1.6.5 Vertical alignment

Vertical alignment specifies the elevation of points along a roadway. The elevation of these roadway points are usually determined by the need to provide an acceptable level of driver safety, driver comfort and proper drainage. A primary concern in vertical alignment is establishing the transition of roadway elevations between two grades. This transition is achieved by means of a vertical curve. One of the most important factors that affect the design of this alignment is the topography of the area through which the proposed road is being passing as presented in Figures 1.30 and 1.31.

Vertical curves are usually parabolic in shape and can be broadly classified into crest vertical curves and sag vertical curves as illustrated in Figures 1.32 and 1.33.

Figure 1.30: Examples of vertical curves

Figure 1.31: Vertical curves in hilly areas

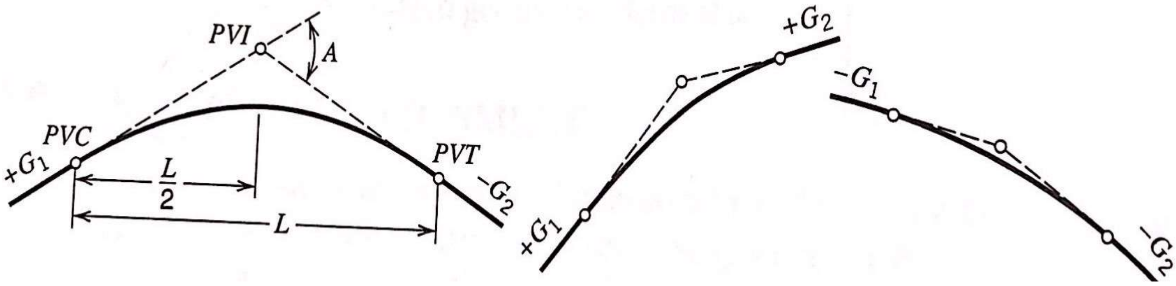

Figure 1.32: Crest vertical curves

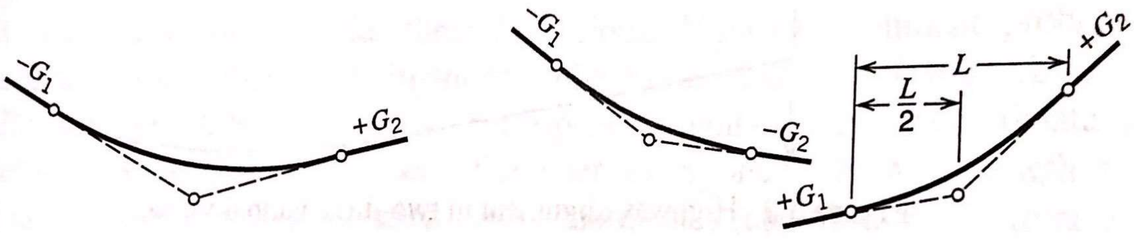

Figure 1.32: Sag vertical curves

1.6.5.1 Maximum grade

Passenger cars are normally less affected by the step grade as compared with the truck or heavy vehicle. Generally, the grade has a great effect on the heavy truck vehicles where a reduction of speed occurs on these grades. It should be noted that the selection of the grade value has a great influence on the volume of earthwork. To reduce this effect, it is customarily adopted to design the highways in such a way that ensure a reduction in the earthwork quantities and hence the cost of the project. Table 1.9 presents recommended maximum values of grades with respect to types of terrain and road.

1.6.5.2 Minimum grade

The minimum grade is generally governed by adopted drainage requirements for roadway being designed. A minimum grade of 0.3% is desirable for high type pavements.

1.6.5.3 Critical length of grade

This critical length can be defined as the maximum length of upgrade on which the design vehicle (almost heavy trucks) can run without a reasonable speed reduction. Figure 1.33 is used to assess the critical length of grade. It should be noted that a speed reduction curve of 15 Km/h is recommended to be used to find the critical length of grade.

