Second :Unsteady- State Radial Flow

ثانيا: الجريان الشعاعي الغير مستقر

Theis's Method

Theis (1935) was the first to develop a formula for unsteady-state flow that introduces the time factor and the storativity. He noted that when a well penetrating an extensive confined aquifer is pumped at a constant rate,the influence of the discharge extends outward with time (Figure). The rate of decline of head, multiplied by the storativity and summed over the area of influence, equals the discharge. The unsteady-state (or Theis) equation, which was derived from the analogy between the flow of groundwater and the conduction of heat, is written as:

$$s = \frac{Q}{4\pi T} \int_{-u}^{u} \frac{e^{-u}}{u} d_u = \frac{Q}{4\pi T} W(u) \quad (1)$$

Where

s= the drawdown in m measured in a piezometer at a distance r (m) from the well,

Q= the constant well discharge in m^3/d

T= the transmissivity of the aquifer in m^2/d

W(u) is the Theis well function, it is equal to:

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4!}{4.4} + \ldots \quad (1a)$$

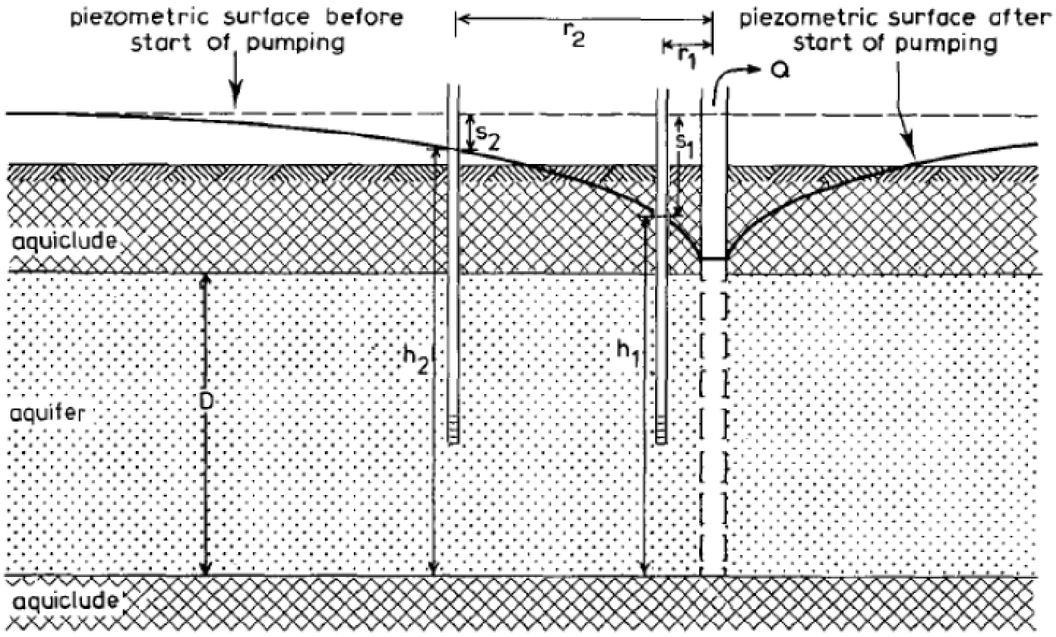
And u is the dimensionless time factor and is equal to

$u = r^2S / 4Tt$ (2)

