5 Force and Motion

These ice boats are a memorable example of the connection between force and motion.

▶ **Looking Ahead** The goal of Chapter 5 is to establish a connection between force and motion.

What Causes Motion?

Kinematics describes *how* an object moves. For the more fundamental issue of understanding *why* an object moves, we now turn our attention to **dynamics**.

Dynamics joins with kinematics to form **mechanics**, the science of motion.

◀ **Looking Back**

Section 1.5 Acceleration

Section 3.2 Vector addition

Force

The fundamental concept of dynamics is that of *force*.

- A force is a push or a pull.
- A force acts on an object.
- A force is a vector.
- A force can be a contact force or a long-range force.

Some important forces that we'll study in this chapter are

Gravity

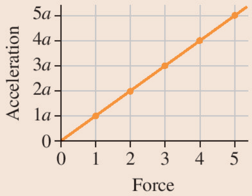
Tension

Friction

Drag

Force and Motion

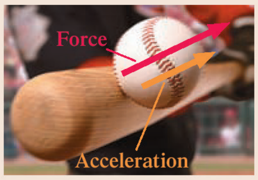
Force causes an object to *accelerate!*



You'll learn that the acceleration of an object is directly proportional to the force exerted on it.

Newton's Laws

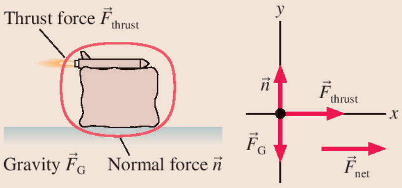
You've likely seen Newton's second law, the famous equation $F = ma$. This is the first of several chapters in which you'll learn to use Newton's *three* laws of motion to solve dynamics problems.



An object accelerates in the same direction as the net force on the object.

Identifying Forces

In this chapter you will learn to *identify* forces and then to represent them on a **free-body diagram.**



Except for the long-range force of gravity, forces act at points of contact.

5.1 Force

The two major issues that this chapter will examine are:

- What is a force?
- What is the connection between force and motion?

We begin with the first of these questions in the table below.

What is a force?

A force is a push or a pull.

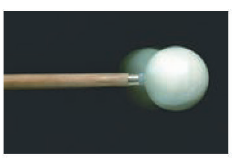
Our commonsense idea of a **force** is that it is a *push* or a *pull.* We will refine this idea as we go along, but it is an adequate starting point. Notice our careful choice of words: We refer to "*a* force," rather than simply "force." We want to think of a force as a very specific *action,* so that we can talk about a single force or perhaps about two or three individual forces that we can clearly distinguish. Hence the concrete idea of "a force" acting on an object.

A force acts on an object.

Implicit in our concept of force is that a **force acts on an object.** In other words, pushes and pulls are applied *to* something—an object. From the object's perspective, it has a force *exerted* on it. Forces do not exist in isolation from the object that experiences them.

A force requires an agent.

Every force has an **agent,** something that acts or exerts power. That is, a force has a specific, identifiable *cause.* As you throw a ball, it is your hand, while in contact with the ball, that is the agent or the cause of the force exerted on the ball. *If* a force is being exerted on an object, you must be able to identify a specific cause (i.e., the agent) of that force. Conversely, a force is not exerted on an object *unless* you can identify a specific cause or agent. Although this idea may seem to be stating the obvious, you will find it to be a powerful tool for avoiding some common misconceptions about what is and is not a force.



A force is a vector.

If you push an object, you can push either gently or very hard. Similarly, you can push either left or right, up or down. To quantify a push, we need to specify both a magnitude *and* a direction. It should thus come as no surprise that a force is a vector quantity. The general symbol for a force is the vector symbol $\vec{F}$. The size or strength of a force is its magnitude F.

A force can be either a contact force . . .

There are two basic classes of forces, depending on whether the agent touches the object or not. **Contact forces** are forces that act on an object by touching it at a point of contact. The bat must touch the ball to hit it. A string must be tied to an object to pull it. The majority of forces that we will examine are contact forces.

. . . or a long-range force.

Long-range forces are forces that act on an object without physical contact. Magnetism is an example of a long-range force. You have undoubtedly held a magnet over a paper clip and seen the paper clip leap up to the magnet. A coffee cup released from your hand is pulled to the earth by the long-range force of gravity.

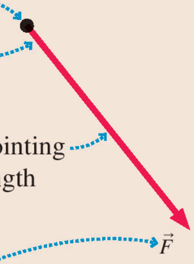
NOTE ▶ In the particle model, objects cannot exert forces on themselves. A force on an object will always have an agent or cause external to the object. Now, there are certainly objects that have internal forces (think of all the forces inside the engine of your car!), but the particle model is not valid if you need to consider those internal forces. If you are going to treat your car as a particle and look only at the overall motion of the car as a whole, that motion will be a consequence of external forces acting on the car. ◀

Force Vectors

We can use a simple diagram to visualize how forces are exerted on objects.

TACTICS BOX 5.1 Drawing force vectors

1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.



Step 2 may seem contrary to what a "push" should do, but recall that moving a vector does not change it as long as the length and angle do not change. The vector $\vec{F}$ is the same regardless of whether the tail or the tip is placed on the particle. FIGURE 5.1 shows three examples of force vectors.

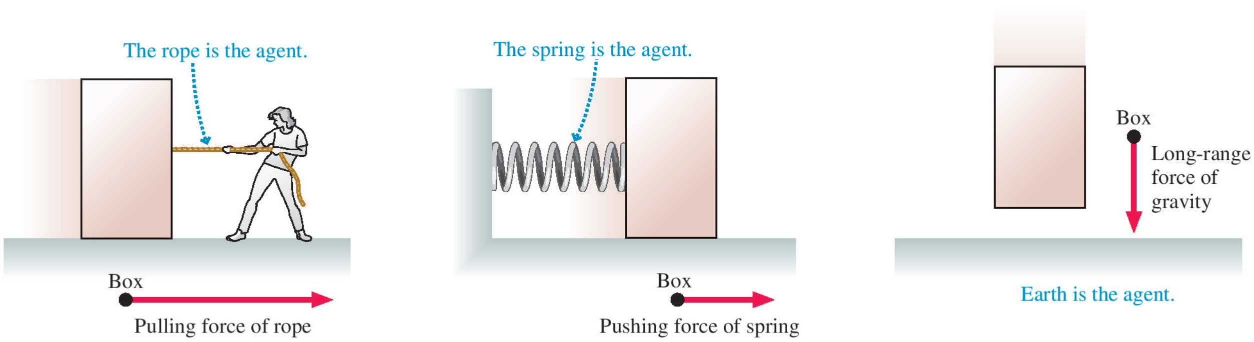


FIGURE 5.1 Three examples of forces and their vector representations.

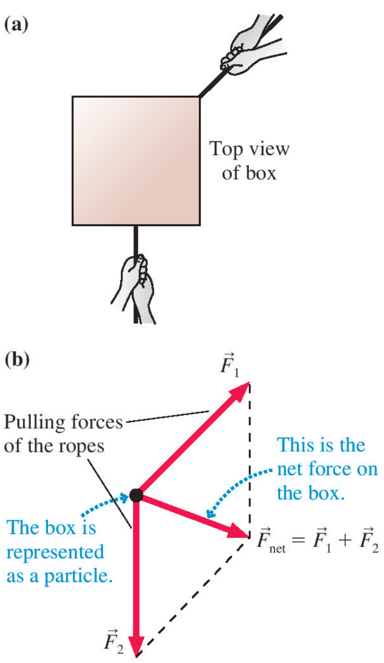
Combining Forces

FIGURE 5.2a shows a box being pulled by two ropes, each exerting a force on the box. How will the box respond? Experimentally, we find that when several forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, . . . are exerted on an object, they combine to form a **net force** given by the *vector* sum of *all* the forces:

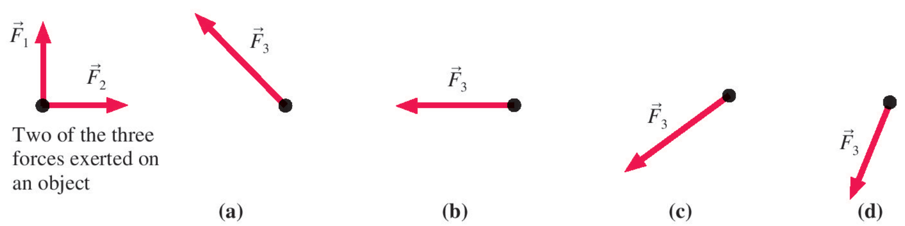
$$\vec{F}_{net} \equiv \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N \qquad (5.1)$$

Recall that $\equiv$ is the symbol meaning "is defined as." Mathematically, this summation is called a **superposition of forces.** FIGURE 5.2b shows the net force on the box.