Table 1.9: Recommended maximum value of grades

Rural Collectors[a]

Design Speed (mi/h)

Type of Terrain	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*
					Grades (%)				
Level	7	7	7	7	7	7	6	6	5
Rolling	10	10	9	9	8	8	7	7	6
Mountainous	12	11	10	10	10	10	9	9	8

Urban Collectors[a]

Design Speed (mi/h)

Type of Terrain	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*
					Grades (%)				
Level	9	9	9	9	9	8	7	7	6
Rolling	12	12	11	10	10	9	8	8	7
Mountainous	14	13	12	12	12	11	10	10	9

Rural Arterials

Design Speed (mi/h)

Type of Terrain	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*
					Grades (%)				
Level	5	5	4	4	3	3	3	3	3
Rolling	6	6	5	5	4	4	4	4	4
Mountainous	8	7	7	6	6	5	5	5	5

Rural and Urban Freeways[b]

Design Speed (mi/h)

Type of Terrain	*50*	*55*	*60*	*65*	*70*	*75*	*80*
				Grades (%)			
Level	4	4	3	3	3	3	3
Rolling	5	5	4	4	4	4	4
Mountainous	6	6	6	5	5	–	–

Urban Arterials

Design Speed (mi/h)

Types of Terrain	*30*	*35*	*40*	*45*	*50*	*55*	*60*
				Grades (%)			
Level	8	7	7	6	6	5	5
Rolling	9	8	8	7	7	6	6
Mountainous	11	10	10	9	9	8	8

