Jacob Methods

(1) t vs. s
(2) s vs. r
(3) \( \frac{t}{r^2} \) vs. s

1- The first method: indicates the relationship between time and the drawdown (direct relationship), yields values of T & S using the late time drawdown data.

\[ T \text{ & } S \]

2- The second method gives the relationship between the drawdown and the distance (inverse relationship). At least 3 wells must be used and can be used to calculate T & S in addition to radius of influence and well loss.

3- The third method similar to the first and relates the drawdown with time divided by the square of the distance.

Jacob methods were based on Theis’s formula and thus have the same assumptions, plus:

1 - The value of u is very small (u <0.01)
2 – Time is long.

**1- First method of Jacob**

According to Theis’s formula:

\[ W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.21} + \frac{u^3}{3.31} - \frac{u^4}{4.41} + \ldots \]

From \( u = \frac{r^2 S}{4Tt} \), it will be seen that u decreases as the time of pumping increases and the distance from the well r decreases. Accordingly, for drawdown observations made in the near vicinity of the well after a
sufficiently long pumping time, the terms beyond \((\ln u)\) in the series become so small that they can be neglected. So for small values of \(u (u < 0.01)\), the drawdown can be approximated by:

\[
W(u) = -0.5772 - \ln \frac{r^2 S}{4Tt} \\
S = \frac{Q}{4\pi T} W(u) = \frac{Q}{4\pi T} \left[-0.5772 - \ln \frac{r^2 S}{4Tt}\right] \\
S = \frac{Q}{4\pi} \left[\log \frac{4Tt}{r^2 S} - 0.5772\right] \\
\ln \frac{1}{x} = -\ln x \\
S = \frac{2.3Q}{4\pi T} \log \frac{4Tt}{1.783r^2 S} \\
\therefore S = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}
\]

\[\text{Diagram:}\]

\[\text{Relationship between time and drawdown}\]

ٍ\(\text{حيث أن:}\)

\[
(2.3Q / 4\pi T) \neq 0
\]

حيث أن لогارتم الواحد يساوي صفرا فيكون:

\[1 = 2.25T_0 / r^2 S\]

\[S = 2.25T_0 / r^2\]

أي أن
The slope of the straight line, i.e. the drawdown difference \( \Delta s \) as per log cycle of time (\( \log t/t_0 = 1 \)), is equal to \( 2.30Q/4\pi T \).

Similarly, it can be shown that, for a fixed time \( t \), a plot of \( \Delta s \) versus \( r \) on semi-log paper forms a straight line and the following equations can be derived:

\[
T = \frac{2.3Q}{4\pi\Delta s} \quad (1)
\]

\[
S = \frac{2.25 Tt_0}{r^2} \quad (2)
\]
The estimates of T and S from log(time)-drawdown and log(distance)-drawdown plots are independent of one another and so are recommended as a check for consistency in data derived from pump tests.

Ideally 4 or 5 observation wells are needed for the distance-drawdown graph and it is recommended that T and S are computed for several different times.
**Example:** \( t = 0.35 \text{ days and } Q = 1100 \text{ m}^3/\text{d} \)

\[
T = \frac{0.366 \times 1100}{3.8} = 106 \text{ m}^2/\text{d}
\]

\[
S = 2.25 \times 106 \times 0.35 / (126 \times 126) = 5.3 \times 10^{-3}
\]

**Determining the well loss**

Well loss is the difference between the head in the aquifer immediately outside the well to the head inside the casing during pumping.

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**Step-drawdown**

- Determine the test rate \( Q_1 \) by pumping for a specified time at a constant rate.
- Measure the drawdown \( s_1 \) and record the data.
- Increase the rate of pumping to \( Q_2 \) and measure the new drawdown \( s_2 \).
- Repeat the process by increasing the rate of pumping to \( Q_3 \) and measuring the new drawdown \( s_3 \).

The well loss can be calculated as:

\[
L = \frac{(s_2 - s_1)}{s_1} \times 100\%
\]
تكرار الزيادة في معدلات التصرف ثلاثة أو خمسة مرات، وقياس الانخفاض الناشئ عن تلك الزيادات. ولتسهيل الحسابات يتم اخذ القياسات على فترات زمنية ثابتة بين كل زيادة بالصرف.