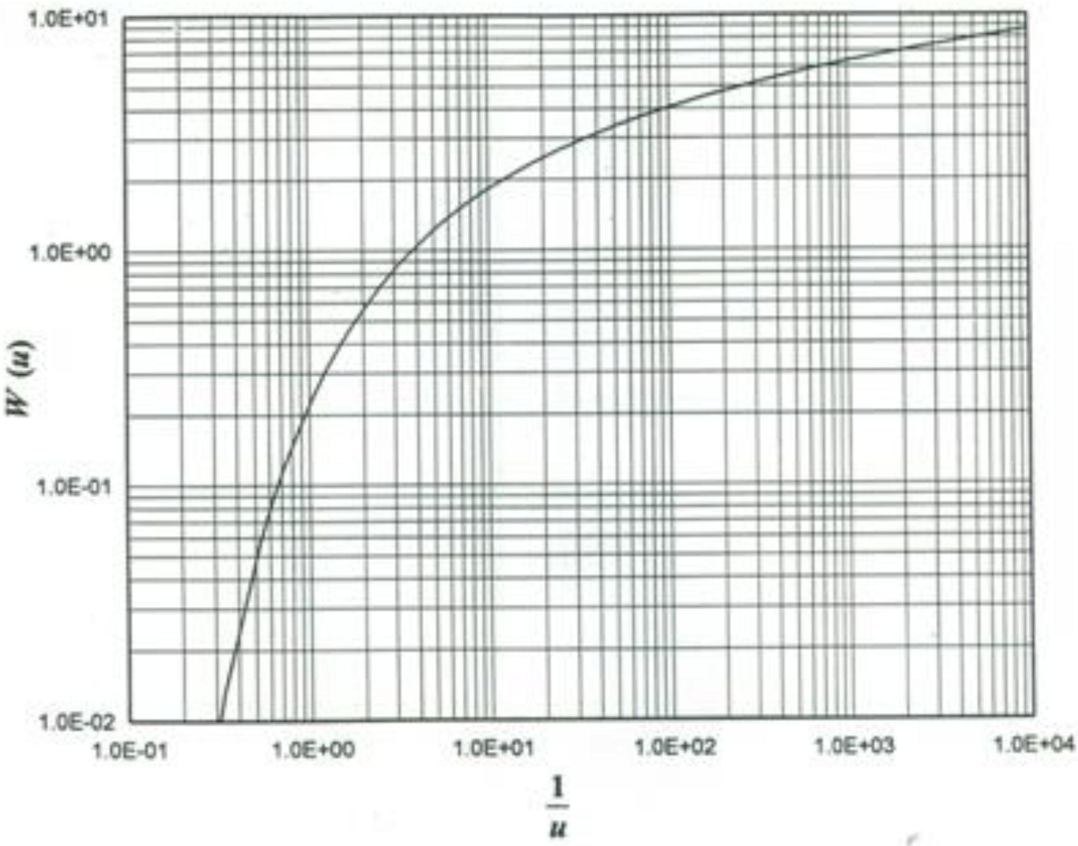
Where S= the dimensionless storativity of the aquifer

t= the time in days since pumping started.

The values of W(u) were calculated by substituting varies values of u in equation(1a), and both were then plotted against each other on a log-log paper. The curve obtained was called ***Theis type curve***.



Cross-section of a pumped well fully penetrating a confined aquifer.



Theis Type Curve for confined aquifers.

Theis method permits the determination of the aquifer properties T and S by means of pumping tests wells. He listed a number of assumptions which should be taken into account prior to applying his method. They are as follows:

1) The aquifer is confined;
2) The aquifer has a seemingly infinite areal extent;
3) The aquifer is homogeneous, isotropic, and of uniform thickness;
4) Prior to pumping, the piezometric surface is horizontal;
5) The aquifer is pumped at a constant discharge rate;
6) The well penetrates the entire thickness of the aquifer and thus receives water horizontal flow;
7) The water removed from storage is discharged instantaneously with decline of head;
8) The diameter of the well is small, i.e. the storage in the well can be neglected.

Application of Theis Method

In order to apply Theis's method a curve matching technique, between field data and Theis type curve, is used.

The step-by-step procedure for determining confined aquifer parameters from time-drawdown data by the Theis Type-Curve method is as follows:

Step 1: Construct Theis Type Curve by plotting W(u) and u on the log-log graph paper as shown in previous Fig.. Alternatively, obtain a copy of this curve from the literature.

Step 2: Plot field-data curve using observed values of **drawdown (s) versus r^2/t** on the **log-log graph** paper having the same scale as the Type Curve.

Step 3: Superimpose the transparent field-data curve on the Type-Curve sheet, keeping coordinate axes of the two graphs parallel to each other. Adjust the field-data curve until a best fit of field data points to the Type Curve.

Step 4: Select an arbitrary 'match point' on the Type Curve and note down the corresponding coordinates (s and r^2/t) from the field-data curve, and W(u) and u from the Type Curve. Note that the selection of (1,1) match point on the Type Curve simplifies the calculation.