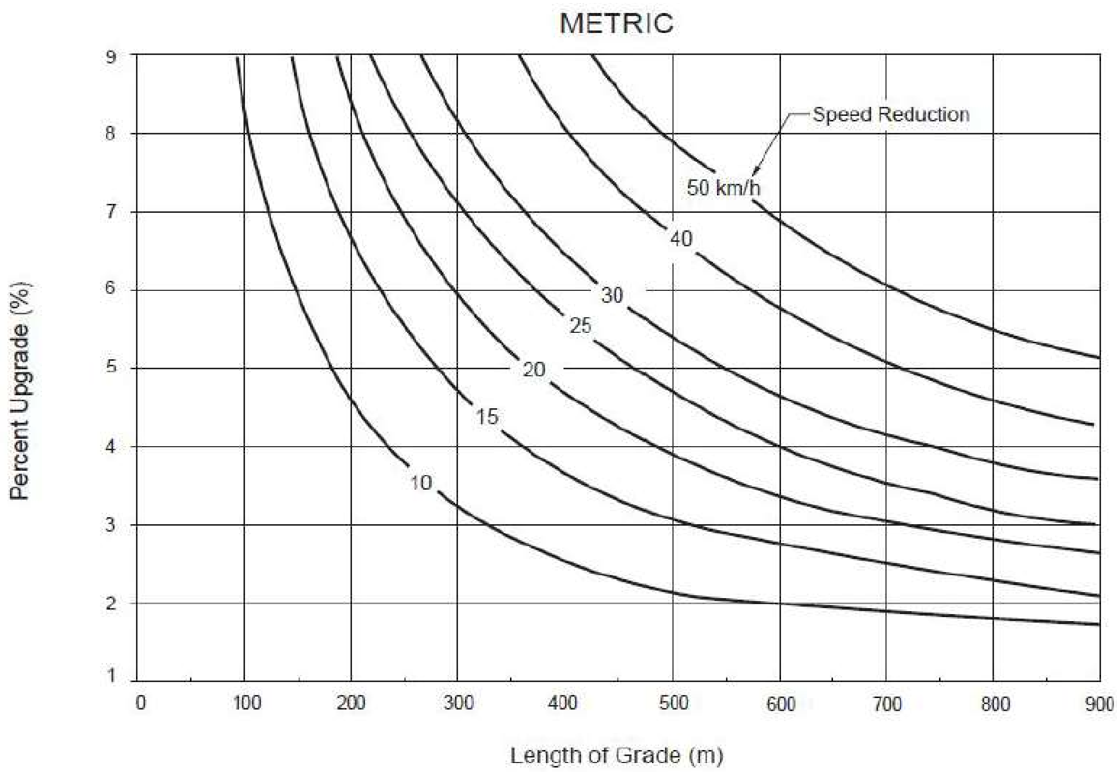

Figure 1.33: Critical length of grade

1.6.5.4 Elements of vertical curves

Elements of vertical curves can be illustrate in Figure 1.34

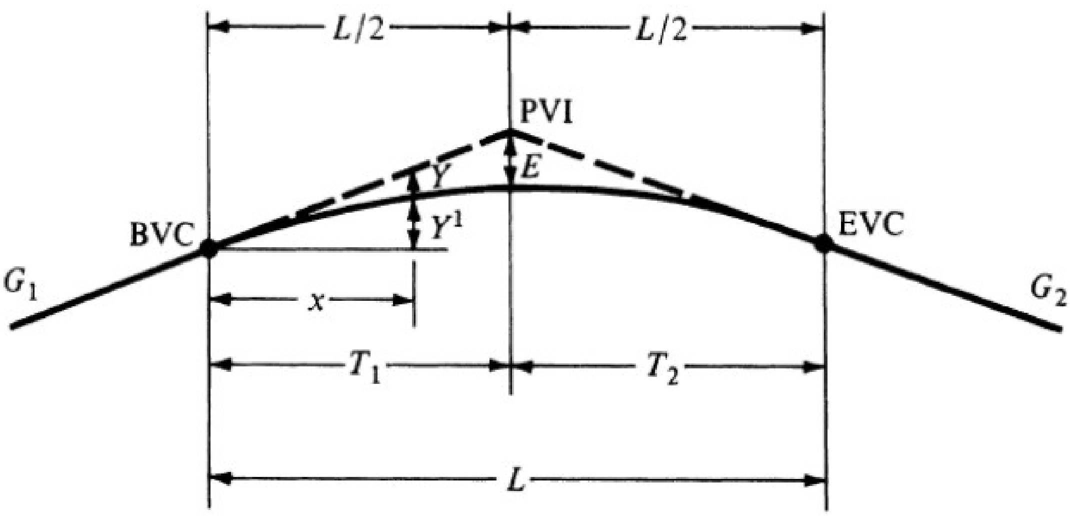

Figure 1.34: Layout and parameters of vertical curve

Where:

G_1, G_2: Grades of tangents %

L: Length of curve

E: External distance

BVC (PVC): beginning of vertical curve

EVC (PVT): End of vertical curve

PVI: point of vertical intersection

A: algebraic difference of grades, G_1-G_2

1.6.5.5 Properties of vertical curves

The determination of vertical curve elevations and elevation of critical points could be computed based on the properties of parabola as shown in equation

$$y = ax^2 + bx + c \quad \text{...}36$$

where

y = elevation of any point on curve.

x= distance from the point of vertical curvature.

a = rate of change of gradient.

b = initial grade

c= elevation of point of curvature

Rate of change of slope = the second derivative

First derivative = 2ax+b

Second derivative = 2a ……………………………………………………..37

But, the rate of change =(G_2-G_1)/100L ………………………………………..38

Equating Eq.37 and Eq.38 gives

2a= (G_2-G_1)/100L

So, $a = \frac{G2-G1}{200L}$

And equation 36 can be rewritten as follows

Elevation of any point on curve= $\frac{G2-G1}{200\,L}$ $x^2 + \frac{G1}{100}$ x +PVC elev.