رسم العلاقة بين معدل التصرف Q والهبوط في مستوى الماء s_w على ورقـة عادية فيلاحظ أن العلاقة على شكل منحنى، حيث أن:

\[ W_{(u)} = \frac{4\pi T \cdot s_w}{Q} \]

فإن

\[ s_w = \frac{W_{(u)} \cdot Q}{4\pi T} \]

إذا فرضنا أن

\[ T = \frac{B \cdot W_{(u)}}{4\pi T} \]

\[ s_w = B \cdot Q \]

بتي

حيث أن B ثابت وسيمـى فقد الطبقة، وهذه العلاقة خطية وتكون فقط في حالة الجريان الخطي.

وفي حالة الجريان العشوائي تصبح العلاقة كما يلي:

\[ s_w = B \cdot Q + CQ^2 \]

حيث C هو فقد البار (Well loss) ويكون BQ هو فقد الطبقة (Aquifer loss) يمثل الجريان العشوائي وهو الفقد في الطبقة الناتج عند عدم كفاءة البئر.

لتحديد قيمة C بقسم المعادلة 2 على Q فتصبح

\[ s_w = \frac{B}{Q} + CQ \]

(3)
Groundwater

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specific capacity is $s_w/Q$ where $s_w$ is the specific yield. The relationship between $Q$ and $s_w/Q$ is linear. The slope of the line is given by $s_w/Q = B$ where $B$ is the intercept. The slope is $C$.
To evaluate well loss a step-drawdown pumping test is required. This consists of pumping a well initially at a low rate until the drawdown within the well essentially stabilize. The discharge is then increased through a successive series of steps as shown by the time-drawdown data in Figure a below. Incremental drawdowns $\Delta s$ for each step are determined from *approximately equal time intervals*. The individual drawdown curves should be extrapolated with a slope proportional to the discharge in order to measure the incremental drawdowns.

From Figure b the well loss coefficient $C$ is given by the slope of the line and the formation loss coefficient $B$ by the intercept $Q=0$. 
\( L_p \) is the ratio of laminar head losses to the total head losses (this parameter can be considered also as well efficiency).

\[ \text{specific capacity (Sc) specific capacity = تعني كمية التصرف لكل وحدة انخفاض.} \]

\[ L_p = \frac{BQ \times 100}{BQ + CQ^2} \]

**Example** For \( Q = 2700 \text{ m}^3/\text{d} \) and \( s = 33.3 \text{ m} \) the \( B = 0.012 \text{ m/m}^3/\text{d} \)

If \( C = 4 \times 10^{-5} \), then \( CQ^2 = 18.2 \text{ m} \)

\[ L_p = \frac{32.4}{(32.4+18.2)} = 65\% \]

**Example**: From a step-drawdown test we have determined the value of \( Sc = 320 \text{ m}^3/\text{d/m} \) of drawdown. And the static water level (SWL) in the borehole lies at \( 5 \text{ m} \) below ground level, and we want at least \( 2 \text{ m} \) of water in the hole above the pump during operation for safety reasons. If the client insists on a yield of \( 2000 \text{ m}^3/\text{d} \). Find the water drawdown in the borehole below ground level.
Therefore, steady drawdown level will be at around \( 5 + 6.25 = 11.25 \text{ m} \) below ground level.

### 3-Third Method of Jacob

If all the drawdown data of all piezometers are used, the values of \( s \) versus \( t/r^2 \) can be plotted on semi-log paper. Subsequently, a straight line can be drawn through the plotted points.

\[
\begin{align*}
  &\text{Example:} \quad \text{The } s = 0 \text{ on the horizontal axis in } (t/r^2)_0 = 2.45 \times 10^{-4} \text{ min/m}^2 \\
  &\quad \text{or } (2.45/1440) \times 10^{-4} \text{ d/m}^2. \quad \text{On the vertical axis, we measure the} \\
  &\quad \text{drawdown difference per log cycle of } t/r^2 \text{ as } \Delta s = 0.33 \text{ m. The discharge} \\
  &\quad \text{rate } Q = 788 \text{ m}^3/\text{d}. \quad \text{Introducing these values into Equation 1 gives:}
\end{align*}
\]

\[
T = \frac{2.3Q}{4\pi\Delta s} = \frac{2.30 \times 788}{4 \times 3.14 \times 0.33} = 437 \text{ m}^2/\text{d}
\]

and into Equation 2:

\[
S = 2.25KD(t/r^2)_0 = 2.25 \times 437 \times \frac{2.45}{1440} \times 10^{-4} = 1.7 \times 10^{-4}
\]