate on the conclusions and skip the intermediate derivations.

We begin by analysing the relationship between elasticity and marginal revenue. Marginal revenue, MR, is given by

$$\text{MR} = \frac{d(\text{TR})}{dQ}$$

Now TR is equal to the product PQ, so we can apply the product rule to differentiate it. If

$$u = P \quad \text{and} \quad v = Q$$

then

$$\frac{\mathrm{d}u}{\mathrm{d}Q} = \frac{\mathrm{d}P}{\mathrm{d}Q} \quad \text{and} \quad \frac{\mathrm{d}v}{\mathrm{d}Q} = \frac{\mathrm{d}Q}{\mathrm{d}Q} = 1$$

By the product rule

$$\begin{aligned}\mathrm{MR} &= u\frac{\mathrm{d}v}{\mathrm{d}Q} + v\frac{\mathrm{d}u}{\mathrm{d}Q}\\ &= P + Q \times \frac{\mathrm{d}P}{\mathrm{d}Q}\\ &= P\left(1 + \frac{Q}{P} \times \frac{\mathrm{d}P}{\mathrm{d}Q}\right)\end{aligned}$$

check this by multiplying out the brackets

Now

$$-\frac{P}{Q} \times \frac{\mathrm{d}Q}{\mathrm{d}P} = E$$

so

$$\frac{Q}{P} \times \frac{\mathrm{d}P}{\mathrm{d}Q} = -\frac{1}{E}$$

turn both sides upside down and multiply by −1

This can be substituted into the expression for MR to get

$$\mathrm{MR} = P\left(1 - \frac{1}{E}\right)$$

The connection between marginal revenue and demand elasticity is now complete, and this formula can be used to justify the intuitive argument that we gave at the beginning of this section concerning revenue and elasticity. Observe that if $E < 1$ then $1/E > 1$, so MR is negative for any value of P. It follows that the revenue function is decreasing in regions where demand is inelastic, because MR determines the slope of the revenue curve. Similarly, if $E > 1$ then $1/E < 1$, so MR is positive for any price, P, and the revenue curve is uphill. In other words, the revenue function is increasing in regions where demand is elastic. Finally, if $E = 1$ then MR is 0, and so the slope of the revenue curve is horizontal at points where demand is unit elastic.

Throughout this section we have taken specific functions and evaluated the elasticity at particular points. It is more instructive to consider general functions and to deduce general expressions for elasticity. Consider the standard linear downward-sloping demand function

$$P = aQ + b$$

when $a < 0$ and $b > 0$. As noted in Section 4.3, this typifies the demand function faced by a monopolist. To transpose this equation for Q, we subtract b from both sides to get

$$aQ = P - b$$

and then divide through by a to get

$$Q = \frac{1}{a}(P - b)$$

Hence

$$\frac{dQ}{dP} = \frac{1}{a}$$

The formula for elasticity of demand is

$$E = -\frac{P}{Q} \times \frac{dQ}{dP}$$

so replacing Q by $(1/a)(P - b)$ and dQ/dP by $1/a$ gives

$$E = \frac{-P}{(1/a)(P-b)} \times \frac{1}{a}$$

$$= \frac{-P}{P-b}$$

$$= \frac{P}{b-P}$$

multiply top and bottom by −1

Notice that this formula involves P and b but not a. Elasticity is therefore independent of the slope of linear demand curves. In particular, this shows that, corresponding to any price P, the elasticities of the two demand functions sketched in Figure 4.20 are identical. This is perhaps a rather surprising result. We might have expected demand to be more elastic at point A than at point B, since A is on the steeper curve. However, the mathematics shows that this is not the case. (Can you explain, in economic terms, why this is so?)

Another interesting feature of the result

$$E = \frac{P}{b-P}$$

is the fact that b occurs in the denominator of this fraction, so that corresponding to any price, P, the larger the value of the intercept, b, the smaller the elasticity. In Figure 4.21, elasticity at C is smaller than that at D because C lies on the curve with the larger intercept.

The dependence of E on P is also worthy of note. It shows that elasticity varies along a linear demand curve. This is illustrated in Figure 4.22. At the left-hand end, $P = b$, so