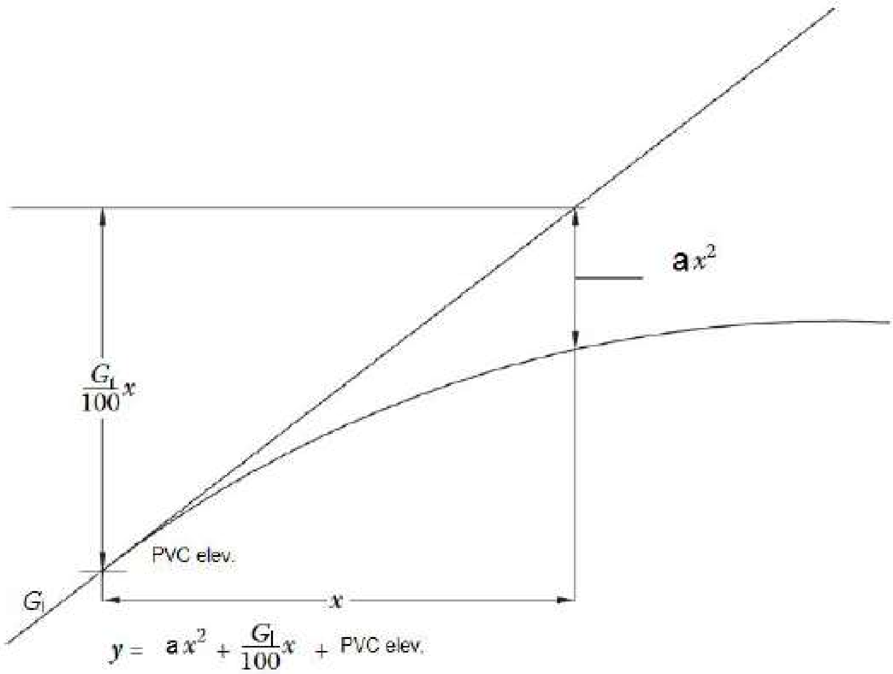

Figure 1.35: Layout and parameters of vertical curve

Offset

As shown in Figure 1.34, Y^1 can be calculated as follows:

$Y^1 = \frac{G1}{100}x - Y$ ……………………………………………………………….39

where $Y = \frac{A}{200L}x^2$ ……………………………………………………………40

A: algebraic difference of grades, G_1-G_2

$$Y^1 = \frac{G1}{100}x - \frac{G1-G2}{200L}x^2$$

$$\frac{dy}{dx} = \frac{G1}{100} - \frac{G1-G2}{100L}x = 0$$

$$X_{high/\ low} = \frac{G1}{G1-G2} L \quad \ldots\ldots 41$$

External distance *E* from the point of vertical intersection (PVI) to the curve is determined by substituting *L*/2 for *x* in Eq. $Y = \frac{A}{200L} x^2$

$$E = \frac{AL}{800} \quad \ldots\ldots 42$$

$$BVC_{\ Station} = PVI_{\ station} - \frac{L}{2} \quad \ldots\ldots 43$$

$$EVC_{\ Station} = BVC_{\ station} + L \quad \ldots\ldots 44$$

$$BVC_{\ Elevation} = PVI_{\ Elevation} - \frac{G_1 L}{200} \quad \ldots\ldots 45$$

$$EVC_{\ Elevation} = PVI_{\ Elevation} - \frac{G_2 L}{200} \quad \ldots\ldots 46$$

1.6.5.6 Design Procedure for Crest and Sag Vertical Curves

Step 1. Determine the minimum length of curve to satisfy sight distance requirements and other criteria for sag curves (sight distance requirements, comfort requirements. appearance requirements, and drainage requirements.

Step 2. Determine from the layout plans the station and elevation of the point where the grades intersect (PVI).

Step 3. Compute the elevations of the beginning of vertical curve, (BVC) and the end of vertical curve (EVC).

Step 4. Compute the offsets, *Y,* (Eq. 40) as the distance between the tangent and the curve. Usually equal distances of 20m (1 station) are used, beginning with the first whole station after the BVC.

Step 5. Compute elevations on the curve for each station.

