Information theory and coding

Introduction

م. فؤاد حمادي

2020 - 2019

Principles of probability theory

Probability can be defined as the chance of an event occurring. Probability is a way of assigning every "event" a value between "0" and "1"

P **- denotes a probability.**
A*, *B*, and *C **- denote specific events.**
P **(*A*) - denotes the probability of event A occurring.**

Basic Concepts

- **Probability Experiment** is a chance process that leads to well-defined results called outcomes.
- An **outcome** is the result of a single trial of a probability experiment.
- An **event** consists of a set of outcomes of a probability experiment.
- **Sample Space** is the set of all possible outcomes of a probability experiment. Sample Space Consists of all possible simple events.

EXPERIMENT	SAMPLE SPACE
Toss one coin	H, T
Roll a die	1, 2, 3, 4, 5, 6
Answer a true-false question	True, False
Toss two coins	HH, HT, TH, TT

The probability of any event *E* is

$$\frac{\textbf{Number of outcomes in } E}{\textbf{Total number of outcomes in the sample space}}$$

This probability is denoted by

$$P(E) = \frac{n(E)}{n(S)}$$

Example1: A coin is tossed 3 times. Find the following:

1. List the sample space
2. Find the probability of tossing heads exactly twice
3. Find the probability of tossing tails at least two.

Solution:

1. The sample space for tossing the coin 3 times has eight outcomes,

S={HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

N(S)= 8 (n is the number of all possible outcomes in sample space)

2. The event A represents the outcomes of exactly two heads H.

Event (A) = { HHT, HTH, THH}

N(A)= 3

P(A)=n(A)/n(S)= 3/8=0.375

3. The event B represents the outcomes of at least two tails T.

Event (B) = { THT, TTH, HTT, TTT}

N(A)= 4

P(A)=$n(A)/n(S)$= 4/8=0.5

Example2: A box contains 3 red balls, 2 blue balls, and 5 white balls. A ball is selected from the box. Find the probability of selecting red balls.

Solution N(S) = 3(red) + 2(blue) + 5(white)

= 10

P(red)=n(red)/n(S)=3/10=0.33

Example3: If a family has three children, find the following:

1. List the sample space
2. Find the probability of a family having at least 1 girl.
3. Find the probability of a family having exactly two boys.

Solution: Use B for boy and G for girl.

1. S = {BBB, BBG, BGB, GBB, GGG, GGB, GBG, BGG}
2. Event (at least one girl) = { BBG, BGB, GBB, GGG, GGB, GBG, BGG }

N(G)= 7

$$\mathrm{P(G)} = \frac{n(G)}{n(S)} = \frac{7}{8} = 0.875$$

3. Event (two boys) = { BBG, BGB, GBB} N(B)= 3

$$\mathrm{P(B)} = \frac{n(B)}{n(S)} = \frac{3}{8} = 0.375$$

Example4: Two dice are rolled. Find the probability of the sum of two dice is equal to 5.

Solution: The sample space of rolling two dice as shown:

Die 1	Die 2					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

n(S) =36

The event A represents the outcomes of the sum of two dice is equal to 5.

Event(A) ={(1,4),(2,3),(3,2),(4,1)}.

n(A) = 4

$$\mathrm{P(A)} = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$