Figure 4.20

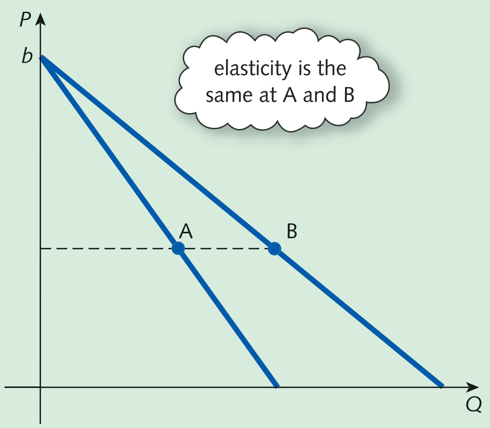


Figure 4.21

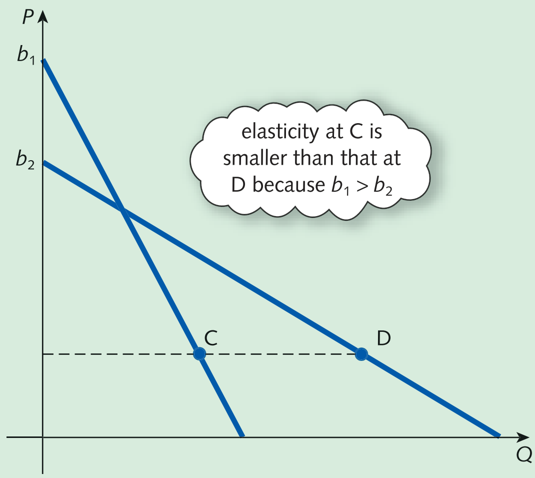
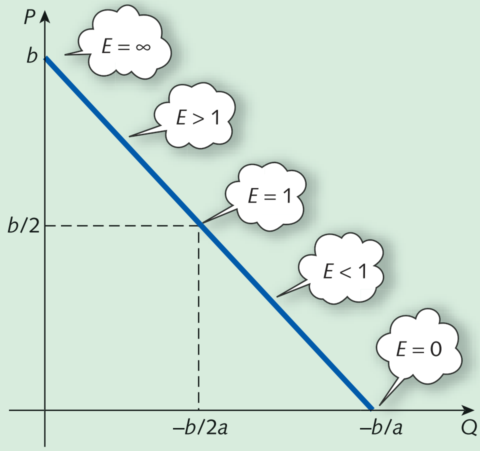


Figure 4.22



$$E = \frac{b}{b - b} = \frac{b}{0} = \infty$$ read 'infinity'

At the right-hand end, $P = 0$, so

$$E = \frac{0}{b - 0} = \frac{0}{b} = 0$$

As you move down the demand curve, the elasticity decreases from ∞ to 0, taking all possible values. Demand is unit elastic when $E = 1$ and the price at which this occurs can be found by solving

$$\frac{P}{b - P} = 1 \text{ for } P$$

$$
\begin{aligned}
P &= b - P && \text{(multiply both sides by } b - P\text{)} \\
2P &= b && \text{(add } P \text{ to both sides)} \\
P &= \frac{b}{2} && \text{(divide both sides by 2)}
\end{aligned}
$$

The corresponding quantity can be found by substituting $P = b/2$ into the transposed demand equation to get

$$Q = \frac{1}{a}\left(\frac{b}{2} - b\right) = -\frac{b}{2a}$$

Demand is unit elastic exactly halfway along the demand curve. To the left of this point $E > 1$ and demand is elastic, whereas to the right $E < 1$ and demand is inelastic.

In our discussion of general demand functions, we have concentrated on those which are represented by straight lines since these are commonly used in simple economic models. There are other possibilities and Practice Problem 11 investigates a class of functions that have constant elasticity.

Key Terms

Arc elasticity Elasticity measured between two points on a curve.

Elastic demand Where the percentage change in demand is more than the corresponding percentage change in price: $E > 1$.

Inelastic demand Where the percentage change in demand is less than the corresponding percentage change in price: $E < 1$.

Price elasticity of demand A measure of the responsiveness of the change in demand due to a change in price: – (percentage change in demand) ÷ (percentage change in price).

Price elasticity of supply A measure of the responsiveness of the change in supply due to a change in price: (percentage change in supply) ÷ (percentage change in price).

Unit elastic demand Where the percentage change in demand is the same as the percentage change in price: $E = 1$.

Practice Problems

5 Given the demand function

$$P = 500 - 4Q^2$$

calculate the price elasticity of demand averaged along an arc joining $Q = 8$ and $Q = 10$.

6 Find the price elasticity of demand at the point $Q = 9$ for the demand function

$$P = 500 - 4Q^2$$

and compare your answer with that of Practice Problem 5.

7 Find the price elasticity of demand at $P = 6$ for each of the following demand functions:

(a) $P = 30 - 2Q$

(b) $P = 30 - 12Q$

(c) $P = \sqrt{(100 - 2Q)}$

8 If the demand equation is

$$Q + 4P = 60$$

find a general expression for the price elasticity of demand in terms of P. For what value of P is demand unit elastic?

9 Consider the supply equation

$$Q = 4 + 0.1P^2$$

(a) Write down an expression for dQ/dP.

(b) Show that the supply equation can be rearranged as

$$P = \sqrt{(10Q - 40)}$$

Differentiate this to find an expression for dP/dQ.

(c) Use your answers to parts (a) and (b) to verify that

$$\frac{dQ}{dP} = \frac{1}{dP/dQ}$$

(d) Calculate the elasticity of supply at the point $Q = 14$.

10 If the supply equation is

$$Q = 7 + 0.1P + 0.004P^2$$

find the price elasticity of supply if the current price is 80.

(a) Is supply elastic, inelastic or unit elastic at this price?

(b) Estimate the percentage change in supply if the price rises by 5%.

11 Show that the price elasticity of demand is constant for the demand functions

$$P = \frac{A}{Q^n}$$

where A and n are positive constants.

12 Find a general expression for the point elasticity of supply for the function,

$$Q = aP + b \qquad (a > 0)$$

Deduce that the supply function is

(a) unit elastic when $b = 0$

(b) inelastic when $b > 0$

Give a brief geometrical interpretation of these results.