Step 6. Compute the location and elevation of the highest (crest) or lowest (sag) point on the curve

1.6.5.7 Determine the minimum length of curve

When length of vertical curves needs to be computed, four scenarios/ criteria should be taken in account. Those includes:

1. Sight distance requirements.
2. Comfort requirements.
3. Appearance requirements.
4. Drainage requirements

The first criteria is only used to design the crest vertical curve; whereas all criteria are taken in account the process of design sag vertical curves.

1.6.5.7.1 Crest vertical Curves

As mentioned previously, crest vertical curves are commonly designed on the basis of sight distance requirements. Two scenarios exist and controls the design. These are when the length of curve is greater than the sight distance (L > S) and when the length of curve is less than the sight distance. Figure 1.36 shows the first case which is the more poplar or common design option.

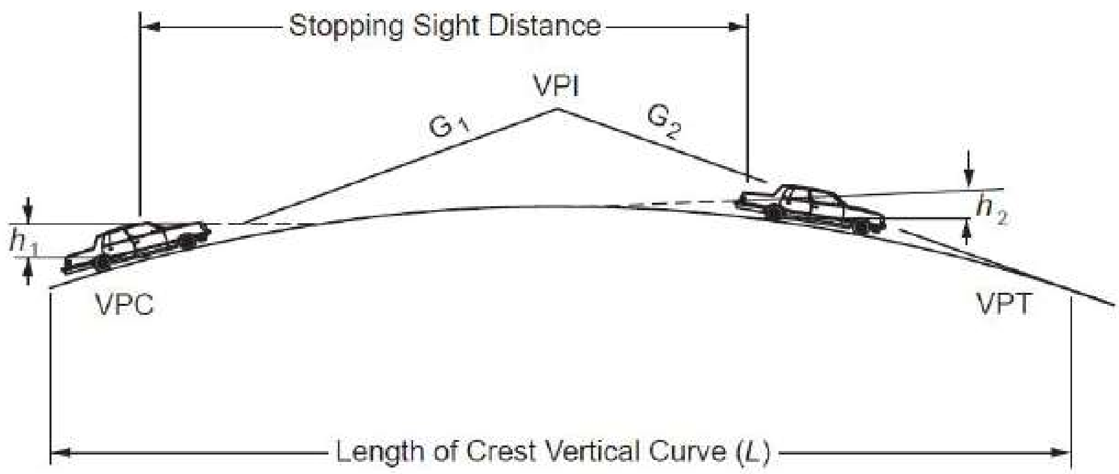

Figure 1.36: Crest vertical curves

The following equations are used to compute minimum length of vertical curve for both design option stated above:

When S is less than L

$$L_{min} = \frac{AS^2}{200\left(\sqrt{h1}+\sqrt{h2}\right)^2} \text{ ....} 47$$

When S is greater than L

$$L_{min} = 2S - \frac{200\left(\sqrt{h1}+\sqrt{h2}\right)^2}{A} \text{ ....} 48$$

Where:

L is length of vertical curve, *m*

A is algebraic difference in grades, *%*

S is sight distance, *m*

h1 is height of eye above roadway surface, *m*

h2 is height of object above roadway surface, *m*

Based on AASHTO's G.D policy, the values of h1 and h2 are 1.08 and 0.6 m, respectively. So by applying these values in equations above results, we get:

When S is less than L

$$L_{min} = \frac{AS^2}{658} \text{ ....} 49$$

When S is greater than L

$$L_{min} = 2S - \frac{658}{A} \text{ ....} 50$$

Design controls: stopping sight distance

Equation 49 (for S is less than L) can be rewritten as follows;

L= K. A ………………………………………………………………….51

Where;

$K = S^2/ 658$ ……………………………………………………………….52

And, K value represent the length of curve for each 1 degree change in the grade.

It should be noted in practice that when S > L, the calculated minimum length will be small and impractical for design consideration. Consequently, the designer should adopt minimum of crest vertical curve of L=0.6V (where L and V represent length of curve and design speed in Km/h, respectively) or use the first equation 49 to compute the design minimum length of curve. Figure 1.37 and Table 1.10 illustrate design controls for crest vertical curves based on stopping sight distance.

