UNIVERSITY OF ANBAR

COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING DEPARTMENT

Power Electronic

Fourth Class

Chapter 02

AC to DC Convertor (Rectifiers)

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CHAPTER 2

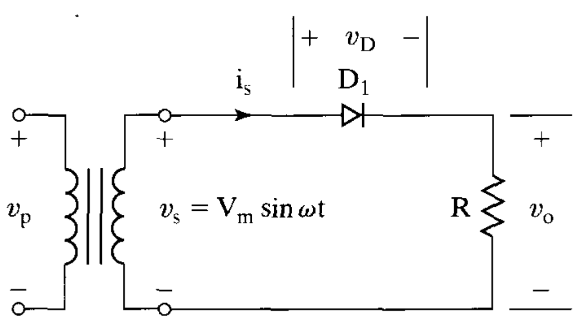
Rectifiers: Converting AC to DC

2.1 Introduction

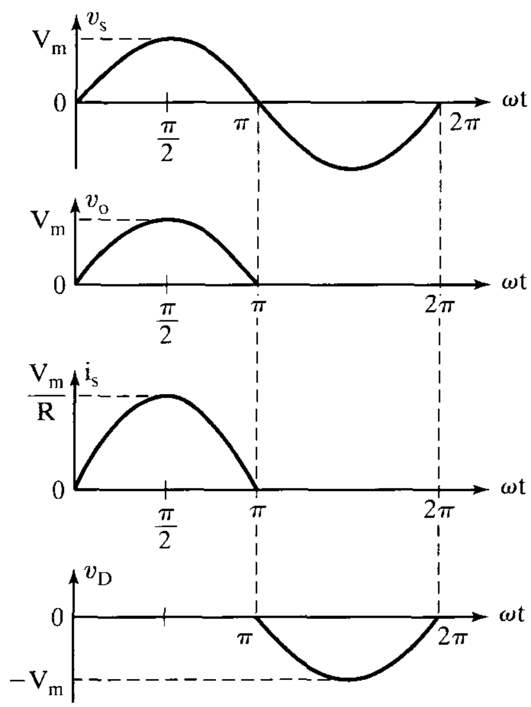
Since the easily available voltage is a sinusoid, which alternates as a function of time, the first task is to convert it into a useful and reliable constant (dc) voltage for the successful operation of electronic circuits and direct current machines. The conversion process is called the rectification. Although there are other semiconductors devices suitable for rectification, diodes are frequently employed. A rectifier is a circuit that converts an ac signal into a dc signal or sometime is called ac to dc converter. The rectifiers are classified into two types, single-phase and three-phase. The typical applications of the rectifier circuits such as dc welder, dc motor drive, Battery charger, dc power supply, High Voltage Direct Current (HVDC).

2.2 Single-Phase Half-Wave Rectifiers

A single-phase half-wave rectifier is the simplest type, but it is not normally used in industrial applications. However, it is useful in understanding the principle of rectifier operation. The circuit diagram with a resistive load is shown in Figure 2.l(a). During the positive half-cycle of the input voltage, diode D_1 conducts and the input voltage appears across the load. During the negative half-cycle of the input voltage, the diode is in a blocking condition and the output voltage is zero. The waveforms for the input voltage and output voltage are shown in Figure 2.1(b).



(a) Circuit diagram



(b) Waveforms

Figure 2.1: Single-phase half-wave rectifier

2.3 Performance Parameters

Although the output voltage as shown in Figure 2.1(b) is dc, it is discontinuous and contains harmonics. A rectifier is a power processor that should give a dc output voltage with a minimum amount of harmonic contents. At the same time, it should maintain the input current as sinusoidal as possible and in phase with the input voltage so that the power factor is near unity. The power-processing quality of a rectifier requires the determination of harmonic contents of the input current, the output voltage, and the output current. Fourier series expansions can be used to find the harmonic contents of voltages and currents. There are different types of rectifier circuits and the performances of a rectifier are normally evaluated in terms of the following parameters:

The average value of the output (load) voltage, V_{dc}
The average value of the output (load) current, I_{dc}
The output dc power,

$$P_{dc} = V_{dc} I_{dc} \tag{2.1}$$

The root-mean-square (rms) value of the output voltage, V_{rms}
The rms value of the output current, I_{rms}
The output ac power

$$P_{ac} = V_{rms} I_{rms} \tag{2.2}$$

The efficiency (or rectification ratio) of a rectifier, which is a figure of merit and permits us to compare the effectiveness, is defined as

$$\eta = \frac{P_{dc}}{P_{ac}} \qquad (2.3)$$

The output voltage can be considered as composed of two components: (1) the dc value, and (2) the ac component or ripple.

The effective (rms) value of the ac component of output voltage is

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2} \qquad (2.4)$$

The form factor, which is a measure of the shape of output voltage, is

$$FF = \frac{V_{rms}}{V_{dc}} \qquad (2.5)$$

The ripple factor, which is a measure of the ripple content, is defined as

$$RF = \frac{V_{ac}}{V_{dc}} \qquad (2.6)$$

Substituting Eq. (2.4) in Eq. (2.6), the ripple factor can be expressed as

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{FF^2 - 1} \qquad (2.7)$$

The transformer utilization factor is defined as

$$TUF = \frac{P_{dc}}{V_s I_s} \qquad (2.8)$$

where V_s and I_s, are the rms voltage and rms current of the transformer secondary, respectively. Consider the waveform of Figure 2.2, where v_s is the sinusoidal input voltage, i_s is the instantaneous input current, and i_{s1} is its fundamental component.

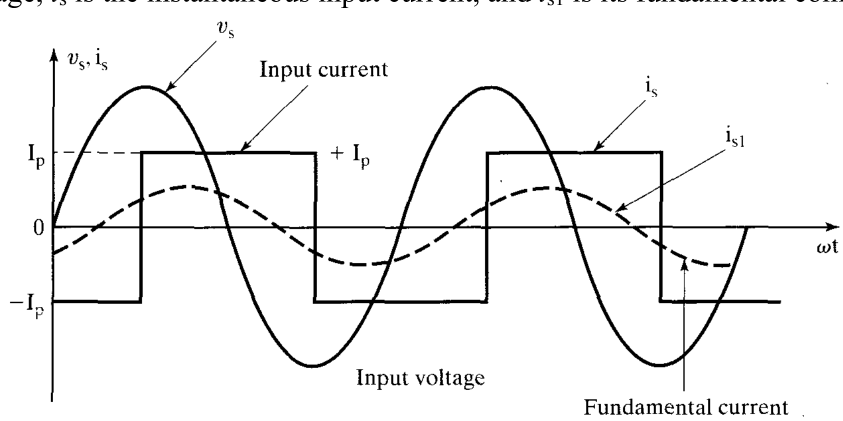


Figure 2.2: Waveforms for input voltage and current

If ϕ is the angle between the fundamental components of the input current and voltage, ϕ is called the displacement angle. The displacement factor is defined as

$$DF = \cos\phi \tag{2.9}$$

The harmonic factor (HF) of the input current is defined as

$$HF = \left(\frac{I_s^2 - I_{s1}^2}{I_{s1}^2}\right)^{1/2} = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} \tag{2.10}$$

where I_{s1} is the fundamental component of the input current I_s. Both I_{s1} and I_s are expressed here in rms. The input power factor (PF) is defined as

$$PF = \frac{V_s I_{s1}}{V_s I_s}\cos\phi = \frac{I_{s1}}{I_s}\cos\phi \tag{2.11}$$

Crest factor (CF), which is a measure of the peak input current $I_{s(peak)}$ as compared with its rms value I_s, is often of interest to specify the peak current ratings of devices and components. CF of the input current is defined by

$$CF = \frac{I_{s(peak)}}{I_s} \tag{2.12}$$

Example 2.1: Finding the performance parameters of a Half-wave Rectifier
The rectifier in Figure 2.l(a) has a purely resistive load of R. Determine,
(a) the efficiency,
(b) the FF,
(c) the RF,
(d) the TUF,
(e) the PIV of diode D_1,
(f) the CF of the input current
(g) input PF.

Solution of Example 2.1
The average output voltage V_{dc} is defined as

$$V_{dc} = \frac{1}{T}\int_0^T v_L(t)\,dt$$

We can notice from Figure 2.1(b) that $v_L(t) = 0$ for $T/2 \le t \le T$. Hence,

$$V_{dc} = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t\, dt = \frac{-V_m}{\omega T}\left(\cos\frac{\omega T}{2} - 1\right)$$

However, the frequency of the source is $f = 1/T$ and $\omega = 2\pi f$. Thus

$$\begin{aligned} V_{dc} &= \frac{V_m}{\pi} = 0.318V_m \\ I_{dc} &= \frac{V_{dc}}{R} = \frac{0.318V_m}{R} \end{aligned} \tag{2.13}$$

The rms value of a periodic waveform is defined as

$$V_{rms} = \left[\frac{1}{T}\int_0^T v_L^2(t)dt\right]^{1/2}$$

For a sinusoidal voltage of $v_0(t)$ = V_m sin ωt for $0 \le t \ge T/2$, the rms value of the output voltage is

$$V_{rms} = \left[\frac{1}{T}\int_0^{T/2}(V_m \sin \omega t)^2\, dt\right]^{1/2} = \frac{V_m}{2} = 0.5V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.5V_m}{R} \qquad (2.14)$$

From Eq. (2.1), $P_{dc} = (0.318V_m)^2/R$, and from Eq. (2.2), $P_{ac} = (0.5V_m)^2/R$

a) From Eq. (2.3), the efficiency $\eta = (0.318V_m)^2/(0.5V_m)^2 = 40.5\%$.
b) From Eq. (2.5), the FF = $0.5V_m/0.318V_m$ = 1.57 or 157%.
c) From Eq. (2.7), the RF = $\sqrt{1.57^2 - 1}$ = 1.21 or 121%.
d) The rms voltage of the transformer secondary is

$$V_s = \left[\frac{1}{T}\int_0^{T}(V_m \sin \omega t)^2\, dt\right]^{1/2} = \frac{Vm}{\sqrt{2}} = 0.707V_m \qquad (2.15)$$

The rms value of the transformer secondary current is the same as that of the load:

$$I_s = \frac{0.5V_m}{R}$$

The volt-ampere rating (VA) of the transformer,
VA = $V_sI_s = 0.707V_m \times 0.5V_m/R$.
From Eq. (2.8) TUF = $P_{ac}/(V_sI_s) = 0.138^2/(0.707 \times 0.5) = 0.286$.