FIGURE 5.2 Two forces applied to a box.



STOP TO THINK 5.1 Two of the three forces exerted on an object are shown. The net force points to the left. Which is the missing third force?



5.2 A Short Catalog of Forces

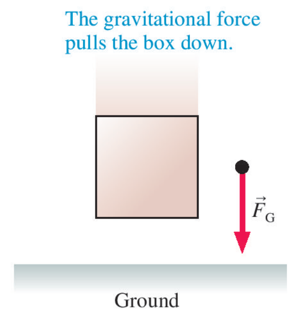
There are many forces we will deal with over and over. This section will introduce you to some of them. Many of these forces have special symbols. As you learn the major forces, be sure to learn the symbol for each.

Gravity

Gravity—the only long-range force we will encounter in the next few chapters—keeps you in your chair, and the planets in their orbits around the sun. We'll have a thorough look at gravity in Chapter 13. For now we'll concentrate on objects on or near the surface of the earth (or other planet).

The pull of a planet on an object on or near the surface is called the **gravitational force.** The agent for the gravitational force is the *entire planet*. Gravity acts on *all* objects, whether moving or at rest. The symbol for gravitational force is $\vec{F}_G$. The gravitational force vector always points vertically downward, as shown in FIGURE 5.3.

FIGURE 5.3 Gravity.

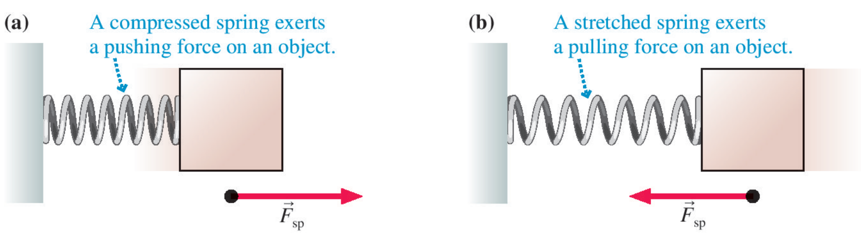


NOTE ▶ We often refer to "the weight" of an object. For an object at rest on the surface of a planet, its weight is simply the magnitude F_G of the gravitational force. However, weight and gravitational force are not the same thing, nor is weight the same as mass. We will briefly examine mass later in the chapter, and we'll explore the rather subtle connections among gravity, weight, and mass in Chapter 6. ◀

Spring Force

Springs exert one of the most common contact forces. A spring can either push (when compressed) or pull (when stretched). FIGURE 5.4 shows the **spring force,** for which we use the symbol $\vec{F}_{sp}$. In both cases, pushing and pulling, the tail of the force vector is placed on the particle in the force diagram.

FIGURE 5.4 The spring force.

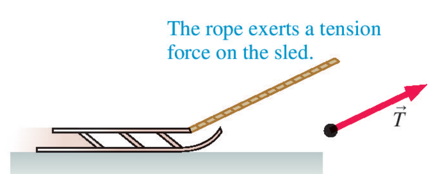


Although you may think of a spring as a metal coil that can be stretched or compressed, this is only one type of spring. Hold a ruler, or any other thin piece of wood or metal, by the ends and bend it slightly. It flexes. When you let go, it "springs" back to its original shape. This is just as much a spring as is a metal coil.

Tension Force

When a string or rope or wire pulls on an object, it exerts a contact force that we call the **tension force,** represented by a capital $\vec{T}$. The direction of the tension force is always in the direction of the string or rope, as you can see in FIGURE 5.5. The commonplace reference to "the tension" in a string is an informal expression for T, the size or magnitude of the tension force.

FIGURE 5.5 Tension.



NOTE ▶ Tension is represented by the symbol T. This is logical, but there's a risk of confusing the tension T with the identical symbol T for the period of a particle in circular motion. The number of symbols used in science and engineering is so large that some letters are used several times to represent different quantities. The use of T is the first time we've run into this problem, but it won't be the last. You must be alert to the *context* of a symbol's use to deduce its meaning. ◀

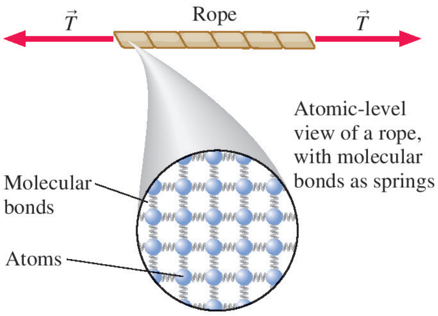


FIGURE 5.6 An atomic model of tension.

If you were to use a very powerful microscope to look inside a rope, you would "see" that it is made of *atoms* joined together by *molecular bonds.* Molecular bonds are not rigid connections between the atoms. They are more accurately thought of as tiny *springs* holding the atoms together, as in **FIGURE 5.6**. Pulling on the ends of a string or rope stretches the molecular springs ever so slightly. The tension within a rope and the tension force experienced by an object at the end of the rope are really the net spring force being exerted by billions and billions of microscopic springs.

This atomic-level view of tension introduces a new idea: a microscopic **atomic model** for understanding the behavior and properties of macroscopic objects. It is a *model* because atoms and molecular bonds aren't really little balls and springs. We're using macroscopic concepts—balls and springs—to understand atomic-scale phenomena that we cannot directly see or sense. This is a good model for explaining the elastic properties of materials, but it would not necessarily be a good model for explaining other phenomena. We will frequently use atomic models to obtain a deeper understanding of our observations.

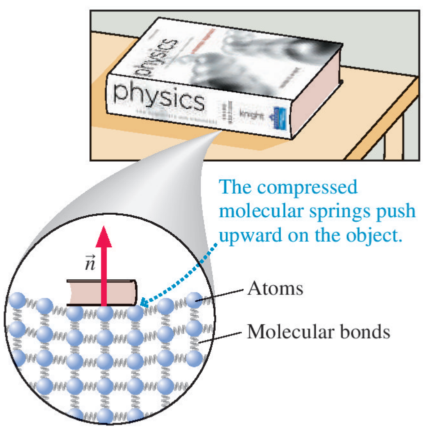


FIGURE 5.7 An atomic model of the force exerted by a table.

Normal Force

If you sit on a bed, the springs in the mattress compress and, as a consequence of the compression, exert an upward force on you. Stiffer springs would show less compression but still exert an upward force. The compression of extremely stiff springs might be measurable only by sensitive instruments. Nonetheless, the springs would compress ever so slightly and exert an upward spring force on you.

FIGURE 5.7 shows an object resting on top of a sturdy table. The table may not visibly flex or sag, but—just as you do to the bed—the object compresses the molecular springs in the table. The size of the compression is very small, but it is not zero. As a consequence, the compressed molecular springs *push upward* on the object. We say that "the table" exerts the upward force, but it is important to understand that the pushing is *really* done by molecular springs. Similarly, an object resting on the ground compresses the molecular springs holding the ground together and, as a consequence, the ground pushes up on the object.

We can extend this idea. Suppose you place your hand on a wall and lean against it, as shown in **FIGURE 5.8**. Does the wall exert a force on your hand? As you lean, you compress the molecular springs in the wall and, as a consequence, they push outward against your hand. So the answer is yes, the wall does exert a force on you.

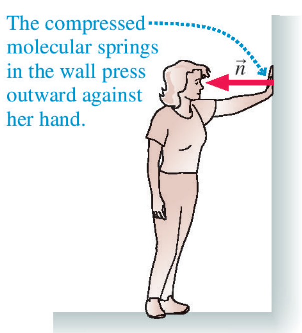


FIGURE 5.8 The wall pushes outward.

The force the table surface exerts is vertical; the force the wall exerts is horizontal. In all cases, the force exerted on an object that is pressing against a surface is in a direction *perpendicular* to the surface. Mathematicians refer to a line that is perpendicular to a surface as being *normal* to the surface. In keeping with this terminology, we define the **normal force** as the force exerted by a surface (the agent) against an object that is pressing against the surface. The symbol for the normal force is $\vec{n}$.