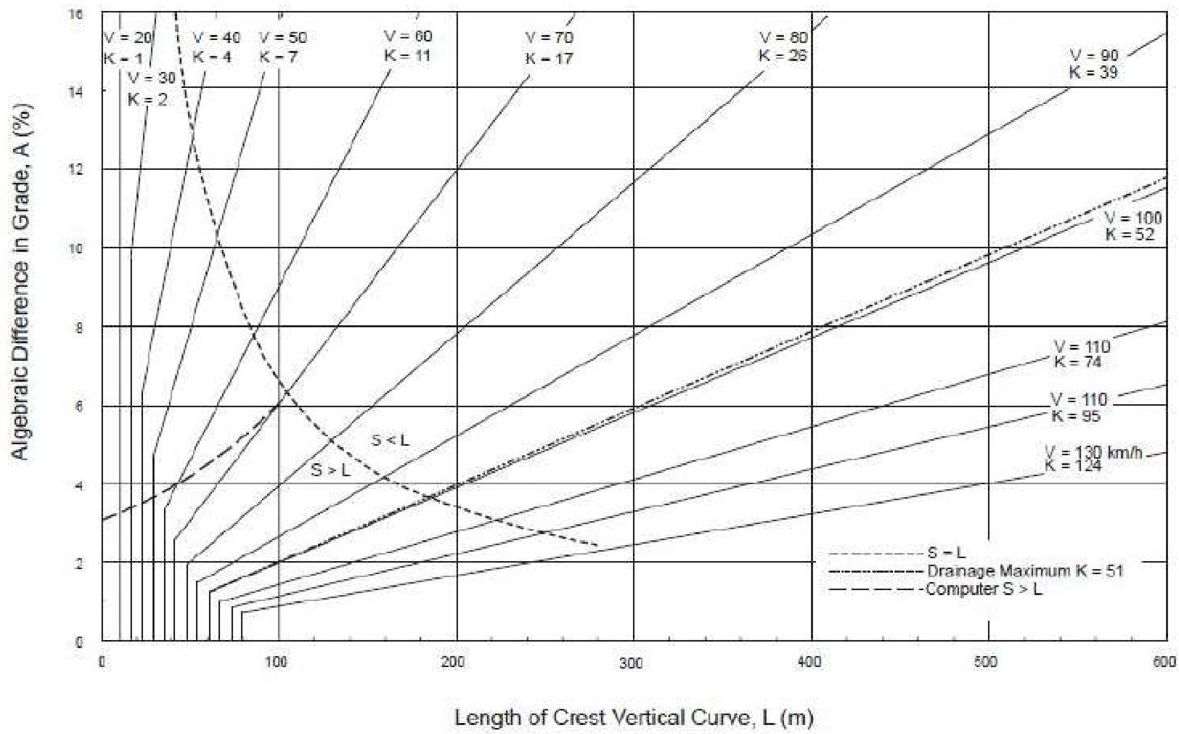

Figure 1.37: Design controls for crest curve

Table 1.10: Design controls for crest vertical curves based on stopping sight distance.

Metric				U.S. Customary			
Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, K^a		Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, K^a	
		Calculated	Design			Calculated	Design
20	20	0.6	1	15	80	3.0	3
30	35	1.9	2	20	115	6.1	7
40	50	3.8	4	25	155	11.1	12
50	65	6.4	7	30	200	18.5	19
60	85	11.0	11	35	250	29.0	29
70	105	16.8	17	40	305	43.1	44
80	130	25.7	26	45	360	60.1	61
90	160	38.9	39	50	425	83.7	84
100	185	52.0	52	55	495	113.5	114
110	220	73.6	74	60	570	150.6	151
120	250	95.0	95	65	645	192.8	193
130	285	123.4	124	70	730	246.9	247
				75	820	311.6	312
				80	910	383.7	384