e) The peak reverse (or inverse) blocking voltage PIV = V_m.
f) $I_{s(peak)} = V_m/R$ and $I_s = 0.5V_m/R$. The CF of the input current is CF = $I_{s(peak)}/I_s$ = 1/0.5 =2.
g) The input PF for a resistive load can be found from

$$PF = \frac{P_{ac}}{VA} = \frac{0.5^2}{0.707 \times 0.5} = 0.707$$

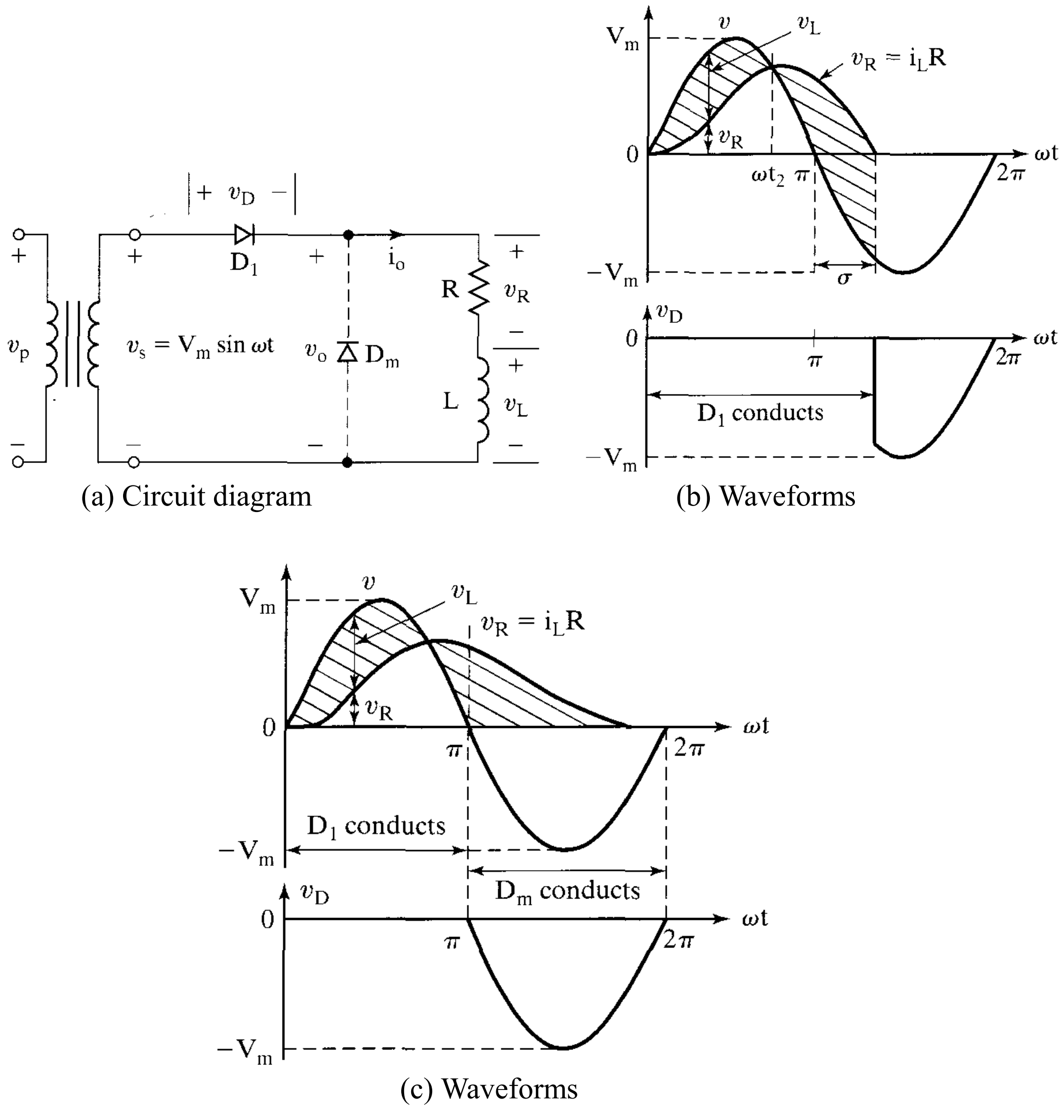


Figure 2.3: Half-wave rectifier with *RL* load

Consider the circuit of Figure 2.l(a) with an *RL* load as shown in Figure 2.3(a). Due to inductive load, the conduction period of diode D_1 will extend beyond 180° until the current becomes zero at $\omega t = \pi + \sigma$. The waveforms for the current and voltage are shown in Figure 2.3(b). It should be noted that the average v_L of the inductor is zero. The average output voltage is

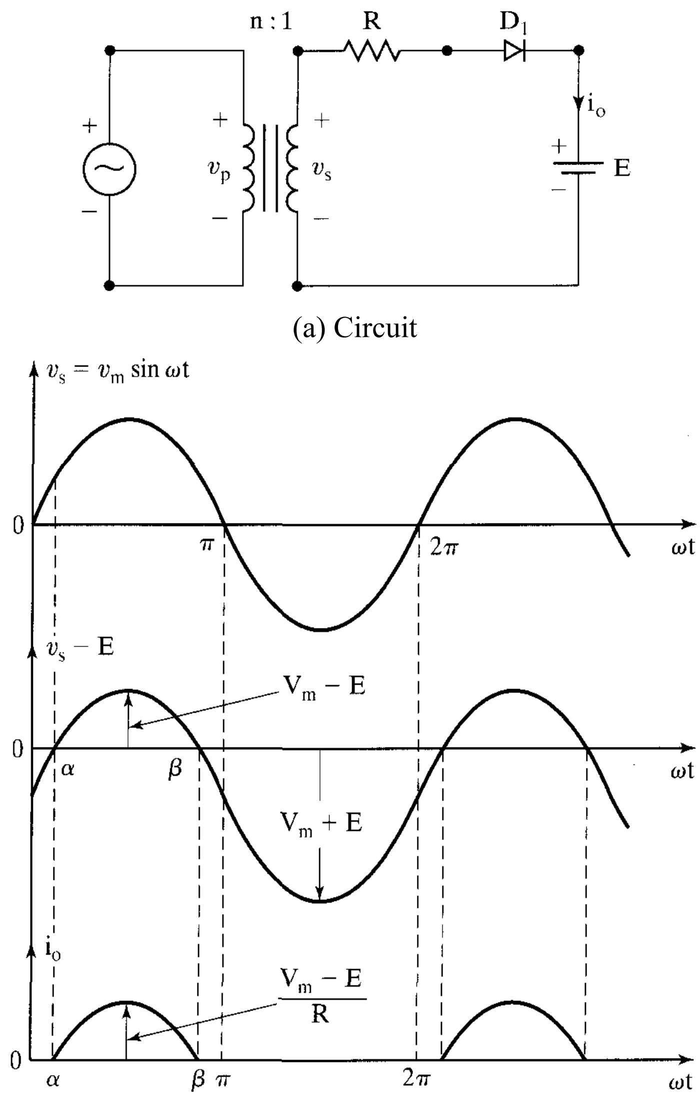
$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\pi+\sigma} \sin \omega t \, d(\omega t) = \frac{V_m}{2\pi}\left[-\cos \omega t\right]_0^{\pi+\sigma} = \frac{V_m}{2\pi}\left[1-\cos(\pi+\sigma)\right] \tag{2.16}$$

The average load current is $I_{dc} = V_{dc}/R$.

It can be noted from Eq. (2.16) that the average voltage and current can be increased by making $\sigma = 0$, which is possible by adding a freewheeling diode D_m as shown in Figure 2.3(a) with dashed lines. The effect of this diode is to prevent a negative voltage appearing across the load; and as a result, the magnetic stored energy is increased. At $t = t_1 = \pi/\omega$, the current from D_1 is transferred to D_m and this process is called commutation of diodes and the waveforms are shown in Figure 2.3(c). Depending on the load time constant, the load current may be discontinuous. Load current i_0 is discontinuous with a resistive load and continuous with a very high inductive load. The continuity of the load current depends on its time constant $\tau = \omega L/R$.

If the output is connected to a battery, the rectifier can be used as a battery charger. This is shown in Figure 2.4(a). For $v_s > E$, diode D_1 conducts. The angle α when the diode starts conducting can be found from the condition.

$$V_m \sin \alpha = E$$



(a) Circuit

(b) Waveforms

Figure 2.4: Battery charger

$$\alpha = \sin^{-1}\frac{E}{V_m} \tag{2.17}$$

Diode D_1 is turned off when $v_s < E$ at

$$\beta = \pi - \alpha$$

The charging current i_L, which is shown in Figure 2.4(b), can be found from

$$i_0 = \frac{v_s - E}{R} = \frac{V_m \sin\omega t - E}{R} \qquad \text{for } \alpha < \omega t < \beta$$

Example 2.2: Finding the Performance Parameters of a Battery Charger

The battery voltage in Figure 2.4(a) is E = 12 V and its capacity is 100 Wh. The average charging current should be I_{dc} = 5 A. The primary input voltage is V_p = 120 V, 60 Hz, and the transformer has a turn ratio of n = 2:1. Calculate

(a) the conduction angle δ of the diode.
(b) the current-limiting resistance R.
(c) the power rating P_R of R.
(d) the charging time h_0 in hours.
(e) the rectifier efficiency η.
(f) the PIV of the diode.

Solution of Example 2.2

E = 12 V, V_p = 120 V, $V_s = V_p/n$ = 120/2 = 60 V, and $V_m = \sqrt{2} \times V_s$, $= \sqrt{2} \times 60$ = 84.85 V.

a) From Eq. (2.17), $\alpha = \sin^{-1}$ (12/84.85) = 8.13° or 0.1419 rad. β = 180 – 8.13 = 171.87°. The conduction angle is δ = β – α = 171.87 – 8.13 = 163.74°.

b) The average charging current I_{dc} is

$$I_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\beta}\frac{V_m \sin\omega t - E}{R}\, d(\omega t) = \frac{1}{2\pi R}\left(2V_m \cos\alpha + 2E\alpha - \pi E\right), \text{ for } \beta = \pi - \alpha \tag{2.18}$$

which gives

$$R = \frac{1}{2\pi I_{dc}}\left(2V_m \cos\alpha + 2E\alpha - \pi E\right)$$

$$= \frac{1}{2\pi \times 5}\left(2\times 84.85\times\cos 8.13^\circ + 2\times 12\times 0.1419 - \pi\times 12\right) = 4.26\,\Omega$$

c) The rms battery current I_{rms} is

$$I_{rms}^2 = \frac{1}{2\pi}\int_{\alpha}^{\beta}\frac{\left(V_m \sin\omega t - E\right)^2}{R^2}\, d(\omega t)$$

$$I_{rms}^2 = \frac{1}{2\pi R^2}\left[\left(\frac{V_m^2}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^2}{2}\sin 2\alpha - 4V_m E\cos\alpha\right] \tag{2.19}$$

$$I_{rms}^2 = 67.4$$

$$I_{rms} = 8.2 \text{ A}$$

The power rating of R is $P_R = 8.2^2 \times 4.26$ = 286.4 W.