We're not using the word *normal* to imply that the force is an "ordinary" force or to distinguish it from an "abnormal force." A surface exerts a force *perpendicular* (i.e., normal) to itself as the molecular springs press *outward.* **FIGURE 5.9** shows an object on an inclined surface, a common situation.

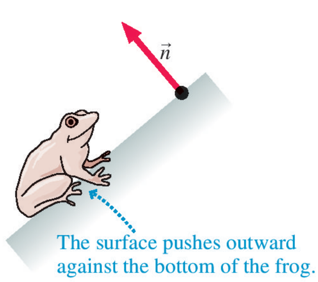


FIGURE 5.9 The normal force.

In essence, the normal force is just a spring force, but one exerted by a vast number of microscopic springs acting at once. The normal force is responsible for the "solidness" of solids. It is what prevents you from passing right through the chair you are sitting in and what causes the pain and the lump if you bang your head into a door.

Friction

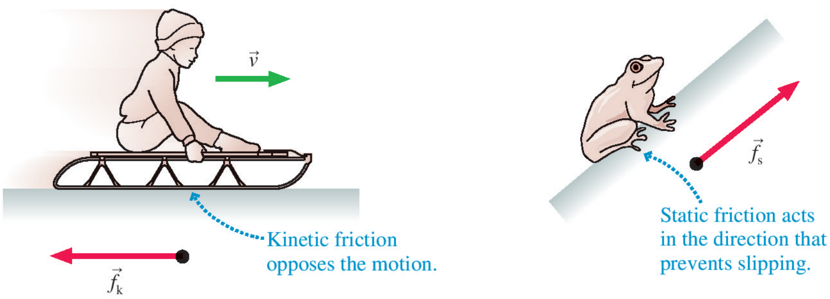
Friction, like the normal force, is exerted by a surface. But whereas the normal force is perpendicular to the surface, the friction force is always *tangent* to the surface. It is useful to distinguish between two kinds of friction:

- *Kinetic friction,* denoted $\vec{f}_k$, appears as an object slides across a surface. This is a force that "opposes the motion," meaning that the friction force vector $\vec{f}_k$ points in a direction opposite the velocity vector $\vec{v}$ (i.e., "the motion").

- *Static friction,* denoted $\vec{f}_s$, is the force that keeps an object "stuck" on a surface and prevents its motion. Finding the direction of $\vec{f}_s$ is a little trickier than finding it for $\vec{f}_k$. Static friction points opposite the direction in which the object *would* move if there were no friction. That is, it points in the direction necessary to *prevent* motion.

FIGURE 5.10 shows examples of kinetic and static friction.

FIGURE 5.10 Kinetic and static friction.



NOTE ▶ A surface exerts a kinetic friction force when an object moves *relative to* the surface. A package on a conveyor belt is in motion, but it does not experience a kinetic friction force because it is not moving relative to the belt. So to be precise, we should say that the kinetic friction force points opposite to an object's motion *relative to* a surface. ◀

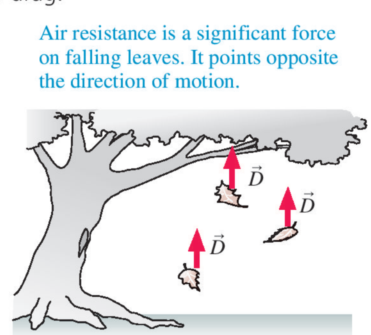
Drag

Friction at a surface is one example of a *resistive force,* a force that opposes or resists motion. Resistive forces are also experienced by objects moving through fluids—gases and liquids. The resistive force of a fluid is called **drag,** with symbol $\vec{D}$. Drag, like kinetic friction, points opposite the direction of motion. **FIGURE 5.11** shows an example.

Drag can be a significant force for objects moving at high speeds or in dense fluids. Hold your arm out the window as you ride in a car and feel how the air resistance against it increases rapidly as the car's speed increases. Drop a lightweight object into a beaker of water and watch how slowly it settles to the bottom.

For objects that are heavy and compact, that move in air, and whose speed is not too great, the drag force of air resistance is fairly small. To keep things as simple as possible, **you can neglect air resistance in all problems unless a problem explicitly asks you to include it.**

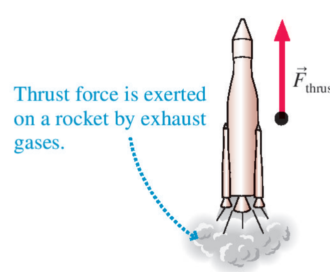
FIGURE 5.11 Air resistance is an example of drag.



Thrust

A jet airplane obviously has a force that propels it forward during takeoff. Likewise for the rocket being launched in **FIGURE 5.12**. This force, called **thrust,** occurs when a jet or rocket engine expels gas molecules at high speed. Thrust is a contact force, with the exhaust gas being the agent that pushes on the engine. The process by which thrust is generated is rather subtle, and we will postpone a full discussion until we study Newton's third law in Chapter 7. For now, we will treat thrust as a force opposite the direction in which the exhaust gas is expelled. There's no special symbol for thrust, so we will call it $\vec{F}_{\text{thrust}}$.

FIGURE 5.12 Thrust force on a rocket.



Electric and Magnetic Forces

Electricity and magnetism, like gravity, exert long-range forces. We will study electric and magnetic forces in detail in Part VI. For now, it is worth noting that the forces holding molecules together—the molecular bonds—are not actually tiny springs. Atoms and molecules are made of charged particles—electrons and protons—and what we call a molecular bond is really an electric force between these particles. So when we say that the normal force and the tension force are due to "molecular springs," or that friction is due to atoms running into each other, what we're really saying is that these forces, at the most fundamental level, are actually electric forces between the charged particles in the atoms.

5.3 Identifying Forces

Force	Notation
General force	$\vec{F}$
Gravitational force	$\vec{F}_G$
Spring force	$\vec{F}_{sp}$
Tension	$\vec{T}$
Normal force	$\vec{n}$
Static friction	$\vec{f}_s$
Kinetic friction	$\vec{f}_k$
Drag	$\vec{D}$
Thrust	$\vec{F}_{thrust}$

Force and motion problems generally have two basic steps:

1. Identify all of the forces acting on an object.
2. Use Newton's laws and kinematics to determine the motion.

Understanding the first step is the primary goal of this chapter. We'll turn our attention to step 2 in the next chapter.

A typical physics problem describes an object that is being pushed and pulled in various directions. Some forces are given explicitly; others are only implied. In order to proceed, it is necessary to determine all the forces that act on the object. The procedure for identifying forces will become part of the *pictorial representation* of the problem.

TACTICS BOX 5.2 Identifying forces

MP

1. **Identify the object of interest.** This is the object whose motion you wish to study.
2. **Draw a picture of the situation.** Show the object of interest and all other objects—such as ropes, springs, or surfaces—that touch it.
3. **Draw a closed curve around the object.** Only the object of interest is inside the curve; everything else is outside.
4. **Locate every point on the boundary of this curve where other objects touch the object of interest.** These are the points where *contact forces* are exerted on the object.
5. **Name and label each contact force acting on the object.** There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.
6. **Name and label each long-range force acting on the object.** For now, the only long-range force is the gravitational force.

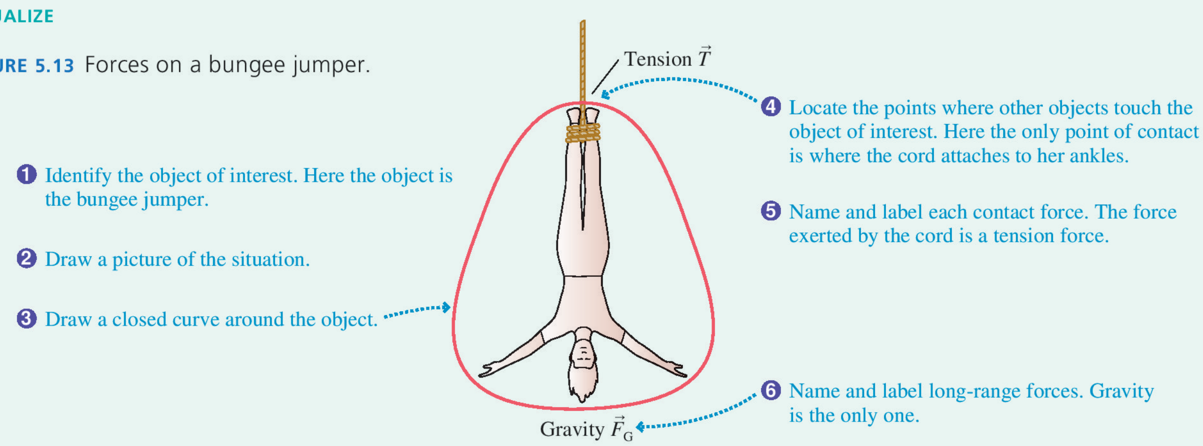
Exercises 3–8

EXAMPLE 5.1 Forces on a bungee jumper

A bungee jumper has leapt off a bridge and is nearing the bottom of her fall. What forces are being exerted on the jumper?