[a] Rate of vertical curvature, *K*, is the length of curve per percent algebraic difference in intersecting grades

Design controls: passing sight distance

Based on AASHTO's G.D policy, both values of h1 and h2 (in case of passing sight distance application as shown in Figure 1.38) should be adopted as 1.08. By applying these values in equations 47 and 48, we get:

When S is less than L

$$L_{min} = \frac{AS^2}{864} \quad \text{......53}$$

When S is greater than L

$$L_{min} = 2S - \frac{864}{A} \quad \text{......54}$$

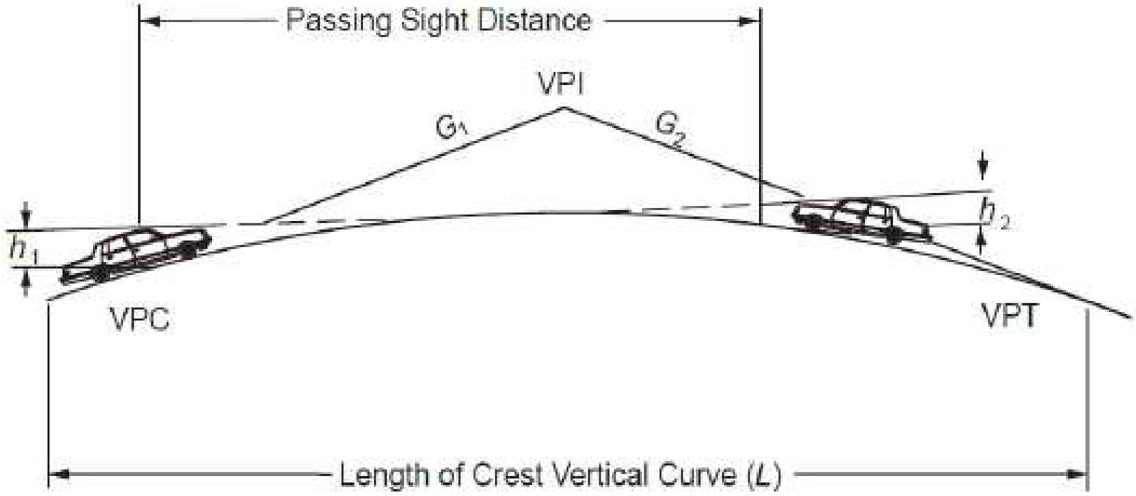

Figure 1.38: Passing sight distance on crest vertical

Table 1.11: Design controls for crest vertical curves based on passing sight distance

Metric			U.S. Customary		
Design Speed (km/h)	Passing Sight Distance (m)	Rate of Vertical Curvature, K^a Design	Design Speed (mph)	Passing Sight Distance (ft)	Rate of Vertical Curvature, K^a Design
30	120	17	20	400	57
40	140	23	25	450	72
50	160	30	30	500	89
60	180	38	35	550	108
70	210	51	40	600	129
80	245	69	45	700	175
90	280	91	50	800	229
100	320	119	55	900	289
110	355	146	60	1000	357
120	395	181	65	1100	432
130	440	224	70	1200	514
			75	1300	604
			80	1400	700

[a] Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). $K = L/A$

1.6.5.7.2 Sag vertical Curves

Having mentioned that the minimum length of sag vertical curve is governed by four criteria, which include:

1. Sight distance requirements.
2. Comfort requirements.
3. Appearance requirements.
4. Drainage requirements

Sag curve minimum length based on sight distance requirements

Sight distance in this type of highways depends on the lighted part of the roadway ahead for the driver as shown in Figures 1.39. This is called as headlight sight distance as previously defined. On day time or on well-lit roadway at night, there is no problem with sight distance on this type of curves. Headlight sight distance is therefore mainly used by most highway department to estimate the length of the sag curve.