d) The power delivered P_{dc} to the battery is

$$P_{dc} = EI_{dc} = 12 \times 5 = 60\ W$$

$$h_o P_{dc} = 100 \quad \text{or} \quad h_o = \frac{100}{P_{dc}} = \frac{100}{60} = 1.667\ h$$

e) The rectifier efficiency η is

$$\eta = \frac{\textit{power delivered to the battery}}{\textit{total input power}} = \frac{P_{dc}}{P_{dc} + P_R} = \frac{60}{60 + 286.4} = 17.32\%$$

f) The peak inverse voltage PIV of the diode is

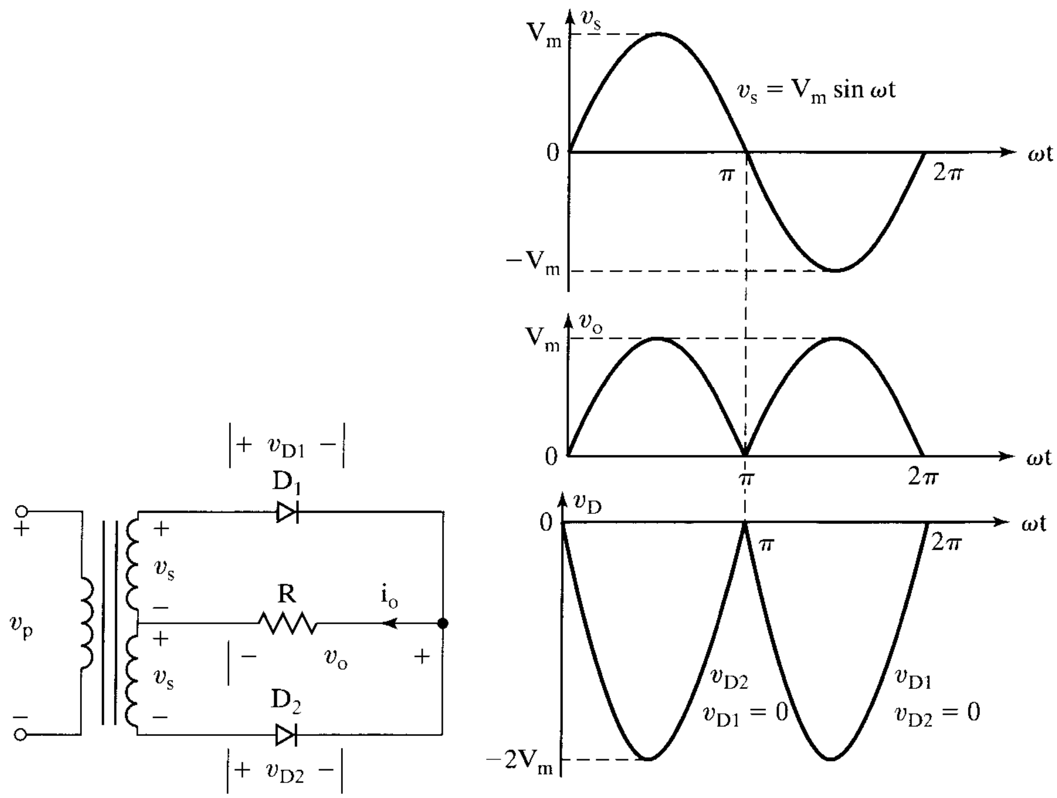
$$PIV = V_m + E = 84.85 + 12 = 96.85\ V$$

2.4 Single-Phase Full-Wave Rectifiers

A full-wave rectifier circuit with a center-tapped transformer is shown in Figure 2.5(a). Each half of the transformer with its associated diode acts as a half-wave rectifier and the output of a full-wave rectifier is shown in Figure 2.5(b). Because there is no dc current flowing through the transformer, there is no dc saturation problem of transformer core. The average output voltage is

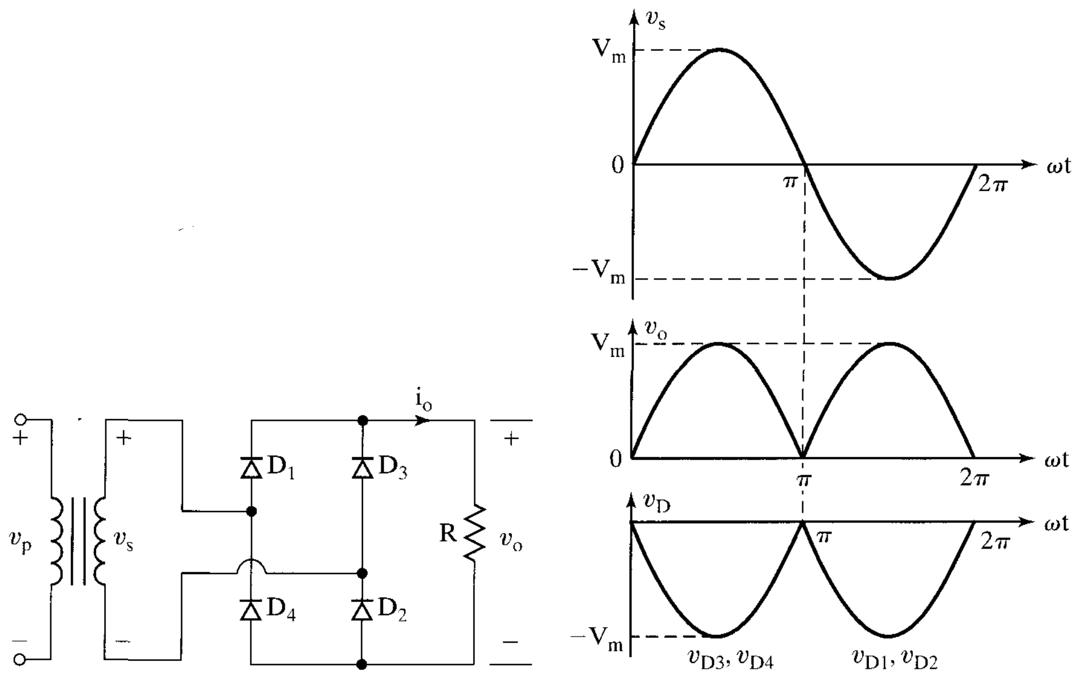
$$V_{dc} = \frac{2}{T} \int_0^{T/2} V_m \sin \omega t \, dt = \frac{2V_m}{\pi} = 0.6366\ V_m \tag{2.20}$$

Instead of using a center-tapped transformer, we could use four diodes, as shown in Figure 2.6(a). During the positive half-cycle of the input voltage, the power is supplied to the load through diodes D_1 and D_2. During the negative cycle, diodes D_3 and D_4 conduct. The waveform for the output voltage is shown in Figure 2.6(b) and is similar to that of Figure 2.5(b). The peak-inverse voltage of a diode is only V_m. This circuit is known as a bridge rectifier, and it is commonly used in industrial applications.



(a) Circuit diagram (b) Waveforms

Figure 2.5: Full-wave rectifier with center-tapped transformer



(a)Circuit diagram (b) Waveforms

Figure 2.6: Full-wave bridge rectifier

Example 2.3: Finding the Performance Parameters of a Full-Wave Rectifier with Center-Tapped Transformer

If the rectifier in Figure 2.5(a) has a purely resistive load of *R,* determine

(a) the efficiency.
(b) the FF.
(c) the RF.
(d) the TUF.
(e) the PIV of diode D_1.
(f) the CF of the input current.

Solution of Example 2.3

From Eq. (2.20), the average output voltage is

$$V_{dc} = \frac{2V_m}{\pi} = 0.6366\,V_m$$

and the average load current is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.6366V_m}{R}$$

The rms value of the output voltage is

$$V_{rms} = \left[\frac{2}{T}\int_0^{T/2}(V_m \sin \omega t)^2\, dt\right]^{1/2} = \frac{V_m}{\sqrt{2}} = 0.707\,V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707V_m}{R}$$

From Eq. (2.1), $P_{dc} = (0.6366V_m)^2/R$, and from Eq. (2.2), $P_{ac} = (0.707V_m)^2/R$

a) From Eq. (2.3), the efficiency $\eta = (0.6366V_m)^2/(0.707V_m)^2 = 81\%$.
b) From Eq. (2.5), the FF = $0.707V_m/0.6366V_m = 1.11$.
c) From Eq. (2.7), the RF = $\sqrt{1.11^2 - 1}$ = 0.482 or 48.2%.
d) The rms voltage of the transformer secondary is $V_s = V_m/\sqrt{2} = 0.707V_m$.The rms value of the transformer secondary current is Is = 0.5Vm/R. The volt-ampere rating (VA) of the transformer, $VA = 2V_sI_s = 2 \times 0.707V_m \times 0.5V_m/R$. From Eq. (2.8)

   TUF = $P_{ac}/(V_sI_s) = 0.6366^2/(2 \times 0.707 \times 0.5) = 0.5732 = 57.32\%$.

e) The peak reverse (or inverse) blocking voltage PIV = 2Vm.
f) $I_{s(peak)} = V_m/R$ and $I_s = 0.707V_m/R$. The CF of the input current is CF = $I_{s(peak)}/I_s = 1/0.707 = \sqrt{2}$.
g) The input PF for a resistive load can be found from

$$PF = \frac{P_{ac}}{VA} = \frac{0.707^2}{2 \times 0.707 \times 0.5} = 0.707$$

2.5 Single-Phase Full-Wave Rectifier With RL Load

With a resistive load, the load current is identical in shape to the output voltage. In practice, most loads are inductive to a certain extent and the load current depends on the values of load resistance *R* and load inductance L. This is shown in Figure 2.7(a). A battery of voltage E is added to develop generalized equations. If $v_s = V_m \sin \omega t = \sqrt{2}\, V_s \sin \omega t$ is the input voltage, the load current i_0 can be found from

$$L\frac{di_0}{dt} + Ri_0 + E = \sqrt{2}\, V_s \sin \omega t, \qquad \text{for } i_0 \geq 0$$

which has a solution of the form

$$i_0 = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R} \tag{2.21}$$

where load impedance $Z = [R^2 + (\omega L)^2]^{1/2}$, load impedance angle $\theta = \tan^{-1}(\omega L/R)$, and V_s is the rms value of the input voltage.

Case 1: continuous load current. This is shown in Figure 2.7(b). The constant A_1 in Eq. (2.21) can be determined from the condition: at $\omega t = \pi$, $i_0 = I_0$.

$$A_1 = \left(I_0 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin\theta\right)e^{(R/L)(\pi/\omega)}$$

Substitution of A i in Eq. (2.21) yields

$$i_0 = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + \left(I_0 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin\theta\right)e^{(R/L)(\pi/\omega - t)} - \frac{E}{R} \tag{2.22}$$

Under a steady-state condition, $i_0(\omega t = 0) = i_0(\omega t = \pi)$. That is, $i_0(\omega t = \pi) = I_0$. Applying this condition, the value of I_0

$$I_0 = \frac{\sqrt{2}V_s}{Z}\sin\theta\frac{1+e^{-(R/L)(\pi/\omega)}}{1-e^{-(R/L)(\pi/\omega)}} - \frac{E}{R} \qquad \text{for } I_0 \geq 0 \qquad (2.23)$$

which, after substituting I_0 in Eq. (2.22) and simplification, gives

$$i_0 = \frac{\sqrt{2}V_s}{Z}\left[\sin(\omega t-\theta) + \frac{2}{1-e^{-(R/L)(\pi/\omega)}}\sin\theta\, e^{-(R/L)t}\right] - \frac{E}{R},$$

$$\text{for } 0 \leq (\omega t - \theta) \leq \pi \text{ and } i_0 \geq 0 \qquad (2.24)$$