VISUALIZE

FIGURE 5.13 Forces on a bungee jumper.

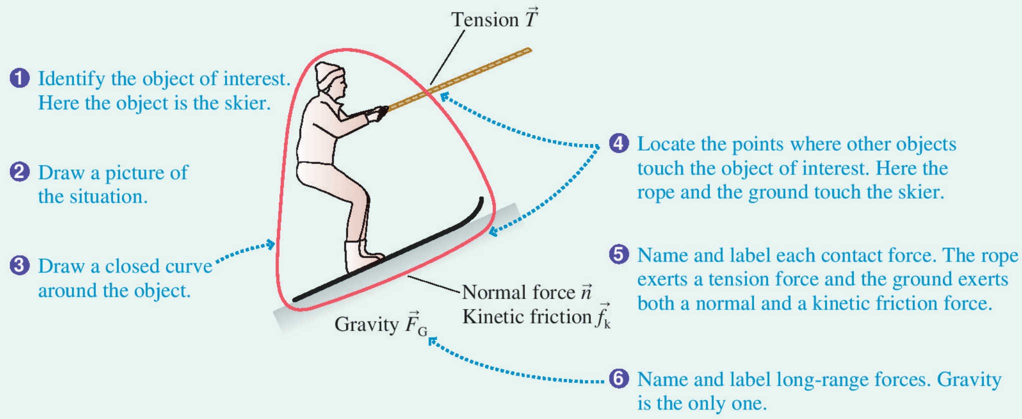


EXAMPLE 5.2 Forces on a skier

A skier is being towed up a snow-covered hill by a tow rope. What forces are being exerted on the skier?

VISUALIZE

FIGURE 5.14 Forces on a skier.



NOTE ▶ You might have expected two friction forces and two normal forces in Example 5.2, one on each ski. Keep in mind, however, that we're working within the particle model, which represents the skier by a single point. A particle has only one contact with the ground, so there is one normal force and one friction force. ◀

EXAMPLE 5.3 Forces on a rocket

A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

VISUALIZE This drawing is much more like the sketch you would make when identifying forces as part of solving a problem.

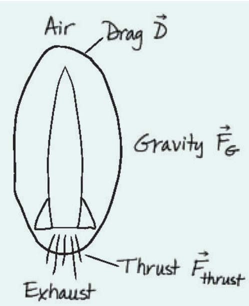


FIGURE 5.15 Forces on a rocket.

STOP TO THINK 5.2 You've just kicked a rock, and it is now sliding across the ground about 2 meters in front of you. Which of these forces act on the rock? List all that apply.

a. Gravity, acting downward.
b. The normal force, acting upward.
c. The force of the kick, acting in the direction of motion.
d. Friction, acting opposite the direction of motion.

5.4 What Do Forces Do? A Virtual Experiment

Having learned to identify forces, we ask the next question: How does an object move when a force is exerted on it? The only way to answer this question is to do experiments. Let's conduct a "virtual experiment," one you can easily visualize. Imagine using your fingers to stretch a rubber band to a certain length—say

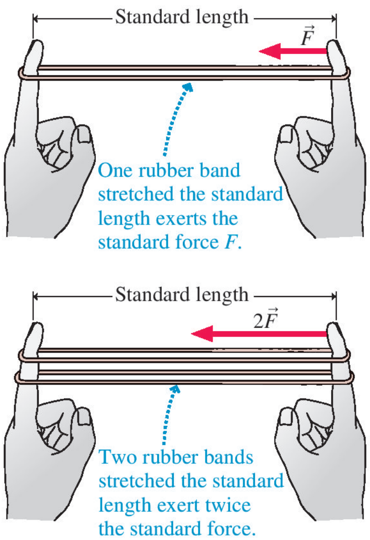


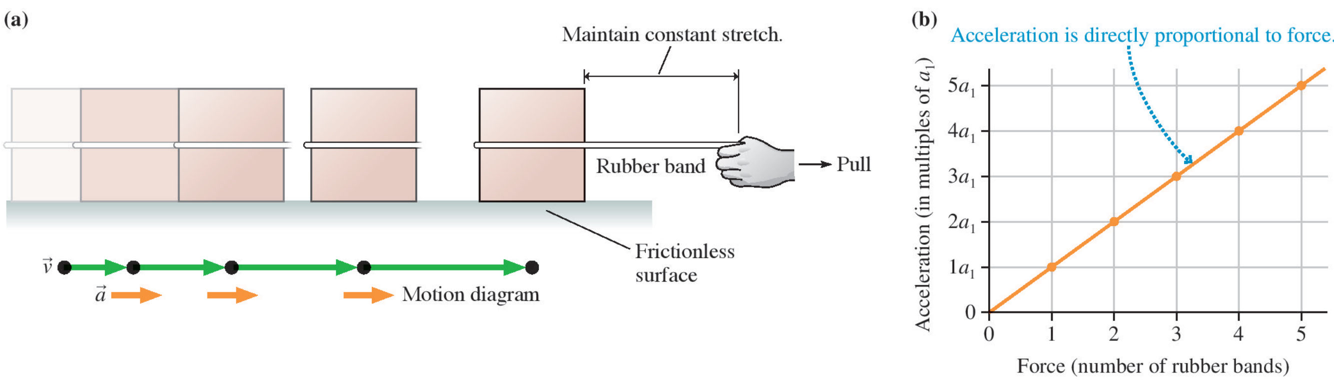
FIGURE 5.16 A reproducible force.

10 centimeters—that you can measure with a ruler, as shown in **FIGURE 5.16**. You know that a stretched rubber band exerts a force—a spring force—because your fingers *feel* the pull. Furthermore, this is a reproducible force; the rubber band exerts the same force every time you stretch it to this length. We'll call this the *standard force F*. Not surprisingly, two identical rubber bands exert twice the pull of one rubber band, and N side-by-side rubber bands exert N times the standard force: $F_{net} = NF$.

Now attach one rubber band to a 1 kg block and stretch it to the standard length. The object experiences the same force F as did your finger. The rubber band gives us a way of applying a known and reproducible force to an object. Then imagine using the rubber band to pull the block across a horizontal, frictionless table. (We can imagine a frictionless table since this is a virtual experiment, but in practice you could nearly eliminate friction by supporting the object on a cushion of air.)

If you stretch the rubber band and then release the object, the object moves toward your hand. But as it does so, the rubber band gets shorter and the pulling force decreases. To keep the pulling force constant, you must *move your hand* at just the right speed to keep the length of the rubber band from changing! **FIGURE 5.17a** shows the experiment being carried out. Once the motion is complete, you can use motion diagrams and kinematics to analyze the object's motion.

FIGURE 5.17 Measuring the motion of an object that is pulled with a constant force.



The first important finding of this experiment is that **an object pulled with a constant force moves with a constant acceleration.** That is, the answer to the question What does a force do? is: A force causes an object to accelerate, and a constant force produces a constant acceleration. This finding could not have been anticipated in advance. It's conceivable that the object would speed up for a while, then move with a steady speed. Or that it would speed up, but that the *rate* of increase, the acceleration, would steadily decline. These are conceivable motions, but they're not what happens. Instead, the object accelerates *with a constant acceleration* a_1 for as long as you pull it with a constant force F.

What happens if you increase the force by using several rubber bands? To find out, use two rubber bands, then three rubber bands, then four, and so on. With N rubber bands, the force on the block is NF. **FIGURE 5.17b** shows the results of this experiment. You can see that doubling the force causes twice the acceleration, tripling the force causes three times the acceleration, and so on. The graph reveals our second important finding: **The acceleration is directly proportional to the force.** This result can be written as

$$a = cF \tag{5.2}$$

where c, called the *proportionality constant,* is the slope of the graph.