Figure 1.39: Sag vertical curve at day and night time

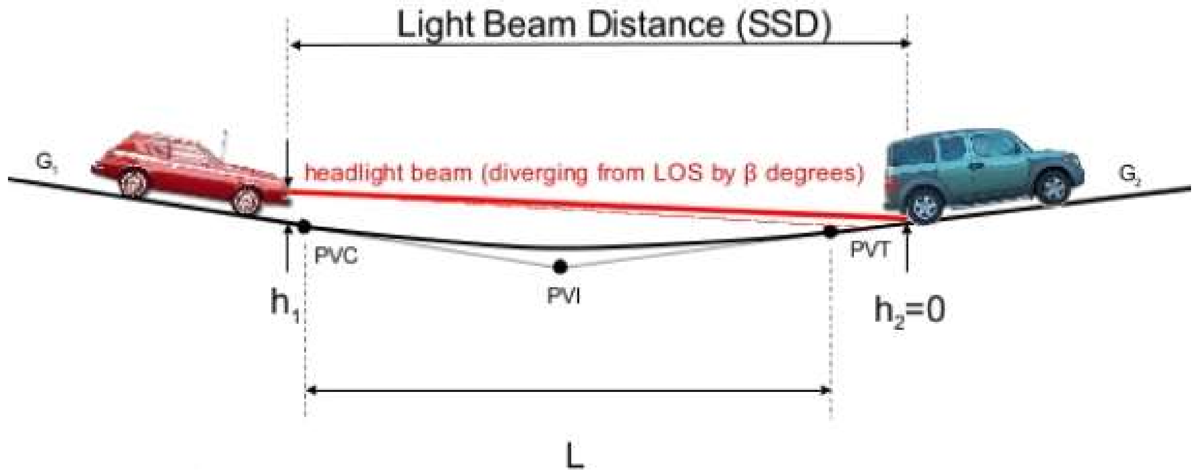

Figure 1.40: headlight (stopping) sight distance on crest vertical

According to sight distance requirement

When S is less than L

$$L_{min} = \frac{AS^2}{200(h+\tan\ )} \quad \text{...}55$$

When S is greater than L

$$L_{min} = 2S - \frac{200(h+\tan\ )}{A} \quad \text{..}56$$

Based on AASHTO's G.D policy, values of h and β are 0.6m and 1° respectively. And by applying these values, we get

When S is less than L

$$L_{min} = \frac{AS^2}{120+3.5\,S} \quad \text{...}57$$

When S is greater than L

$$L_{min} = 2S - \frac{120+3.5\,S}{A} \quad \text{..}58$$

Table 1.13 presents design controls for sag vertical curves based on stopping sight distance.

Sag curve minimum length based on driver comfort

Unlike on crest vertical curves, vehicle on sag curve is under a combination of gravitational and centrifugal forces. This combination may apply discomfort to the driver on this type of curves. To satisfy this criterion, the minimum length of curve should be estimated from the following formula.

$$L = \frac{AV^2}{395} \qquad \ldots\ldots 59$$

Sag curve minimum length based on general appearance

Vertical curves are normally provided at all change in grade. However, for the slight change in grade (small A values), high K values are frequently provided to make sure that an appropriate appearance exist. Table 1.12 illustrates the maximum change in gradient that do not require a vertical curves and also the minimum length of curves for satisfactory appearance.

Table 1.12: Appearance requirement requirements

Design speed (km/h)	Maximum gradient change without vertical curve (%)	Minimum length of vertical curve for satisfactory appearance (m)
40	1.0	30
60	0.8	50
80	0.6	80
100	0.4	100
120	0.2	150

Sag curve minimum length based on drainage requirements

This criterion has to be considered in the case of curbed roads. In this scenario, the requirement is normally focuses on the maximum length whereas minimum lengths for other criteria are required. To satisfy this criterion, the maximum length should ensure that there is a minimum grade of 0.35 at the lowest 15 m of the curve. The maximum length to meet this requirement is normally equal the minimum length for other criterion for speed over 60 km/h.