The rms diode current can be found from Eq. (2.24) as

$$I_r = \left[\frac{1}{2\pi}\int_0^{\pi} i_0^2\, d(\omega t)\right]^{1/2}$$

and the rms output current can then be determined by combining the rms current of each diode as

$$I_{rms} = \left(I_r^2 + I_r^2\right)^{1/2} = \sqrt{2}I_r$$

The average diode current can also be found from Eq. (2.24) as

$$I_d = \frac{1}{2\pi}\int_0^{\pi} i_0\, d(\omega t)$$

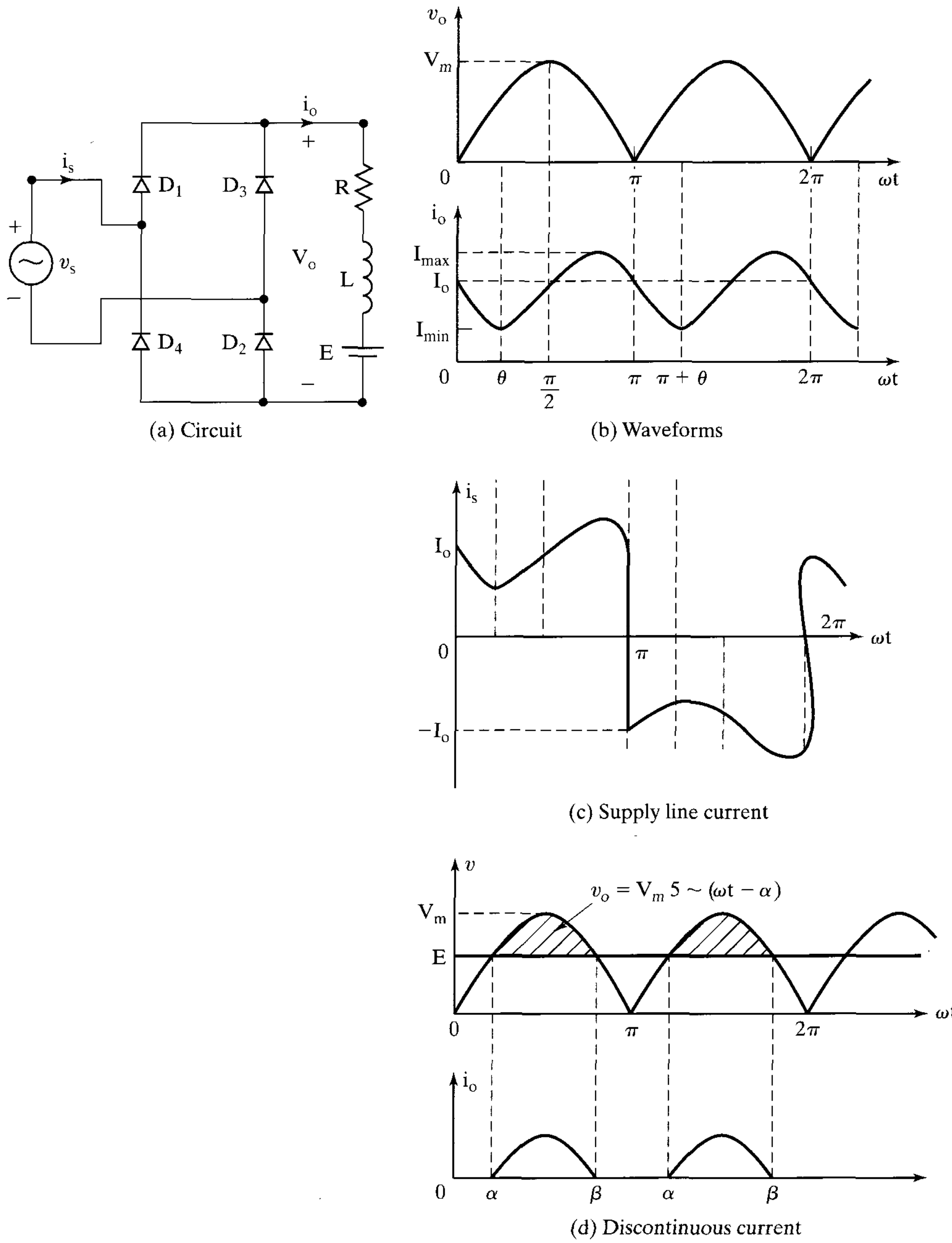


Figure 2.7: Full-bridge rectifier with RL load

Case 2: discontinuous load current. This is shown in Figure 2.7(d). The load current flows only during the period $\alpha \le \omega t \le \beta$. Let us define $x = E/V_m = E/\sqrt{2}V_s$ as the load battery (emf) constant, called the voltage ratio. The diodes start to conduct at $\omega t = \alpha$ given by

$$\alpha = \sin^{-1}\frac{E}{V_m} = \sin^{-1}(x)$$

At $\omega t = \alpha$, $i_0(\omega t) = 0$ and Eq. (2.21) gives

$$A_1 = \left[\frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\theta)\right]e^{(R/L)(\alpha/\omega)}$$

which, after substituting in Eq. (2.21), yields the load current

$$i_0 = \frac{\sqrt{2}V_s}{Z}\sin(\omega t-\theta) + \left[\frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\theta)\right]e^{(R/L)(\alpha/\omega - t)} - \frac{E}{R} \tag{2.25}$$

At $\omega t = \beta$, the current falls to zero, and $i_0(\omega t = \beta) = 0$. That is,

$$\frac{\sqrt{2}V_s}{Z}\sin(\omega t-\theta) + \left[\frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\theta)\right]e^{(R/L)(\alpha-\beta)/\omega} - \frac{E}{R} = 0 \tag{2.26}$$

Dividing Eq. (2.26) by $\sqrt{2}V_s/Z$, and substituting $R/Z = \cos\theta$ and $\omega L/R = \tan\theta$,

$$\sin(\beta-\theta) + \left(\frac{x}{\cos(\theta)} - \sin(\alpha-\theta)\right)e^{\frac{(\alpha-\beta)}{\tan(\theta)}} - \frac{x}{\cos(\theta)} = 0 \tag{2.27}$$

β can be determined from this transcendental equation by an iterative (trial and error) method of solution. Start with $\beta = 0$, and increase its value by a very small amount until the left-hand side of this equation becomes zero.

The rms diode current can be found from Eq. (2.25) as

$$I_r = \left[\frac{1}{2\pi}\int_0^{\beta} i_0^2\, d(\omega t)\right]^{1/2}$$

The average diode current can also be found from Eq. (2.25) as

$$I_d = \frac{1}{2\pi}\int_0^{\beta} i_0\, d(\omega t)$$

Boundary conditions: The condition for the discontinuous current can be found by setting I_0 in Eq. (2.23) to zero.

$$0 = \frac{V_s\sqrt{2}}{Z}\sin(\theta)\left[\frac{1+e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}{1-e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}\right] - \frac{E}{R}$$

which can be solved for the voltage ratio $x = E/(\sqrt{2}V_s)$ as

$$x(\theta) := \left[\frac{1+e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}{1-e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}\right]\sin(\theta)\cos(\theta) \tag{2.28}$$

The plot of the voltage ratio x against the load impedance angle θ is shown in Figure 2.8. The load angle θ cannot exceed π/2. The value of x is 63.67% at θ = 1.5567 rad, 43.65% at θ = 0.52308 rad (30°) and 0% at θ = 0.

Example 2.4: Finding the Performance Parameters of a Full-Wave Rectifier with an *RL* Load

The single-phase full-wave rectifier of Figure 2.7(a) has L = 6.5 mH, R = 2.5 H, and E = 10 V. The input voltage is V_s = 120 V at 60 Hz.

(a) Determine

(1) the steady-state load current I_0 at ωt = 0,

(2) the average diode current I_d.

(3) the rms diode current I_r.

(4) the rms output current I_{rms}.

(b) Use PSpice to plot the instantaneous output current i_0.

Assume diode parameters IS = 2.22E - 15, BV = 1800 V.

Solution of Example 3.4

It is not known whether the load current is continuous or discontinuous. Assume that the load current is continuous and proceed with the solution. If the assumption is not correct, the load current is zero and then moves to the case for a discontinuous current.

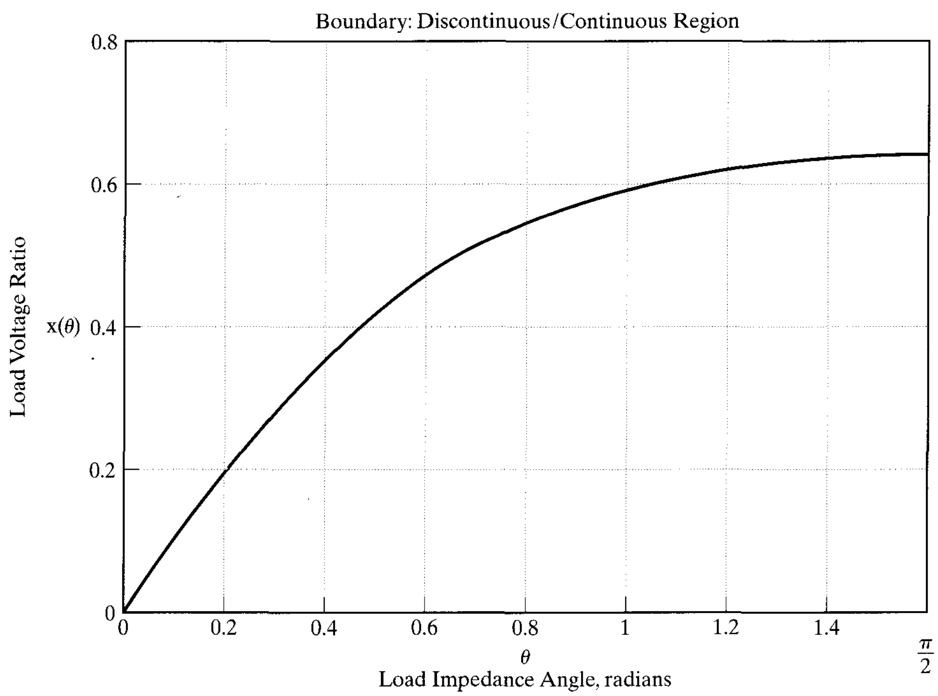


Figure 2.8: Boundary of continuous and discontinuous regions for single-phase rectifier

a. R = 2.5 Ω, L = 6.5 mH, f = 60 Hz, ω = 2π × 60 = 377 rad/s, V_s = 120 V, $Z = [R^2 + (\omega L)^2]^{1/2}$ = 3.5 Ω, and $\theta = \tan^{-1}(\omega L/R)$ = 44.43°.

(1) The steady-state load current at $\omega t = 0$, $I_0 = 32.8$ A. Because $I_0 > 0$, the load current is continuous and the assumption is correct.
(2) The numerical integration of i_0 in Eq. (2.24) yields the average diode current as $I_d = 19.61$ A.
(3) By numerical integration of i_0^2 between the limits $\omega t = 0$ and π, we get the rms diode current as $I_r = 28.5$ A.
(4) The rms output current $I_{rms} = \sqrt{2}I_r = \sqrt{2} \times 28.50 = 40.3$ A.