MATHEMATICAL ASIDE Proportionality and proportional reasoning

The concept of **proportionality** arises frequently in physics. A quantity symbolized by u is *proportional* to another quantity symbolized by v if

$$u = cv$$

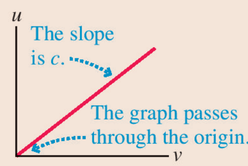
where c (which might have units) is called the **proportionality constant.** This relationship between u and v is often written

$$u \propto v$$

where the symbol $\propto$ means "is proportional to."

If v is doubled to $2v$, then u is doubled to $c(2v) = 2(cv) = 2u$. In general, if v is changed by any factor f, then u changes by the same factor. This is the essence of what we *mean* by proportionality.

A graph of u versus v is a straight line *passing through the origin* (i.e., the y-intercept is zero) with slope $= c$. Notice that proportionality is a much more specific relationship between u and v than mere linearity. The linear equation $u = cv + b$ has a straight-line graph, but it doesn't pass through the origin (unless b happens to be zero) and doubling v does not double u.



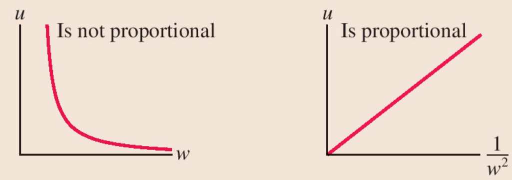
u is proportional to v.

If $u \propto v$, then $u_1 = cv_1$ and $u_2 = cv_2$. Dividing the second equation by the first, we find

$$\frac{u_2}{u_1} = \frac{v_2}{v_1}$$

By working with *ratios,* we can deduce information about u without needing to know the value of c. (This would not be true if the relationship were merely linear.) This is called **proportional reasoning.**

Proportionality is not limited to being linearly proportional. The graph on the left below shows that u is clearly not proportional to w. But a graph of u versus $1/w^2$ *is* a straight line passing through the origin, thus, in this case, u is proportional to $1/w^2$, or $u \propto 1/w^2$. We would say that "u is proportional to the inverse square of w."



u is proportional to the inverse square of w.

EXAMPLE u is proportional to the inverse square of w. By what factor does u change if w is tripled?

SOLUTION This is an opportunity for proportional reasoning; we don't need to know the proportionality constant. If u is proportional to $1/w^2$, then

$$\frac{u_2}{u_1} = \frac{1/w_2^2}{1/w_1^2} = \frac{w_1^2}{w_2^2} = \left(\frac{w_1}{w_2}\right)^2$$

Tripling w, with $w_2/w_1 = 3$, and thus $w_1/w_2 = \frac{1}{3}$, changes u to

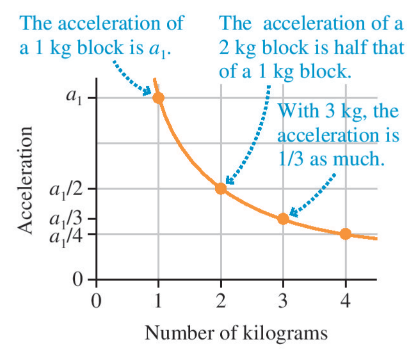
$$u_2 = \left(\frac{w_1}{w_2}\right)^2 u_1 = \left(\frac{1}{3}\right)^2 u_1 = \frac{1}{9} u_1$$

Tripling w causes u to become $\frac{1}{9}$ of its original value.

Many *Student Workbook* and end-of-chapter homework questions will require proportional reasoning. It's an important skill to learn.

The final question for our virtual experiment is: How does the acceleration depend on the mass of the object being pulled? To find out, apply the *same force*—for example, the standard force of one rubber band—to a 2 kg block, then a 3 kg block, and so on, and for each measure the acceleration. Doing so gives you the results shown in **FIGURE 5.18**. An object with twice the mass of the original block has only half the acceleration when both are subjected to the same force.

FIGURE 5.18 Acceleration is inversely proportional to mass.



Mathematically, the graph of Figure 5.18 is one of *inverse proportionality*. That is, **the acceleration is inversely proportional to the object's mass,** which we can write as

$$a = \frac{c'}{m} \tag{5.3}$$

where c' is another proportionality constant.

Force causes an object to *accelerate!* The results of our experiment are that the acceleration is directly proportional to the force applied and inversely proportional to the object's mass. We can combine these into the single statement

$$a = \frac{F}{m} \tag{5.4}$$

if we define the basic unit of force as the force that causes a 1 kg mass to accelerate at 1 m/s^2. That is,

$$1 \text{ basic unit of force} \equiv 1 \text{ kg} \times 1 \frac{\text{m}}{\text{s}^2} = 1 \frac{\text{kg m}}{\text{s}^2}$$

TABLE 5.1 Approximate magnitude of some typical forces

Force	Approximate magnitude (newtons)
Weight of a U.S. quarter	0.05
Weight of a 1 pound object	5
Weight of a 110 pound person	500
Propulsion force of a car	5,000
Thrust force of a rocket motor	5,000,000

This basic unit of force is called a newton:

One **newton** is the force that causes a 1 kg mass to accelerate at 1 m/s^2. The abbreviation for newton is N. Mathematically, $1\ N = 1\ kg\,m/s^2$.

Table 5.1 lists some typical forces. As you can see, "typical" forces on "typical" objects are likely to be in the range 0.01–10,000 N.

Mass

We've been using the term *mass* without a clear definition. As we learned in Chapter 1, the SI unit of mass, the kilogram, is based on a particular metal block kept in a vault in Paris. This suggests that *mass* is the amount of matter an object contains, and that is certainly our everyday concept of mass. Now we see that a more precise way of defining an object's mass is in terms of its acceleration in response to a force. Figure 5.18 shows that an object with twice the amount of matter accelerates only half as much in response to the same force. The more matter an object has, the more it *resists* accelerating in response to a force. You're familiar with this idea: Your car is much harder to push than your bicycle. The tendency of an object to resist a *change* in its velocity (i.e., to resist acceleration) is called **inertia.** Consequently, the mass used in Equation 5.4, a measure of an object's resistance to changing its motion, is called **inertial mass.** We'll meet a different concept of mass, *gravitational mass,* when we study Newton's law of gravity in Chapter 13.

STOP TO THINK 5.3 Two rubber bands stretched to the standard length cause an object to accelerate at 2 m/s^2. Suppose another object with twice the mass is pulled by four rubber bands stretched to the standard length. The acceleration of this second object is

a. 1 m/s^2 b. 2 m/s^2 c. 4 m/s^2 d. 8 m/s^2 e. 16 m/s^2

Hint: Use proportional reasoning.

5.5 Newton's Second Law

Equation 5.4 is an important finding, but our experiment was limited to looking at an object's response to a single applied force. Realistically, an object is likely to be subjected to several distinct forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ that may point in different directions. What happens then? In that case, it is found experimentally that the acceleration is determined by the *net* force.

Newton was the first to recognize the connection between force and motion. This relationship is known today as Newton's second law.

Newton's second law An object of mass m subjected to forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ will undergo an acceleration $\vec{a}$ given by

$$\vec{a} = \frac{\vec{F}_{net}}{m} \tag{5.5}$$

where the net force $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$ is the vector sum of all forces acting on the object. The acceleration vector $\vec{a}$ points in the same direction as the net force vector $\vec{F}_{net}$.

The significance of Newton's second law cannot be overstated. There was no reason to suspect that there should be any simple relationship between force and acceleration. Yet there it is, a simple but exceedingly powerful equation relating the two. The critical idea is that **an object accelerates in the direction of the net force vector $\vec{F}_{net}$.**

We can rewrite Newton's second law in the form

$$\vec{F}_{net} = m\vec{a} \qquad (5.6)$$

which is how you'll see it presented in many textbooks. Equations 5.5 and 5.6 are mathematically equivalent, but Equation 5.5 better describes the central idea of Newtonian mechanics: A force applied to an object causes the object to accelerate.