Step 5: Finally, substitute the values of these coordinates and the value of Q in Eqn.

$$T = \frac{Q}{4\pi s} W(u)$$

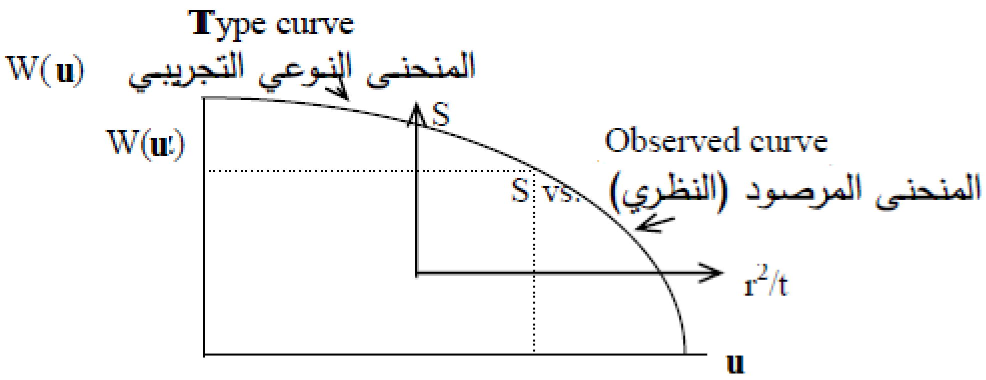
to calculate T. Thereafter, substitute the values of the known variables in Eqn

$S = \frac{4Ttu}{r^2}$ to obtain S.

Matching Method طريقه المضاهاة

للحصول على ثابتي التشكيل T و S من الضخ الإختباري (الهبوط مقابل الزمن) اقترح سيز Theis الطريقة البيانية التالية:

- يجهز رسم u مقابل $W(u)$ في ورق Log Log يسمى المنحنى النوعي (Type curve) –



Matching method

- يرسم في ورق آخر مماثل لوغرثمي المحورين Log Log قيم الهبوط S مقابل لقيمة $\left(\frac{r^2}{t}\right)$ ويسمى بالمنحنى المرصود (Observed curve)
- يوضع المنحنى المرصود فوق المنحنى النوعي (الأول يرسم في ورق عادي والثاني يرسم في ورق شفاف) مع وزن الإحداثيات لتكون متوازية إلى أن يتم تطابق القيم في المنحنيين بأكبر قدر (بالتجربة)
- تحدد نقط اختبارية في منطقة التطابق وتؤخذ قيم الإحداثيات من كلا المنحنيين $\frac{r^2}{t}$ و S و u و $W(u)$ ومن ثم تحدد قيم S و T من المعادلتين **المذكورتيين سابقا**

From eq.1:

$$S = \frac{Q}{4\pi T} W(u)$$

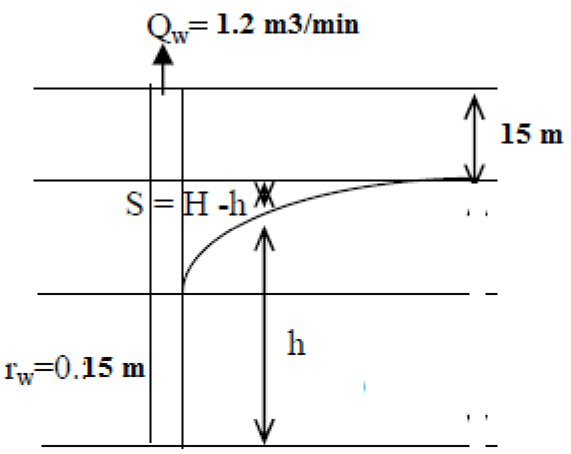
W(u) can be found from the following table:

W(u) for different values of u

9	8	7	6	5	4	3	2	1	***u***
0.000012	0.000038	0.00012	0.00036	0.00114	0.0038	0.013	0.049	0.219	X1
0.26	0.31	0.37	0.45	0.56	0.7	0.91	1.22	1.82	X10^{-1}
1.92	2.03	2.15	2.3	2.48	2.68	2.96	3.35	4.04	X10^{-2}
4.14	4.16	4.39	4.54	4.73	4.95	5.23	5.64	6.33	X10^{-3}
6.44	6.55	6.69	6.84	7.02	7.25	7.53	7.94	8.63	X10^{-4}
8.74	8.86	8.99	9.14	9.33	9.55	9.84	10.24	10.95	X10^{-5}
11.04	11.16	11.29	11.45	11.63	11.85	12.14	12.55	13.24	X10^{-6}
13.34	13.46	13.6	13.75	13.93	14.15	14.44	14.85	15.54	X10^{-7}
15.65	15.76	15.9	16.05	16.23	16.46	16.74	17.15	17.84	X10^{-8}
17.95	18.07	18.2	18.35	18.54	18.76	19.05	19.45	20.15	X10^{-9}
20.25	20.37	20.5	20.66	20.84	21.06	21.35	21.76	22.45	X10^{-10}
22.55	22.67	22.81	22.96	23.14	23.36	23.65	24.06	24.75	X10^{-11}
24.86	24.97	25.11	25.26	25.44	25.66	25.95	26.36	27.05	X10^{-12}
27.16	27.28	27.41	27.56	27.75	27.97	28.26	28.66	29.36	X10^{-13}
29.46	29.58	29.71	29.87	30.05	30.27	30.56	30.97	31.66	X10^{-14}
31.76	31.88	32.02	32.17	32.35	32.58	32.86	33.27	33.96	X10^{-15}

Example : A 30cm diameter well penetrates vertically through an aquifer has thickness 18m depth and static water table level is 15 m from the ground surface. S=0.01, Q= 1.2 m^3/min, T= 218 m^3/d/m. Find the water drawdown calculated from ground surface in the end of the first, second and third year respectively.

Solution:



S=0.01 , r_w =0.15 m , T= 218 m^3/d/m, Q= 1.2 m^3/min , D = 18 m

$$u = r^2S / 4Tt$$

end of 1st year $u1=(0.15)^2 *0.01/4*218*365 = 7*10^{-10}$

end of 2nd year $u2= u1/2 = 3.5*10^{-10}$

end of 3rd year $u3= u1/3= 2.35*10^{-10}$

from the table:

W(u1) = 20.5 , W(u2) = 21.2 , W(u3) =21.6

$$S = \frac{Q}{4\pi T} W(u)$$

$Q=1.2*60*24=1728\ m^3/d$

s1= 12.9 m + 15 m (from the surface) = 27.9 m

s2= 13.35 m + 15 m (from the surface) = 28.35 m

s3= 13.6 m + 15 m (from the surface) = 28.6 m

الاجراءات الواجب اتباعها قبل بدء اختبار الضخ

1- Make sure that the water level in the steady-state. This is done by taking several water level readings at different times and then calculating the relative error (∞), by using the following equation:
∞ = (h1-h2 / h1) * 100
Where h1= the higher reading (m),and h2 the smaller reading (m).

If ∞ is equal to 5% or less, then we can consider the flow is in steady condition. If not then we must wait until it becomes constant.

2 -Measure the static water level.
3- Measure the total depth of the well.
4 - Measure the diameter of the well and the distance between the main welland observation wells, if available.
5 – Prepare the stopwatch and the data sheet to register the data.

طريقة إجراء الاختبار:

1-يتم تشغيل المضخة بسرعة معتدلة .
2-يبدأ تشغيل الساعة مباشرة عند بدأ الضخ.
3-يتم قياس مستوى الماء على فترات معينة من الزمن وتسجيل البيانات في جدول.

4- عند الوصول لمستوى ثابت للماء توقف المضخة ويبدأ عندها قياس مستوى الرجوع.
5- يجب أخذ عدة قياسات لمعدل التصريف على فترات مختلفة أثناء الضخ ثم يحسب المتوسط لها:

$$Q = \frac{Q_1 + Q_2 + Q_3 + \dots + Q_n}{n}$$

6- توضع البيانات الخاصة بالتجربة في جدول خاص.