2.6 Three-Phase Bridge Rectifiers

A three-phase bridge rectifier is commonly used in high-power applications and it is shown in Figure 2.10. This is a full-wave rectifier. It can operate with or without a transformer and gives six-pulse ripples on the output voltage. The diodes are numbered in order of conduction sequences and each one conducts for 120°. The conduction sequence for diodes is $D_1 - D_2$, $D_3 - D_2$, $D_3 - D_4$, $D_5 - D_4$, $D_5 - D_6$ and $D_1 - D_6$. The pair of diodes which are connected between that pair of supply lines having the highest amount of instantaneous line-to-line voltage will conduct. The line-to-line voltage is $\sqrt{3}$ times the phase voltage of a three-phase Y-connected source. The waveforms and conduction times of diodes are shown in Figure 2.11. If V_m is the peak value of the phase voltage, then the instantaneous phase voltages can be described by

$$v_{an} = V_m \sin(\omega t), \qquad v_{bn} = V_m \sin(\omega t - 120^o), \qquad v_{cn} = V_m \sin(\omega t - 240^o)$$

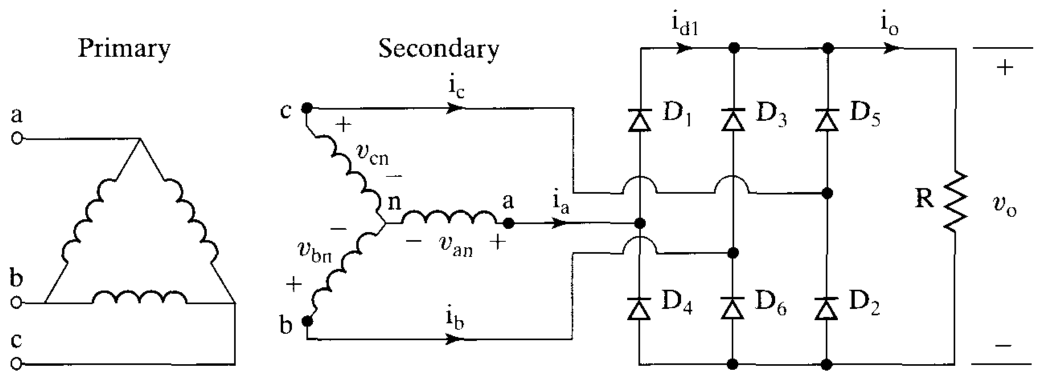


Figure 2.10: Three-phase bridge rectifier

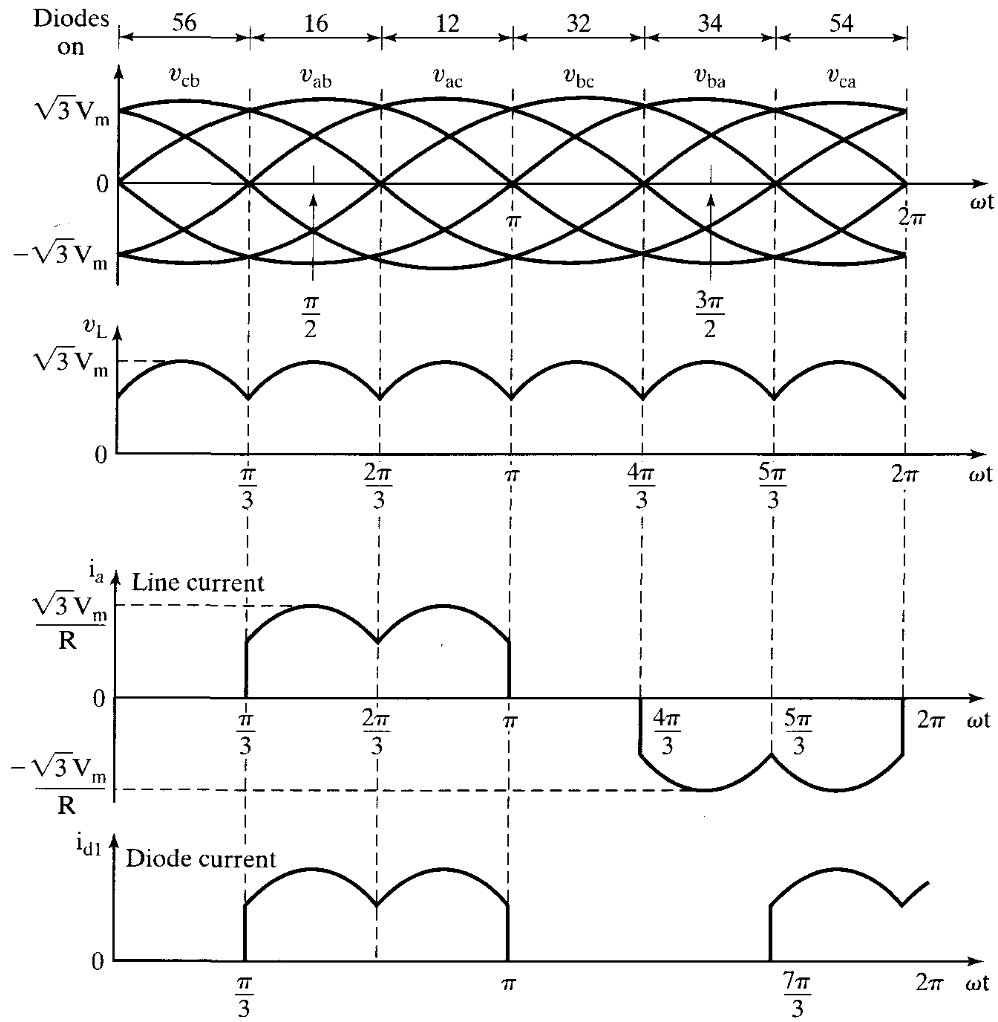


Figure 2.11: Waveforms and conduction times of diodes

Because the line-line voltage leads the phase voltage by 30°, the instantaneous line-line voltages can be described by

$v_{ab} = \sqrt{3}V_m \sin(\omega t + 30^o)$,

$v_{bc} = \sqrt{3}V_m \sin(\omega t - 90^o)$,

$v_{ca} = \sqrt{3}V_m \sin(\omega t - 210^0)$

The average output voltage is found from

$$V_{dc} = \frac{2}{2\pi/6}\int_0^{\pi/6}\sqrt{3}V_m \cos\omega t \, d(\omega t)$$

$$= \frac{3\sqrt{3}}{\pi}V_m \qquad (2.29)$$

$$= 1.654V_m$$

where V_m, is the peak phase voltage.

The rms output voltage is

$$V_{rms} = \left[\frac{2}{2\pi/6}\int_0^{\pi/6} 3V_m^2 \cos^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$= \left(\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\right)^{1/2} V_m \qquad (2.30)$$

$$= 1.6554V_m$$

If the load is purely resistive, the peak current through a diode is $I_m = \sqrt{3}V_m/R$ and the rms value of the diode current is

$$I_r = \left[\frac{4}{2\pi}\int_0^{\pi/6} I_m^2 \cos^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$= I_m\left[\frac{1}{\pi}\left(\frac{\pi}{6} + \frac{1}{2}\sin\frac{2\pi}{6}\right)\right]^{1/2} \qquad (2.32)$$

$$= 0.5518I_m$$

and the rms value of the transformer secondary current,

$$I_s = \left[\frac{8}{2\pi}\int_0^{\pi/6} I_m^2 \cos^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$= I_m\left[\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{1}{2}\sin\frac{2\pi}{6}\right)\right]^{1/2} \qquad (2.33)$$

$$= 0.7804I_m$$

where I_m is the peak secondary line current.

Example 2.5: Finding the Performance Parameters of a Three-Phase Bridge Rectifier

A three-phase bridge rectifier has a purely resistive load of R. Determine
(a) the efficiency.
(b) the FF.
(c) the RF.
(d) the TUF.
(e) the peak inverse (or reverse) voltage (PIV) of each diode.
(f) the peak current through a diode.
The rectifier delivers I_{dc} = 60 A at an output voltage of V_{dc} = 280.7 V and the source frequency is 60 Hz.

Solution of Example 2.5

a. From Eq. (2.29), $V_{dc} = 1.654V_m$ and $I_{dc} = 1.654V_m/R$.
   From Eq. (2.30), $V_{rms} = 1.6554V_m$, and $I_{rms} = 1.6554V_m/R$.
   From Eq. (2.1), $P_{dc} = (1.654V_m)^2/R$.
   From Eq. (2.2), $P_{ac} = (1.6554\ V_m)^2/R$.
   From Eq. (2.3) the efficiency

$$\eta = \frac{(1.654V_m)^2}{(1.6554V_m)^2} = 99.83\%$$

b. From Eq. (2.5), the FF = 1.6554/1.654 = 1.0008 = 100.08%.

c. From Eq. (2.6), the RF = $\sqrt{1.0008^2 - 1}$ = 0.04 = 4%.

d. From Eq. (2.15), the rms voltage of the transformer secondary, $V_s = 0.707V_m$.
From Eq. (2.33), the rms current of the transformer secondary,

$$I_s = 0.7804I_m = 0.7804 \times \sqrt{3}\frac{V_m}{R}$$

The VA rating of the transformer,

$$VA = 3V_sI_s = 3 \times 0.707I_m \times 0.7804 \times \sqrt{3}\frac{V_m}{R}$$

From Eq. (2.8),

$$TUF = \frac{1.654^2}{3 \times \sqrt{3} \times 0.707 \times 0.7804} = 0.9542$$

e. From Eq. (2.29), the peak line-to-neutral voltage is V_m = 280.7/1.654 = 169.7 V. The peak inverse voltage of each diode is equal to the peak value of the secondary line-to-line voltage, PIV = $\sqrt{3}V_m = \sqrt{3} \times 169.7$ = 293.9 V.

f. The average current through each diode is

$$I_d = \frac{4}{2\pi}\int_0^{\pi/6} I_m \cos\omega t \, d(\omega t) = I_m \frac{2}{\pi}\sin\frac{\pi}{6} = 0.3183I_m$$

The average current through each diode is I_d = 60/3 = 20 A; therefore, the peak current is I_m = 20/0.3183 = 62.83 A.

2.7 Three-Phase Bridge Rectifier With RL Load

Equations that are derived in Section 2.5 can be applied to determine the load current of a three-phase rectifier with an *RL* load (similar to Figure 2.12). It can be noted from Figure 2.11 that the output voltage becomes

$v_{ab} = \sqrt{2}\, V_{ab} \sin \omega t$ for $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$

where V_{ab} is the line-to-line rms input voltage. The load current i_0 can be found from

$$L\frac{di_0}{dt} + Ri_0 + E = \sqrt{2}V_{ab}\sin\omega t$$

which has a solution of the form

$$i_0 = \frac{\sqrt{2}V_{ab}}{Z}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R} \qquad (2.34)$$

where load impedance $Z = \left[R^2 + (\omega L)^2\right]^{1/2}$ and load impedance angle = tan^{-1} (ωL/R). The constant A_1 in Eq. (2.34) can be determined from the condition: at $\omega t = \pi/3$, $i_0 = I_0$.

$$A_1 = \left[I_0 + \frac{E}{R} - \frac{\sqrt{2}V_{ab}}{Z} \sin\left(\frac{\pi}{3} - \theta\right) \right] e^{(R/L)(\pi/3\omega)}$$

Substitution of A_1 in Eq. (3.45) yields

$$i_0 = \frac{\sqrt{2}V_{ab}}{Z} \sin(\omega t - \theta) + \left[I_0 + \frac{E}{R} - \frac{\sqrt{2}V_{ab}}{Z} \sin\left(\frac{\pi}{3} - \theta\right) \right] e^{(R/L)(\pi/3\omega - t)} - \frac{E}{R} \qquad (2.35)$$

Under a steady-state condition, $i_0(\omega t = 2\pi/3) = i_0(\omega t = \pi/3)$. That is, $i_0(\omega t = 2\pi/3) = I_0$. Applying this condition, we get the value of I_0 as

$$I_0 = \frac{\sqrt{2}V_{ab}}{Z} \frac{\sin(2\pi/3 - \theta) - \sin(\pi/3 - \theta) e^{-(R/L)(\pi/3\omega)}}{1 - e^{-(R/L)(\pi/3\omega)}} - \frac{E}{R} \quad \text{for } I_0 \geq 0 \qquad (2.36)$$

which, after substitution in Eq. (2.35) and simplification, gives

$$i_0 = \frac{\sqrt{2}V_{ab}}{Z} \left[\sin(\omega t - \theta) + \frac{\sin(2\pi/3 - \theta) - \sin(\pi/3 - \theta)}{1 - e^{-(R/L)(\pi/3\omega - t)}} e^{(R/L)(\pi/3\omega - t)} \right] - \frac{E}{R}$$

for $\pi/3 \leq \omega t \leq 2\pi/3$ and $i_0 \geq 0$ (2.37)