It's also worth noting that **the object responds only to the forces acting on it *at this instant*.** The object has no memory of forces that may have been exerted at earlier times. This idea is sometimes called **Newton's zeroth law.**

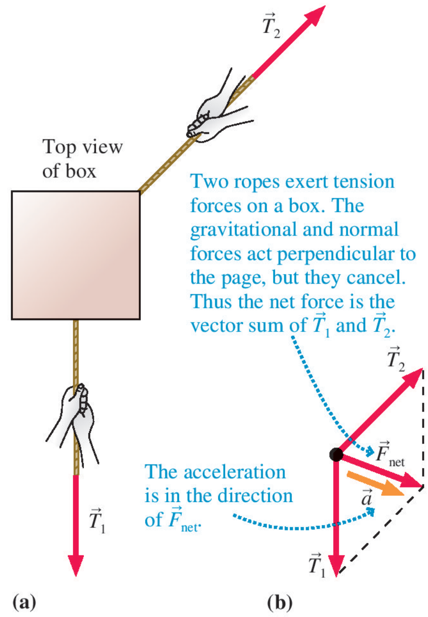
NOTE ▶ Be careful not to think that one force "overcomes" the others to determine the motion. Forces are not in competition with each other! It is $\vec{F}_{net}$, the sum of *all* the forces, that determines the acceleration $\vec{a}$. ◀

As an example, **FIGURE 5.19a** shows a box being pulled by two ropes. The ropes exert tension forces $\vec{T}_1$ and $\vec{T}_2$ on the box. **FIGURE 5.19b** represents the box as a particle, shows the forces acting on the box, and adds them graphically to find the net force $\vec{F}_{net}$. The box will accelerate in the direction of $\vec{F}_{net}$ with acceleration

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{\vec{T}_1 + \vec{T}_2}{m}$$

NOTE ▶ The acceleration is *not* $(T_1 + T_2)/m$. You must add the forces as *vectors,* not merely add their magnitudes as scalars. ◀

FIGURE 5.19 Acceleration of a pulled box.

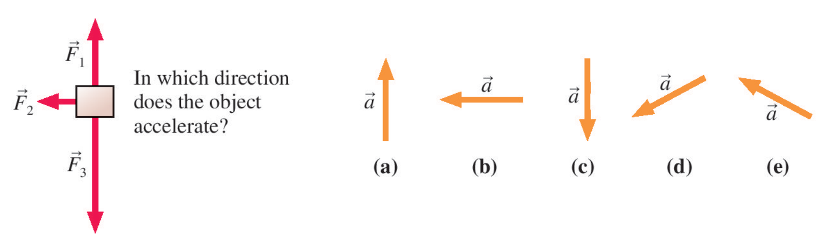


Forces Are Interactions

There's one more important aspect of forces. If you push against a door (the object) to close it, the door pushes back against your hand (the agent). If a tow rope pulls on a car (the object), the car pulls back on the rope (the agent). In general, if an agent exerts a force on an object, the object exerts a force on the agent. We really need to think of a force as an *interaction* between two objects. This idea is captured in Newton's third law—that for every action there is an equal but opposite reaction.

Although the interaction perspective is a more exact way to view forces, it adds complications that we would like to avoid for now. Our approach will be to start by focusing on how a single object responds to forces exerted on it. Then, in Chapter 7, we'll return to Newton's third law and the larger issue of how two or more objects interact with each other.

STOP TO THINK 5.4 Three forces act on an object. In which direction does the object accelerate?



5.6 Newton's First Law

Aristotle and his contemporaries in the world of ancient Greece were very interested in motion. One question they asked was: What is the "natural state" of an object if left to itself? It is easy to see that every moving object on earth, if left to itself, eventually comes to rest. Aristotle concluded that the natural state of an earthly object is to be at rest. An object at rest requires no explanation. A moving object, though, is not in its natural state and thus requires an explanation: Why is this object moving? What keeps it going and prevents it from being in its natural state?

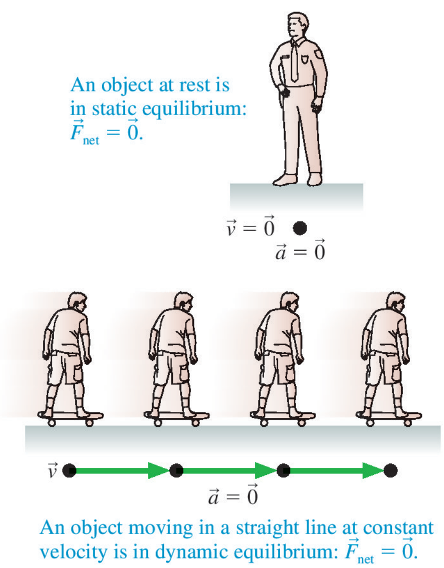
Galileo reopened the question of the "natural state" of objects. He suggested focusing on the *limiting case* in which resistance to the motion (e.g., friction or air resistance) is zero. Many careful experiments in which he minimized the influence of friction led Galileo to a conclusion that was in sharp contrast to Aristotle's belief that rest is an object's natural state.

Galileo found that an external influence (i.e., a force) is needed to make an object accelerate—to *change* its velocity. In particular, a force is needed to put an object in motion. In the absence of friction or air resistance, a moving object would continue to move along a straight line forever with no loss of speed. In other words, the natural state of an object—its behavior if free of external influences—is *uniform motion* with constant velocity! This does not happen in practice because friction or air resistance prevents the object from being left alone. "At rest" has no special significance in Galileo's view of motion; it is simply uniform motion that happens to have $\vec{v} = \vec{0}$.

It was left to Newton to generalize this result, and today we call it Newton's first law of motion.

> **Newton's first law** An object that is at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force acting on the object is zero.

FIGURE 5.20 Two examples of mechanical equilibrium.



Newton's first law is also known as the *law of inertia.* If an object is at rest, it has a tendency to stay at rest. If it is moving, it has a tendency to continue moving with the *same velocity.*

NOTE ▶ The first law refers to *net* force. An object can remain at rest, or can move in a straight line with constant velocity, even though forces are exerted on it as long as the *net* force is zero. ◀

Notice the "if and only if" aspect of Newton's first law. If an object is at rest or moves with constant velocity, then we can conclude that there is no net force acting on it. Conversely, if no net force is acting on it, we can conclude that the object will have constant velocity, not just constant speed. The direction remains constant, too!

An object on which the net force is zero, $\vec{F}_{net} = \vec{0}$, is said to be in **mechanical equilibrium.** There are two distinct forms of mechanical equilibrium:

1. The object is at rest. This is **static equilibrium.**
2. The object is moving in a straight line with constant velocity. This is **dynamic equilibrium.**

Two examples of mechanical equilibrium are shown in **FIGURE 5.20**. Both share the common feature that the acceleration is zero: $\vec{a} = \vec{0}$.

What Good Is Newton's First Law?

The first law completes our definition of force. It answers the question: What is a force? If an "influence" on an object disturbs a state of equilibrium by causing the object's velocity to change, the influence is a force.

Newton's first law changes the question the ancient Greeks were trying to answer: What causes an object to move? Newton's first law says **no cause is needed for an object to move!** Uniform motion is the object's natural state. Nothing at all is required for it to remain in that state. The proper question, according to Newton, is: What causes an object to *change* its velocity? Newton, with Galileo's help, also gave us the answer. **A *force* is what causes an object to change its velocity.**

The preceding paragraph contains the essence of Newtonian mechanics. This new perspective on motion, however, is often contrary to our common experience. We all know perfectly well that you must keep pushing an object—exerting a force on it—to keep it moving. Newton is asking us to change our point of view and to consider motion *from the object's perspective* rather than from our personal perspective. As far as the object is concerned, our push is just one of several forces acting on it. Others might include friction, air resistance, or gravity. Only by knowing the *net* force can we determine the object's motion.

Newton's first law may seem to be merely a special case of Newton's second law. After all, the equation $\vec{F}_{net} = m\vec{a}$ tells us that an object moving with constant velocity ($\vec{a} = \vec{0}$) has $\vec{F}_{net} = \vec{0}$. The difficulty is that the second law assumes that we already know what force is. The purpose of the first law is to *identify* a force as something that disturbs a state of equilibrium. The second law then describes how the object responds to this force. Thus from a *logical* perspective, the first law really is a separate statement that must precede the second law. But this is a rather formal distinction. From a pedagogical perspective it is better—as we have done—to use a commonsense understanding of force and start with Newton's second law.

Inertial Reference Frames

If a car stops suddenly, you may be "thrown" into the windshield if you're not wearing your seat belt. You have a very real forward acceleration *relative to the car,* but is there a force pushing you forward? A force is a push or a pull caused by an identifiable agent in contact with the object. Although you *seem* to be pushed forward, there's no agent to do the pushing.

This guy thinks there's a force hurling him into the windshield. What a dummy!