Pumping Test Data

Well Owner | Area
Well No | Well Diameter
Distance to Observation Well | Total Depth of Well
Casing Length | Casing Type
Initial depth to water level | Final depth to water Level
Start time | Stop time
Observer | Date

Time (min)	Water level Meter below reference point	Drawdown (m)	Discharge rate (m^3 / day)
1			
2			
قراءة كل دقيقة الى 10			
10			
12			
14			
16			
20			
25			
30			
40			
50			
60			
70			
80			
90			
100			
120			
150			
180			
210			
240			
270			
300			
360			
ثم كل ساعة إلى النهاية			

Necessary equipment for the pumping test: المعدات الضرورية لفحص الضخ

1 – One main (pumped) well and one or two observation wells (or piezometers),
2 – A pump
3-One or two stop watches,
4-A tape Meter,
.5 - A device for measuring the depth to the water level,
6- A device for measuring the rate of discharge of the well.

Office Work of pumping test

The time-drawdown plots obtained from the pumping tests are plotted on semi-logarithmic and /or log-lag papers. Each plot serves certain urposes:

1. Semi-logarithmic paper:

When using semi-log papers the pumping test data fall along straight lines. They benefit the following purposes:

a- Determining the characteristics of the aquifers (T and S) using straight-line methods.

b- Indicating the well-diameter effect.

C -Determining the well loss.

D – Indicating the possibility of application of Darcy law.

2. Logarithmic paper: Log-log paper

When using log-log papers the pumping test data form curves. They benefit the following purposes:

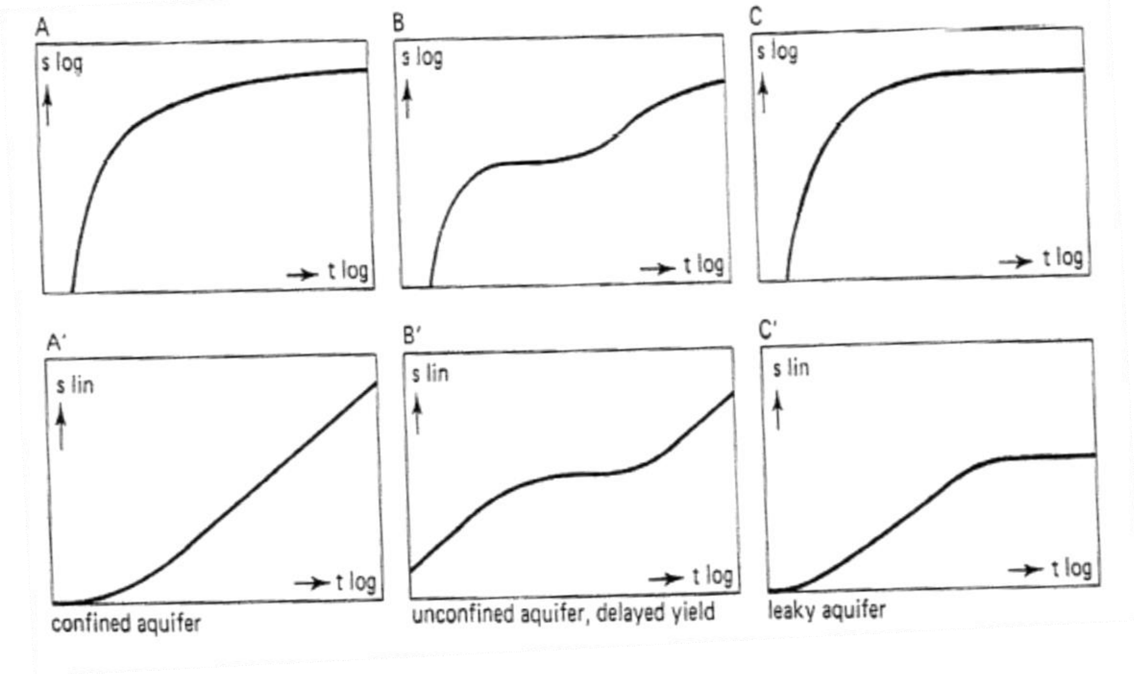
a- Determine the aquifer characteristics (T &S) of the water-bearing layer by curve matching techniques.

b- Estimating the nature of the homogeneity of aquifer.

c- If the initial readings fill on a straight line, the well has a large diameter well.

d- If the final readings fill on a straight line, the aquifer is fractured.

e- Determining the aquifer type from the shape of the curve.



Log-log and semi-log plots of the theoretical time-drawdown relationships of unconsolidated aquifers:
Parts A and A': Confined aquifer Parts B and B': Unconfined aquifer Parts C and C': Leaky aquifer