The rms diode current can be found from Eq. (2.37) as

$$I_r = \left[\frac{2}{2\pi} \int_{\pi/3}^{2\pi/3} i_0^2 \, d(\omega t) \right]^{1/2}$$

and the rms output current can then be determined by combining the rms current of each diode as

$$I_{rms} = \left(I_r^2 + I_r^2 + I_r^2 \right)^{1/2} = \sqrt{3} I_r$$

The average diode current can also be found from Eq. (2.36) as

$$I_d = \frac{2}{2\pi} \int_{\pi/3}^{2\pi/3} i_0 \, d(\omega t)$$

Boundary conditions: The condition for the discontinuous current can be found by setting I_0 in Eq. (2.36) to zero.

$$\frac{\sqrt{2}V_{AB}}{Z} \left[\frac{\sin\left(\frac{2\pi}{3} - \theta\right) - \sin\left(\frac{\pi}{3} - \theta\right) e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{3\omega}\right)}}{1 - e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{3\omega}\right)}} \right] - \frac{E}{R} = 0$$

which can be solved for the voltage ratio $x = E/\left(\sqrt{2}V_{AB}\right)$ as

$$x(\theta) := \left[\frac{\sin\left(\frac{2\pi}{3} - \theta\right) - \sin\left(\frac{\pi}{3} - \theta\right) e^{-\left(\frac{\pi}{3\tan(\theta)}\right)}}{1 - e^{-\left(\frac{\pi}{3\tan(\theta)}\right)}} \right] \cos(\theta) \qquad (2.38)$$

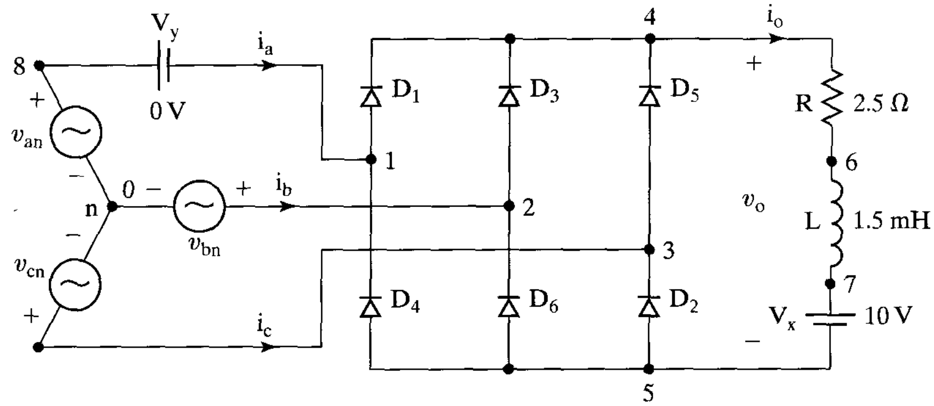
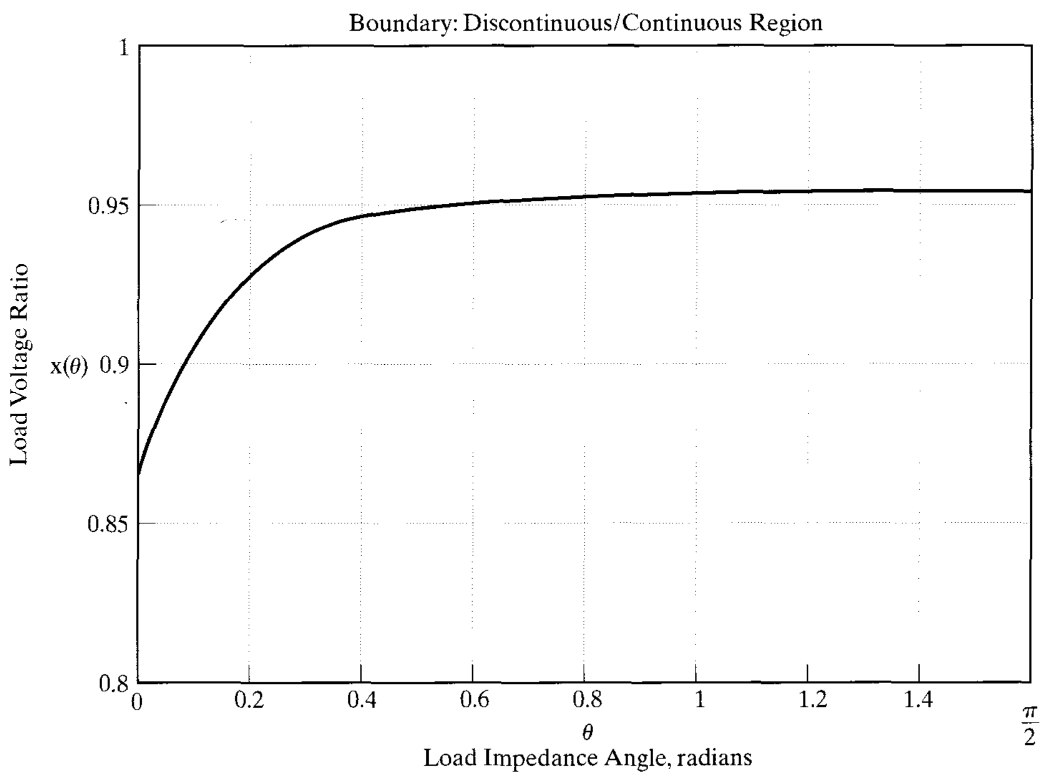


Figure 2.12: Three-phase bridge rectifier for PSpice simulation



2.13: Boundary of continuous and discontinuous for three-phase rectifier

2.8 Principle of Phase-controlled Converter Operation – Single-phase Half-wave Controlled Converter

During the positive half-cycle of input voltage, the thyristor anode is positive with respect to its cathode and the thyristor is said to be forward biased. When thyristor T_1 is fired at $\omega t = \alpha$, thyristor T_1 conducts and the input voltage appears across the load. When the input voltage starts to be negative at $\omega t = \pi$, the thyristor is negative with respect to its cathode and thyristor T_1 said to be reverse biased and it is turned off. The time after the input voltage starts to go negative until the thyristor is fired at $\omega t = \alpha$ is called the delay or firing angle α.

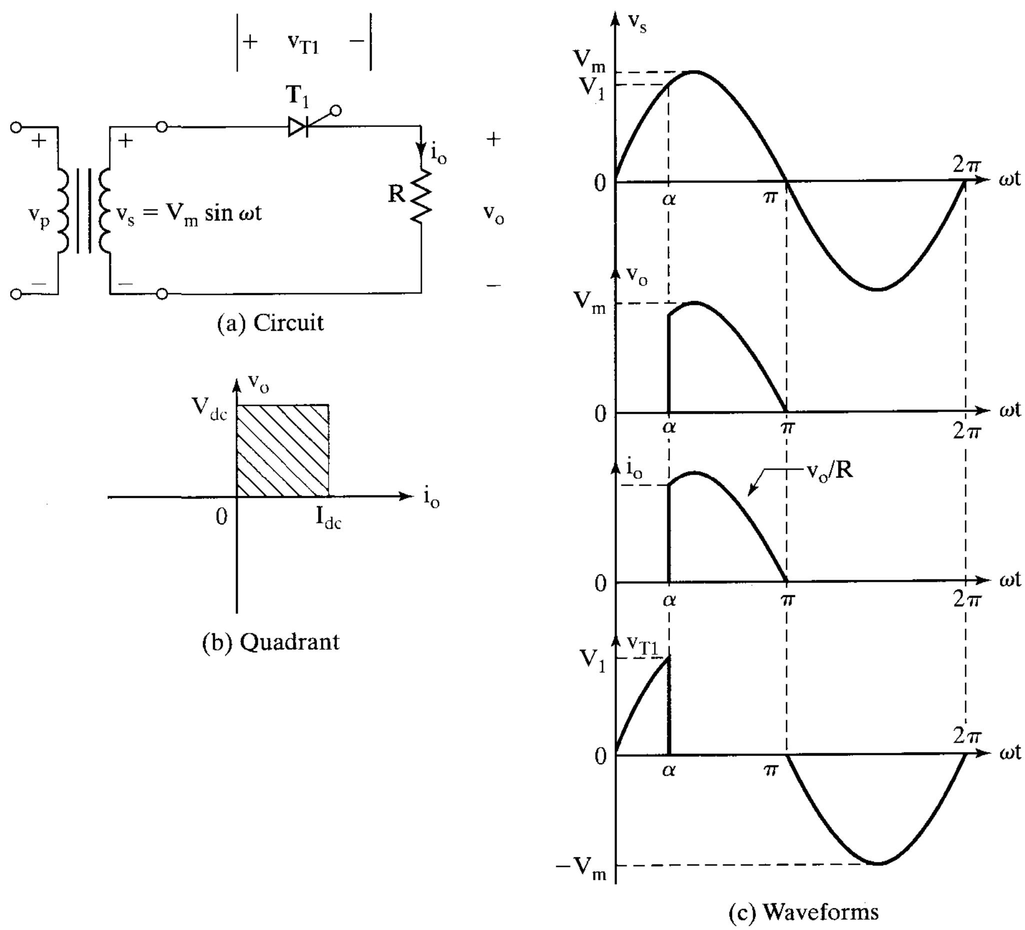


Figure 2.14: Single-phase thyristor converter with a resistive load

Figure 2.14(b) shows the region of converter operation, where the output voltage and current have one polarity. Figure 2.14(c) shows the waveforms for input voltage, output voltage, load current, and voltage across T_1. This converter is not normally used in industrial applications because its output has high ripple content and low ripple frequency. However, it explains the principle of the single-phase thyristor converter. If f_s is the frequency of input supply, the lowest frequency of output ripple voltage is f_s.

If V_m is the peak input voltage, the average output voltage V_{dc} can be found from

$$V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t d(\omega t) = \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi} = \frac{V_m}{2\pi}\left(1+\cos\alpha\right) \qquad (2.39)$$

and V_{dc} can be varied from V_m/π to 0 by varying *a* from 0 to π. The average output voltage becomes maximum when a = 0 and the maximum output voltage V_{dm} is

$$V_{dm} = \frac{V_m}{\pi} \qquad (2.40)$$

Normalizing the output voltage with respect to V_{dm}, the normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right) \qquad (2.41)$$

The root-mean-square (rms) output voltage is given by

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d(\omega t)\right]^{1/2} = \left[\frac{V_m^2}{4\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)d(\omega t)\right]^{1/2} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2} \qquad (2.42)$$

Gating sequence. The gating sequence for the thyristor switch is as follows:

1. Generate a pulse-signal at the positive zero crossing of the supply voltage v_s.
2. Delay the pulse by the desired angle α and apply it between the gate and cathode terminals of T_1 through a gate-isolating circuit.

Example 2.6: Finding the Performances of a Single-Phase Controlled Converter

If the converter of Figure 2.14(a) has a purely resistive load of *R* and the delay angle is α = π/2. Determine

(a) the rectification efficiency.
(b) the form factor (FF).
(c) the ripple factor (RF).
(d) the TUF.
(e) the peak inverse voltage (PIV) of thyristor T_1.