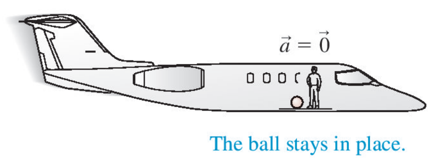
The difficulty—an acceleration without an apparent force—comes from using an inappropriate reference frame. Your acceleration measured in a reference frame attached to the car is not the same as your acceleration measured in a reference frame attached to the ground. Newton's second law says $\vec{F}_{net} = m\vec{a}$. But which $\vec{a}$? Measured in which reference frame?

We define an **inertial reference frame** as a reference frame in which Newton's laws are valid. The first law provides a convenient way to test whether a reference frame is inertial. If $\vec{a} = \vec{0}$ (an object is at rest or moving with constant velocity) only when $\vec{F}_{net} = \vec{0}$, then the reference frame in which $\vec{a}$ is measured is an inertial reference frame.

Not all reference frames are inertial reference frames. **FIGURE 5.21a** shows a physics student cruising at constant velocity in an airplane. If the student places a ball on the floor, it stays there. There are no horizontal forces, and the ball remains at rest relative to the airplane. That is, $\vec{a} = \vec{0}$ in the airplane's reference frame when $\vec{F}_{net} = \vec{0}$. Newton's first law is satisfied, so this airplane is an inertial reference frame.

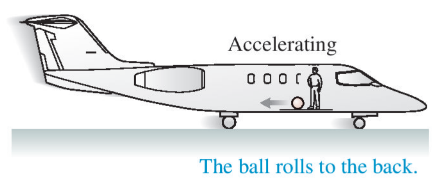
FIGURE 5.21 Reference frames.

(a)



A ball with no horizontal forces stays at rest in an airplane cruising at constant velocity. The airplane is an inertial reference frame.

(b)



The ball rolls to the back of the plane during takeoff. An accelerating plane is not an inertial reference frame.

The physics student in **FIGURE 5.21b** conducts the same experiment during takeoff. He carefully places the ball on the floor just as the airplane starts to accelerate down the runway. You can imagine what happens. The ball rolls to the back of the plane as the passengers are being pressed back into their seats. Nothing exerts a horizontal contact force on the ball, yet the ball accelerates *in the plane's reference frame.* This violates Newton's first law, so the plane is *not* an inertial reference frame during takeoff.

In the first example, the plane is traveling with constant velocity. In the second, the plane is accelerating. **Accelerating reference frames are not inertial reference frames.** Consequently, Newton's laws are not valid in an accelerating reference frame.

The earth is not exactly an inertial reference frame because the earth rotates on its axis and orbits the sun. However, the earth's acceleration is so small that violations of Newton's laws can be measured only in high-precision experiments. We will treat the earth and laboratories attached to the earth as inertial reference frames, an approximation that is exceedingly well justified.

To understand the motion of the passengers in a braking car, you need to measure velocities and accelerations *relative to the ground.* From the perspective of an observer on the ground, the body of a passenger in a braking car tries to continue moving forward with constant velocity, exactly as we would expect on the basis of Newton's first law, while his immediate surroundings are decelerating. The passenger is not "thrown" into the windshield. Instead, the windshield runs into the passenger!

Thinking About Force

It is important to identify correctly all the forces acting on an object. It is equally important not to include forces that do not really exist. We have established a number of criteria for identifying forces; the three critical ones are:

- A force has an agent. Something tangible and identifiable causes the force.
- Forces exist at the point of contact between the agent and the object experiencing the force (except for the few special cases of long-range forces).
- Forces exist due to interactions happening *now*, not due to what happened in the past.

There's no "force of motion" or any other forward force on this arrow. It continues to move because of inertia.

We all have had many experiences suggesting that a force is necessary to keep something moving. Consider a bowling ball rolling along on a smooth floor. It is very tempting to think that a horizontal "force of motion" keeps it moving in the forward direction. But *nothing contacts the ball* except the floor. No agent is giving the ball a forward push. According to our definition, then, there is *no* forward "force of motion" acting on the ball. So what keeps it going? Recall our discussion of the first law: *No* cause is needed to keep an object moving at constant velocity. It continues to move forward simply because of its inertia.

One reason for wanting to include a "force of motion" is that we tend to view the problem from our perspective as one of the agents of force. You certainly have to keep pushing to move a box across the floor at constant velocity. If you stop, it stops. Newton's laws, though, require that we adopt the object's perspective. The box experiences your pushing force in one direction *and* a friction force in the opposite direction. The box moves at constant velocity if the *net* force is zero. This will be true as long as your pushing force exactly balances the friction force. When you stop pushing, the friction force causes an acceleration that slows and stops the box.

A related problem occurs if you throw a ball. A pushing force was indeed required to accelerate the ball *as it was thrown*. But that force disappears the instant the ball loses contact with your hand. The force does not stick with the ball as the ball travels through the air. Once the ball has acquired a velocity, *nothing* is needed to keep it moving with that velocity.

5.7 Free-Body Diagrams

Having discussed at length what is and is not a force, we are ready to assemble our knowledge about force and motion into a single diagram called a *free-body diagram*. You will learn in the next chapter how to write the equations of motion directly from the free-body diagram. Solution of the equations is a mathematical exercise—possibly a difficult one, but nonetheless an exercise that could be done by a computer. The *physics* of the problem, as distinct from the purely calculational aspects, are the steps that lead to the free-body diagram.

A **free-body diagram,** part of the *pictorial representation* of a problem, represents the object as a particle and shows *all* of the forces acting on the object.

TACTICS BOX 5.3 Drawing a free-body diagram MP

1. **Identify all forces acting on the object.** This step was described in Tactics Box 5.2.
2. **Draw a coordinate system.** Use the axes defined in your pictorial representation.
3. **Represent the object as a dot at the origin of the coordinate axes.** This is the particle model.
4. **Draw vectors representing each of the identified forces.** This was described in Tactics Box 5.1. Be sure to label each force vector.
5. **Draw and label the *net force* vector $\vec{F}_{net}$.** Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{net} = \vec{0}$. Then check that $\vec{F}_{net}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Exercises 24–29

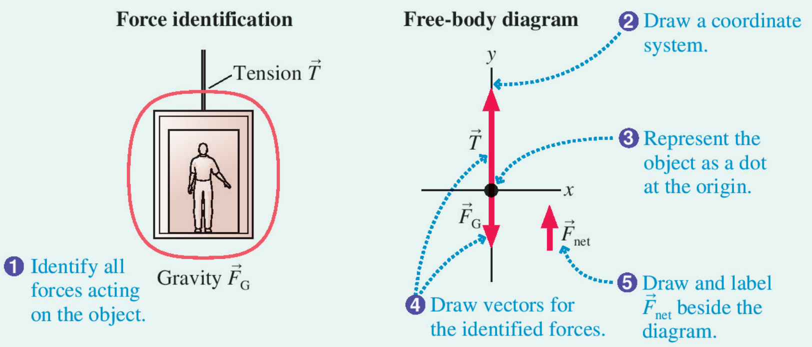
EXAMPLE 5.4 **An elevator accelerates upward**

An elevator, suspended by a cable, speeds up as it moves upward from the ground floor. Identify the forces and draw a free-body diagram of the elevator.

MODEL Treat the elevator as a particle.

VISUALIZE

FIGURE 5.22 Free-body diagram of an elevator accelerating upward.



ASSESS The coordinate axes, with a vertical y-axis, are the ones we would use in a pictorial representation of the motion. The elevator is accelerating upward, so $\vec{F}_{net}$ must point upward. For this to be true, the magnitude of $\vec{T}$ must be larger than the magnitude of $\vec{F}_G$. The diagram has been drawn accordingly.

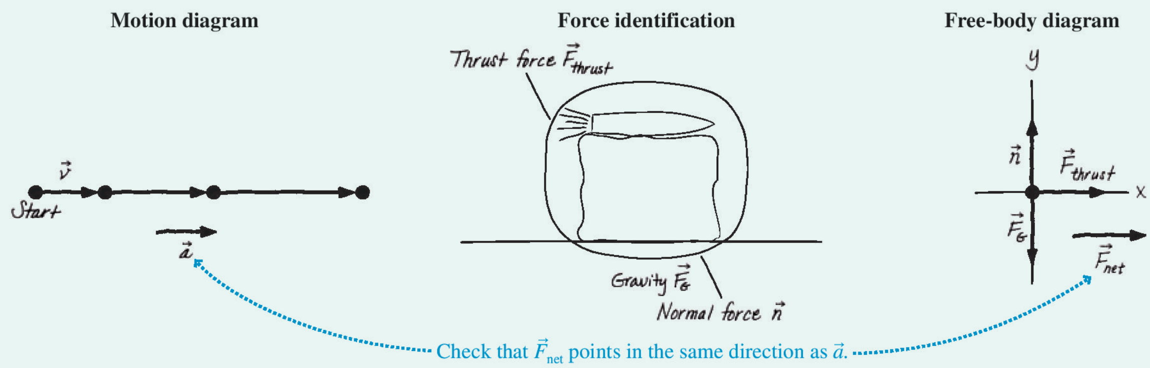
EXAMPLE 5.5 **An ice block shoots across a frozen lake**

Bobby straps a small model rocket to a block of ice and shoots it across the smooth surface of a frozen lake. Friction is negligible. Draw a pictorial representation of the block of ice.