Solution of Example 2.6

The delay angle α = π/2. From Eq. (2.39), V_{dc} = 0.1592 V_m and I_{dc} = 0.1592V_m/R.
Form Eq. (2.41), V_n = 0.5,
From Eq. (2.42) V_{rms} = 0.3536V_m and I_{rms} = 0.3536V_m/R.
From Eq. (2.1) $P_{dc} = V_{dc}I_{dc} = (0.1592V_m)^2/R$
From Eq. (2.2) $P_{ac} = V_{rms}I_{rms} = (0.3536V_m)^2/R$

a. From Eq. (2.3) the rectification efficiency

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{(0.1592V_m)^2}{(0.3536V_m)^2} = 20.27\%$$

b. From Eq. (2.3), The FF

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{0.3536V_m}{0.1592V_m} = 2.221 \quad or \quad 222.1\%$$

c. From Eq. (2.3), the RF

$$RF = \sqrt{FF^2 - 1} = \left(2.221^2 - 1\right)^{1/2} = 1.983 \quad or \quad 198.3\%$$

d. The rms voltage of the transformer secondary

$$V_s = V_m / \sqrt{2} = 0.707V_m$$

The rms voltage of the transformer secondary current is the same as that of the load

$$I_s = 0.3536V_m / R$$

The volt-ampere rating (VA) of the transformer,

$$VA = V_s I_s = 0.707V_m \times 0.3536V_m / R$$

From Eq. (2.8),

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{0.1592^2}{0.707 \times 0.3536} = 0.1014 \quad and \quad \frac{1}{TUF} = 9.86$$

The PF is approximately equal to TUF. Thus PF = 0.1014

e. The PIV = V_m.

2.9 Single-Phase Full-wave Controlled Converter

The circuit arrangement of a single-phase full converter is shown in Figure 2.15(a) with a highly inductive load so that the load current is continuous and ripples free. During the positive half-cycle, thyristors T_1 and T_2 are forward biased; and when these two thyristors are fired simultaneously at $\omega t = \alpha$, the load is connected to the input supply through T_1 and T_2. Due to the inductive load, thyristors T_1 and T_2 continue to conduct beyond $\omega t = \pi$, even though the input voltage is already negative. During the negative half-cycle of the input voltage, thyristors T_1 and T_2 are forward biased; and firing of thyristors T_1 and T_2 applies the supply voltage across thyristors T_1 and T_2 as reverse blocking voltage. T_1 and T_2 are turned off due to line or natural commutation and the load current is transferred from T_1 and T_2 to T_3 and T_4. Figure 2.15(b) shows the regions of converter operation and Figure 2.15(c) shows the waveforms for input voltage, output voltage, and input and output currents.

During the period from α to π, the input voltage v_s and input current i_s are positive and the power flows from the supply to the load. The converter is said to be operated in rectification mode. During the period from π to $\pi + \alpha$, the input voltage v_s is negative and the input current i_s, is positive and reverse power flows from the load to the supply. The converter is said to be operated in inversion mode. This converter is extensively used in industrial applications. Depending on the value of α*,* the average output voltage could be either positive or negative and it provides two-quadrant operation.

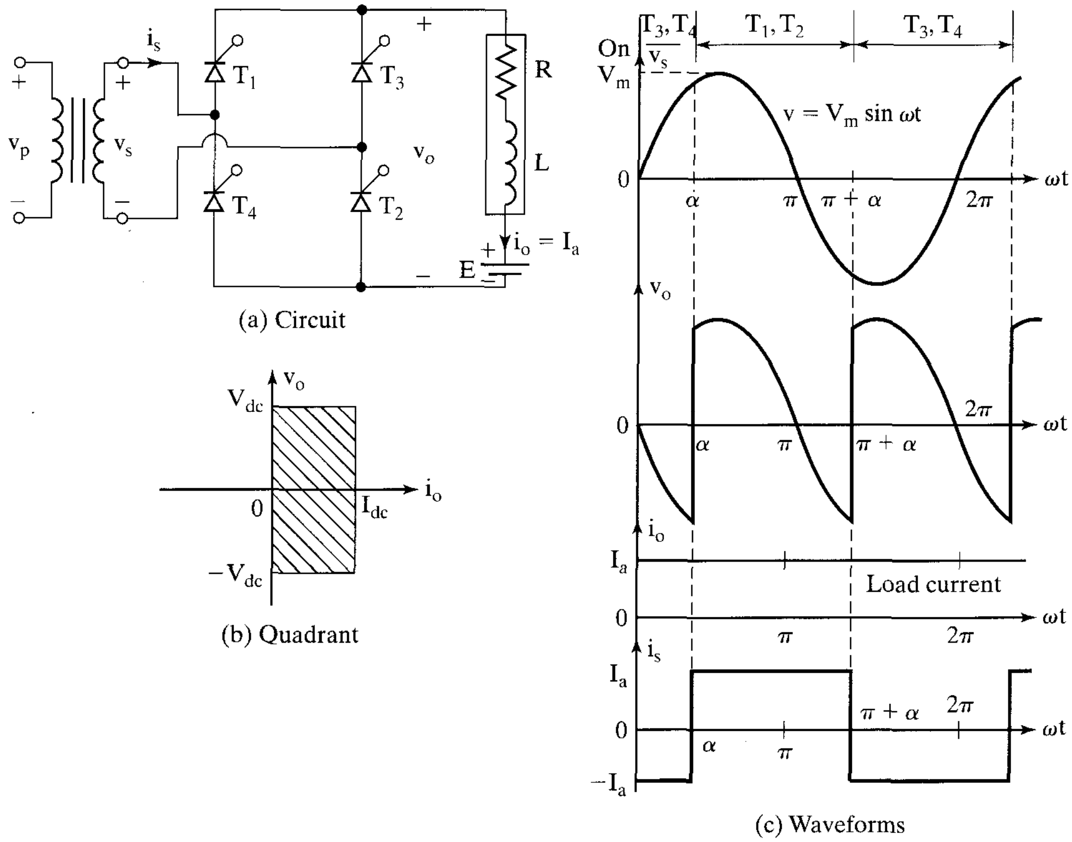


Figure 2.15: Single-phase Full-wave Converter

The average output voltage can be found from

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\, d(\omega t) = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha} = \frac{2V_m}{\pi}\cos\alpha \qquad (2.43)$$

and V_{dc} can be varied from $2V_m/\pi$ to $-2V_m/\pi$ by varying α from 0 to π. The maximum average output voltage is $V_{dm} = 2V_m/\pi$ and the normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha \qquad (2.44)$$

The rms value of the output voltage is given by

$$V_{rms} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi+\alpha} (1-\cos 2\omega t)\, d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}} = V_s \quad (2.45)$$

With a purely resistive load, thyristors T_1 and T_2 can conduct from α to π, and thyristors T_3 and T_4 can conduct from $\alpha + \pi$ to 2π.

2.9.1 Single-Phase Full-wave Controlled Converter with RL Load

The operation of the converter in Figure 2.15(a) can be divided into two identical modes: mode 1 when T_1 and T_2 conduct and mode 2 when T_3 and T_4 conduct. The output currents during these modes are similar and we need to consider only one mode to find the output current i_L.

Mode 1 is valid for $\alpha \leq \omega t \leq (\alpha + \pi)$. If v_s, $= \sqrt{2}$ Vs sin ωt is the input voltage, the load current i_L during mode 1 can be found from

$$L\frac{di_L}{dt} + Ri_L + E = \sqrt{2}V_s \sin \omega t \qquad \text{for} \quad i_L \geq 0$$

whose solution is of the form

$$i_L = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R} \qquad \text{for} \quad i_L \geq 0$$

where load impedance $Z = [R^2 + (\omega L)^2]^{1/2}$ and load angle $\theta = \tan^{-1}(\omega L/R)$. Constant A_1, which can be determined from the initial condition: at $\omega t = \alpha$, $i_L = I_{L0}$ is found as

$$A_1 = \left[I_{L0} + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha - \theta)\right]e^{(R/L)(\alpha/\omega)}$$

Substitution of A_1gives $i_L$as

$$i_L = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) - \frac{E}{R} + \left[I_{L0} + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha - \theta)\right]e^{(R/L)(\alpha/\omega - t)} \qquad (2.46)$$

At the end of mode 1 in the steady-state condition $i_L(\omega t = \pi + \alpha) = I_{L1} = I_{L0}$. Applying this condition to Eq. (2.46) and solving for I_{L0} get

$$I_{L0} = I_{L1} = \frac{\sqrt{2}V_s}{Z}\,\frac{-\sin(\alpha - \theta) - \sin(\alpha - \theta)e^{-(R/L)(\pi)/\omega}}{1 - e^{-(R/L)(\pi/\omega)}} - \frac{E}{R} \qquad \text{for} \quad I_{L0} \geq 0 \qquad (2.47)$$

The critical value of α at which I_0 becomes zero can be solved for known values of θ, R, L, E, and V_s, by an iterative method. The rms current of a thyristor can be found from Eq. (2.46) as

$$I_R = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} i_L^2\, d(\omega t)\right]^{1/2}$$

The rms output current can then be determined from

$$I_{rms} = \left(I_R^2 + I_R^2\right)^{1/2} = \sqrt{2}\, I_R$$

The average current of a thyristor can also be found from Eq. (2.46) as

$$I_A = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} i_L\, d(\omega t)$$

The average output current can be determined from

$$I_{dc} = I_A + I_A = 2I_A$$

Discontinuous load current. The critical value of α_c at which I_{L0} becomes zero can be solved. Dividing Eq. (2.47) by $\sqrt{2}V_s/Z$, and substituting R/Z = cos θ and ωL/R = tan θ, get

$$0 = \frac{V_s\sqrt{2}}{Z}\sin(\alpha-\theta)\left[\frac{1+e^{-(R/L)(\pi/\omega)}}{1-e^{-(R/L)(\pi/\omega)}}\right]+\frac{E}{R}$$

which can be solved for the critical value of a as

$$\alpha_c = \theta - \sin^{-1}\left[\frac{1-e^{-(\pi/\tan(\theta))}}{1+e^{-(\pi/\tan(\theta))}}\frac{x}{\cos(\theta)}\right] \tag{2.48}$$

where $x = E/\sqrt{2}V_s$ is the voltage ratio, and θ is the load impedance angle. For $\alpha \geq \alpha_c$, $I_{L0} = 0$. The load current that is described by Eq. (2.46) flows only during the period, $\alpha \leq \omega t \leq \beta$. At ωt = ω, the load current falls to zero again.

Gating sequence. The gating sequence is as follows:

1. Generate a pulse signal at the positive zero crossing of the supply voltage v_s. Delay the pulse by the desired angle α and apply the same pulse between the gate and cathode terminals of T_1 and T_2 through gate-isolating circuits.
2. Generate another pulse of delay angle α + π and apply the same pulse between the gate and source terminals of T_3 and T_4 through gate-isolating circuits.

Example 2.7: Finding the Current Ratings of Single-Phase Controlled Full Converter with an *RL* load

The single-phase full converter of Figure 215(a) has a *RL* load having L = 6.5 mH, *R* = 0.5 Ω, and *E* = 10 V. The input voltage is V_s = 120 V at (rms) 60 Hz. Determine

(a) the load current I_{L0} at ωt = α = 60°.
(b) the average thyristor current I_A.
(c) the rms thyristor current I_R.
(d) the rms output current I_{rms}.
(e) the average output current I_{dc}.
(f) the critical delay angle α_c.