MODEL Treat the block of ice as a particle. The pictorial representation consists of a motion diagram to determine $\vec{a}$, a force-identification picture, and a free-body diagram. The statement of the situation implies that friction is negligible.

VISUALIZE

FIGURE 5.23 Pictorial representation for a block of ice shooting across a frictionless frozen lake.



ASSESS The motion diagram tells us that the acceleration is in the positive x-direction. According to the rules of vector addition, this can be true only if the upward-pointing $\vec{n}$ and the downward-pointing $\vec{F}_G$ are equal in magnitude and thus cancel each other $\left((F_G)_y = -n_y\right)$. The vectors have been drawn accordingly, and this leaves the net force vector pointing toward the right, in agreement with $\vec{a}$ from the motion diagram.

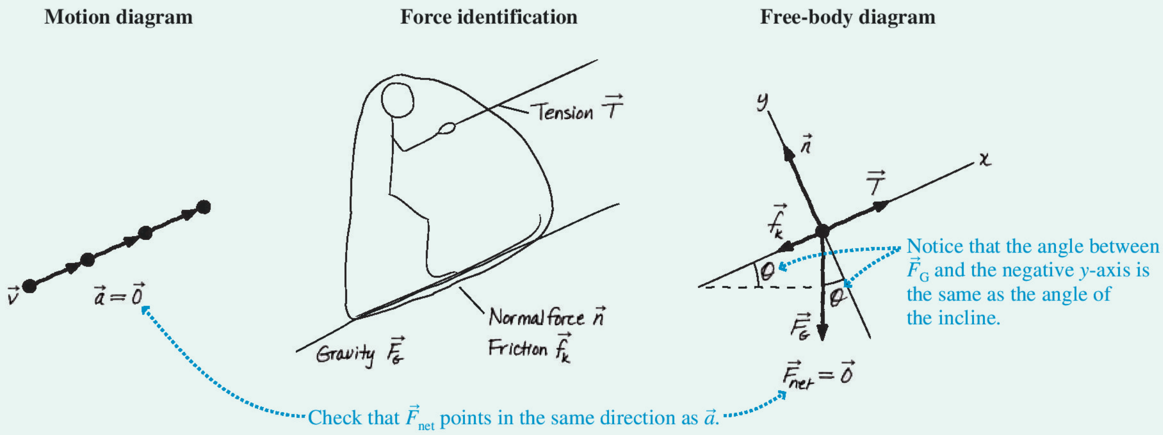
EXAMPLE 5.6 A skier is pulled up a hill

A tow rope pulls a skier up a snow-covered hill at a constant speed. Draw a pictorial representation of the skier.

MODEL This is Example 5.2 again with the additional information that the skier is moving at constant speed. The skier will be treated as a particle in *dynamic equilibrium*. If we were doing a kinematics problem, the pictorial representation would use a tilted coordinate system with the x-axis parallel to the slope, so we use these same tilted coordinate axes for the free-body diagram.

VISUALIZE

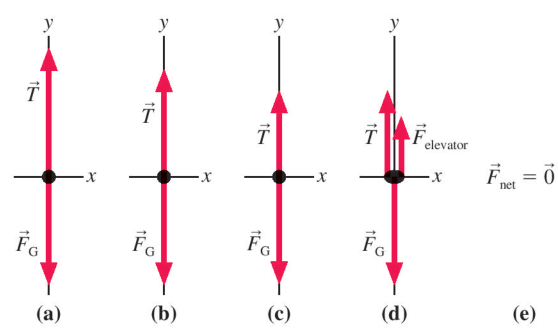
FIGURE 5.24 Pictorial representation for a skier being towed at a constant speed.



ASSESS We have shown $\vec{T}$ pulling parallel to the slope and $\vec{f}_k$, which opposes the direction of motion, pointing down the slope. $\vec{n}$ is perpendicular to the surface and thus along the y-axis. Finally, and this is important, the gravitational force $\vec{F}_G$ is *vertically* downward, *not* along the negative y-axis. In fact, you should convince yourself from the geometry that the angle θ between the $\vec{F}_G$ vector and the negative y-axis is the same as the angle θ of the incline above the horizontal. The skier moves in a straight line with constant speed, so $\vec{a} = \vec{0}$ and, from Newton's first law, $\vec{F}_{net} = \vec{0}$. Thus we have drawn the vectors such that the y-component of $\vec{F}_G$ is equal in magnitude to $\vec{n}$. Similarly, $\vec{T}$ must be large enough to match the negative x-components of both $\vec{f}_k$ and $\vec{F}_G$.

Free-body diagrams will be our major tool for the next several chapters. Careful practice with the workbook exercises and homework in this chapter will pay immediate benefits in the next chapter. Indeed, it is not too much to assert that a problem is half solved, or even more, when you complete the free-body diagram.

STOP TO THINK 5.5 An elevator suspended by a cable is moving upward and slowing to a stop. Which free-body diagram is correct?



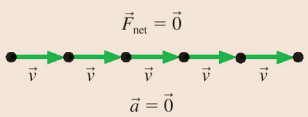
SUMMARY

The goal of Chapter 5 has been to establish a connection between force and motion.

General Principles

Newton's First Law

An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.



The first law tells us that no "cause" is needed for motion. Uniform motion is the "natural state" of an object.

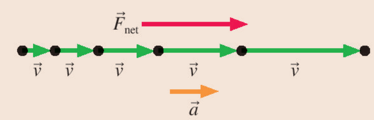
Newton's laws are valid only in inertial reference frames.

Newton's Second Law

An object with mass m will undergo acceleration

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$ is the vector sum of all the individual forces acting on the object.



The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

Important Concepts

Acceleration is the link to kinematics.

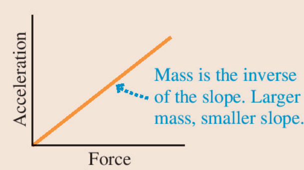
From $\vec{F}_{net}$, find $\vec{a}$.
From a, find v and x.

$\vec{a} = \vec{0}$ is the condition for **equilibrium.**

Static equilibrium if $\vec{v} = \vec{0}$.
Dynamic equilibrium if $\vec{v} =$ constant.

Equilibrium occurs if and only if $\vec{F}_{net} = \vec{0}$.

Mass is the resistance of an object to acceleration. It is an intrinsic property of an object.



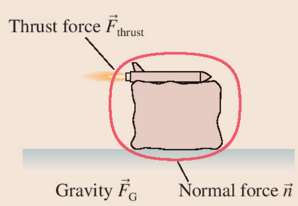
Force is a push or a pull on an object.

- Force is a vector, with a magnitude and a direction.
- Force requires an agent.
- Force is either a contact force or a long-range force.

Key Skills

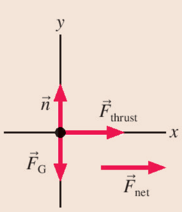
Identifying Forces

Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.



Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



Terms and Notation

dynamics
mechanics
force, $\vec{F}$
agent
contact force
long-range force
net force, $\vec{F}_{net}$
superposition of forces
gravitational force, $\vec{F}_G$
spring force, $\vec{F}_{sp}$
tension force, $\vec{T}$
atomic model
normal force, $\vec{n}$
friction, $\vec{f}_k$ or $\vec{f}_s$
drag, $\vec{D}$
thrust, $\vec{F}_{thrust}$
proportionality
proportionality constant
proportional reasoning
newton, N
inertia
inertial mass, m
Newton's second law
Newton's zeroth law
Newton's first law
mechanical equilibrium
static equilibrium
dynamic equilibrium
inertial reference frame
free-body diagram