Solution of Example 2.7

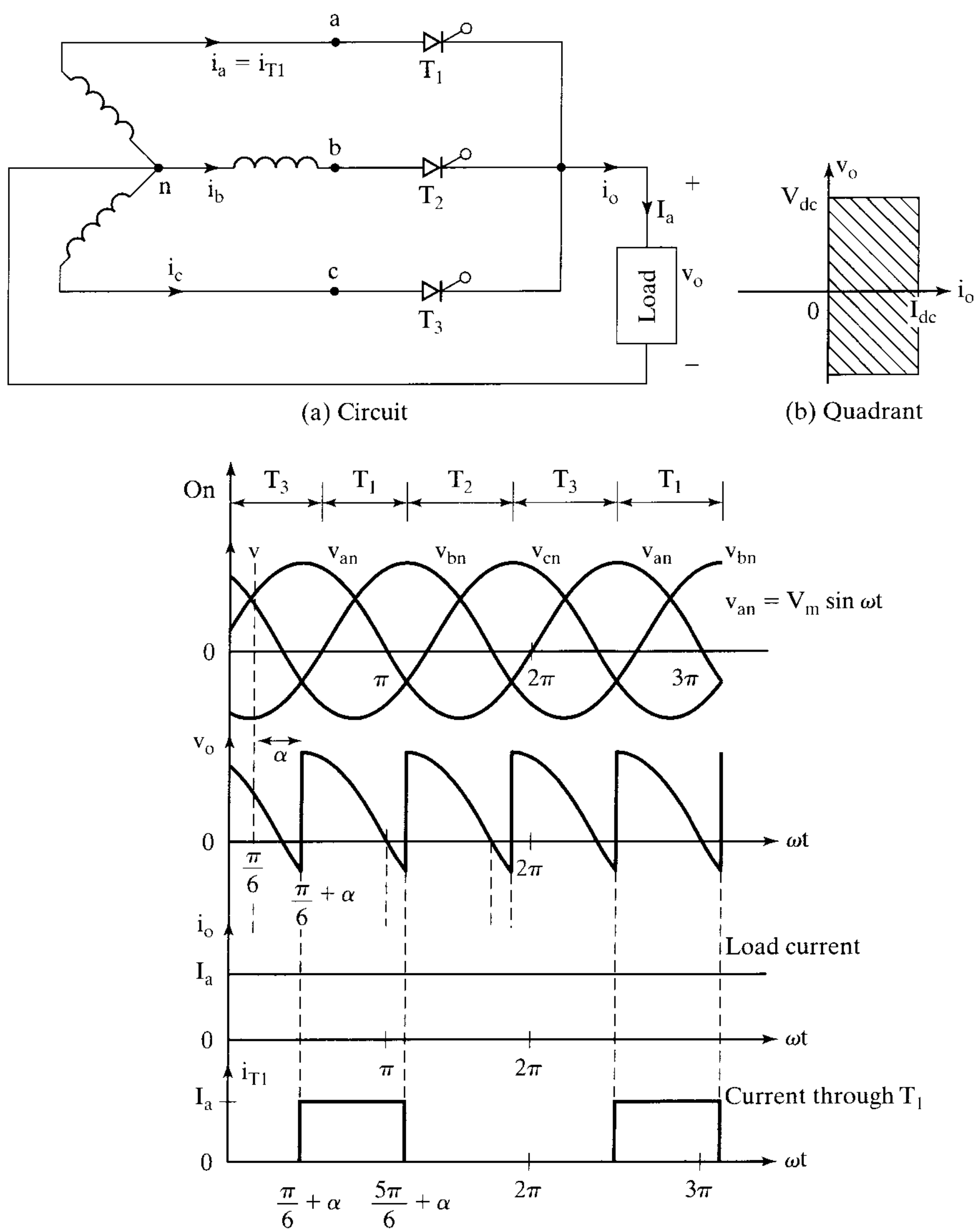
α = 60°, R = 0.5 Ω, L = 6.5 mH, *f* = 60 Hz, ω = 2π × 60 = 377 rad/s, V_s = 120V, and $\theta = \tan^{-1}(\omega L/R) = 78.47°$.

a. The steady-state load current at ωt = α, I_{L0} = 49.34 A.
b. The numerical integration of i_L in Eq. (2.46) yields the average thyristor current as I_A = 44.05 A.
c. By numerical integration of i_L^2 between the limits ωt = α to π + α, get the rms thyristor current as I_R = 63.71 A.
d. The rms output current $I_{rms} = \sqrt{2}I_R = \sqrt{2} \times 63.71 = 90.1$ A.

e. The average output current $I_{dc} = 2\ I_A = 2 \times 44.04 = 88.1$ A.
   From Eq. (2.48), by iteration we find the critical delay angle $\alpha_c = 73.23°$.

2.10 Principle of Three-phase Half-wave Controlled Converter

Three-phase converters provide higher average output voltage and in addition the frequency of the ripples on the output voltage is higher compared with that of single-phase converters. As a result, the filtering requirements for smoothing out the load current and load voltage are simpler. For these reasons, three-phase converters are used extensively in high-power variable-speed drives. Three single-phase half-wave converters in Figure 2.14(a) can be connected to form a three-phase half-wave converter, as shown in Figure 2.16(a).



(c) For inductive load

Figure 2.16: Three-phase half-wave converter

When thyristor T_1 is fired at $\omega t = \pi/6 + \alpha$, the phase voltage v_{an} appears across the load until thyristor T_2 is fired at $\omega t = 5\pi/6 + \alpha$. When thyristor T_2 is fired, thyristor T_1 is reverse biased, because the line-to-line voltage, v_{an} $(=v_{an} - v_{bn})$, is negative and T_1 is turned off. The phase voltage v_{bn} appears across the load until thyristor T_3 is fired at $\omega t = 3\pi/2 + \alpha$. When thyristor T_3, is fired, T_2 is turned off and v_{cn} appears across the load until T_1 is fired again at the beginning of next cycle. Figure 2.16(b) shows the $v - i$ characteristics of the load and this is a two-quadrant converter. Figure 2.16(c) shows the input voltages, output voltage, and the current through thyristor T_1 for a highly inductive load. For a resistive load and $\alpha > \pi/6$, the load current would be discontinuous and each thyristor is self-commutated when the polarity of its phase voltage is reversed. The frequency of output ripple voltage is $3f_s$. This converter is not normally used in practical systems, because the supply currents contain dc components. However, this converter explains the principle of the three-phase thyristor converter.

If the phase voltage is $v_{an} = V_m \sin \omega t$ average output voltage for a continuous load current is

$$V_{dc} = \frac{3}{2\pi}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin \omega t\, d(\omega t) = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha \tag{2.49}$$

where V_m is the peak phase voltage. The maximum average output voltage that occurs at delay angle, $\alpha = 0$ is

$$V_{dm} = \frac{3\sqrt{3}V_m}{2\pi}$$

and the normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha \tag{2.50}$$

The rms output voltage is found from

$$V_{rms} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \sqrt{3}V_m\left(\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos\alpha\right)^{1/2} \tag{2.51}$$

For a resistive load and $\alpha \geq \pi/6$:

$$V_{dc} = \frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi} V_m \sin \omega t\, d(\omega t) = \frac{\sqrt{3}V_m}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right] \tag{2.52}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{\sqrt{3}}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right] \tag{2.53}$$

$$V_{rms} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi} V_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \sqrt{3}V_m\left[\frac{5}{24}-\frac{\alpha}{4\pi}+\frac{1}{8\pi}\sin\left(\frac{\pi}{3}+2\alpha\right)\right]^{1/2} \tag{2.54}$$

Gating sequence.

The gating sequence is as follows:

1. Generate a pulse signal at the positive zero crossing of the phase voltage v_{an}. Delay the pulse by the desired angle $\alpha + \pi/6$ and apply it to the gate and cathode terminals of T_1 through a gate-isolating circuit.
2. Generate two more pulses of delay angles $\alpha + 5\pi/6$ and $\alpha + 9\pi/6$ for gating T_2 and T_3, respectively, through gate-isolating circuits.

Example 2.8 Finding the Performances of a Three-Phase Controlled Half-Wave Converter

A three-phase half-wave converter in Figure 2.16(a) is operated from a three-phase Y-connected 208-V, 60-Hz supply and the load resistance is $R = 10\ \Omega$. If it is required to obtain an average output voltage of 50% of the maximum possible output voltage, calculate

(a) the delay angle α.
(b) the rms and average output currents.
(c) the average and rms thyristor currents.
(d) the rectification efficiency.
(e) the TUF.
(f) the input PF.

Solution of Example 2.8

The phase voltage is V, = 208/√3 = 120.1 V, $V_m = \sqrt{2}V_s = 169.83$ V, $V_n = 0.5$, and R = 10 Ω. The maximum output voltage is

$$V_{dm} = \frac{3\sqrt{3}V_m}{2\pi} = 3\sqrt{3} \times \frac{169.83}{2\pi} = 140.45\,\text{V}$$

The average output voltage, $V_{dc} = 0.5 \times 140.45 = 70.23$ V.

a. For a resistive load, the load current is continuous if $\alpha \le \pi/6$ and Eq. (2.50) gives $V_n \ge \cos(\pi/6) = 86.6\%$. With a resistive load and 50% output, the load current is discontinuous. From Eq. (2.53), $0.5 = (1/\sqrt{3})[1 + \cos(\pi/6 + \alpha)]$, which gives the delay angle as $\alpha = 67.7°$.

b. The average output current, $I_{dc} = V_{dc}/R = 70.23/10 = 7.02$ A. From Eq. (2.54), $V_{rms} = 94.74$ V and the rms load current, $I_{rms} = 94.74/10 = 9.47$ A.

c. The average current of a thyristor is $I_A = 1_{dc}/3 = 7.02/3 = 2.34$ A and the rms current of a thyristor is $I_R = I_{rms}/\sqrt{3} = 9.47/V3 = 5.47$ A.

d. From Eq. (2.3) the rectification efficiency is $\eta = V_{dc}I_{dc}/V_{rms}I_{rms} = 70.23 \times 7.02/(94.74 \times 9.47) = 54.95\%$.

e. The rms input line current is the same as the thyristor rms current, and the input volt-ampere rating (VAR), $VI = 3V_sI_s = 3 \times 120.1 \times 5.47 = 1970.84$ W. From Eq. (2.8), $TUF = V_{dc}I_{dc}/VI = 70.23 \times 7.02/1970.84 = 0.25$ or 25%.

f. The output power $P_0 = I_{rms}^2 R = 9.47^2 \times 10 = 896.81$ VA. The input PF = P_0/VI = 896.81/1970.84 = 0.455 (lagging).

2.11 Three-Phase Full-wave Controlled Converters

Three-phase converters are extensively used in industrial applications up to the 120-kW level, where a two-quadrant operation is required. Figure 2.17(a) shows a full-converter circuit with a highly inductive load. This circuit is known as a three-phase bridge. The thyristors are fired at an interval of $\pi/3$. The frequency of output ripple voltage is $6f_s$ and the filtering requirement is less than that of half-wave converters. At $\omega t = \pi/6 + \alpha$, thyristor T_6 is already conducting and thyristor T_1 is turned on. During interval $(\pi/6 + \alpha) \leq \omega t \leq (\pi/2 + \alpha)$, thyristors T_1 and T_6 conduct and the line-to-line voltage $v_{ab}(= v_{an} - v_{bn})$ appears across the load. At $\omega t = \pi/2 + \alpha$, thyristor T_2 is fired and thyristor T_6 is reversed biased immediately. T_6 is turned off due to natural commutation. During interval $(\pi/2 + \alpha) \leq \omega t \leq (5\pi/6 + \alpha)$, thyristors T_1 and T_2 conduct and the line-to-line voltage v_{ac} appears across the load. If the thyristors are numbered, as shown in Figure 2.17(a), the firing sequence is 12, 23, 34, 45, 56, and 61. Figure 2.17(b) shows the waveforms for input voltage, output voltage, input current, and currents through thyristors.

If the line-to-neutral voltages are defined as

$$v_{an} = V_m \sin \omega t$$

$$v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

the corresponding line-to-line voltages are

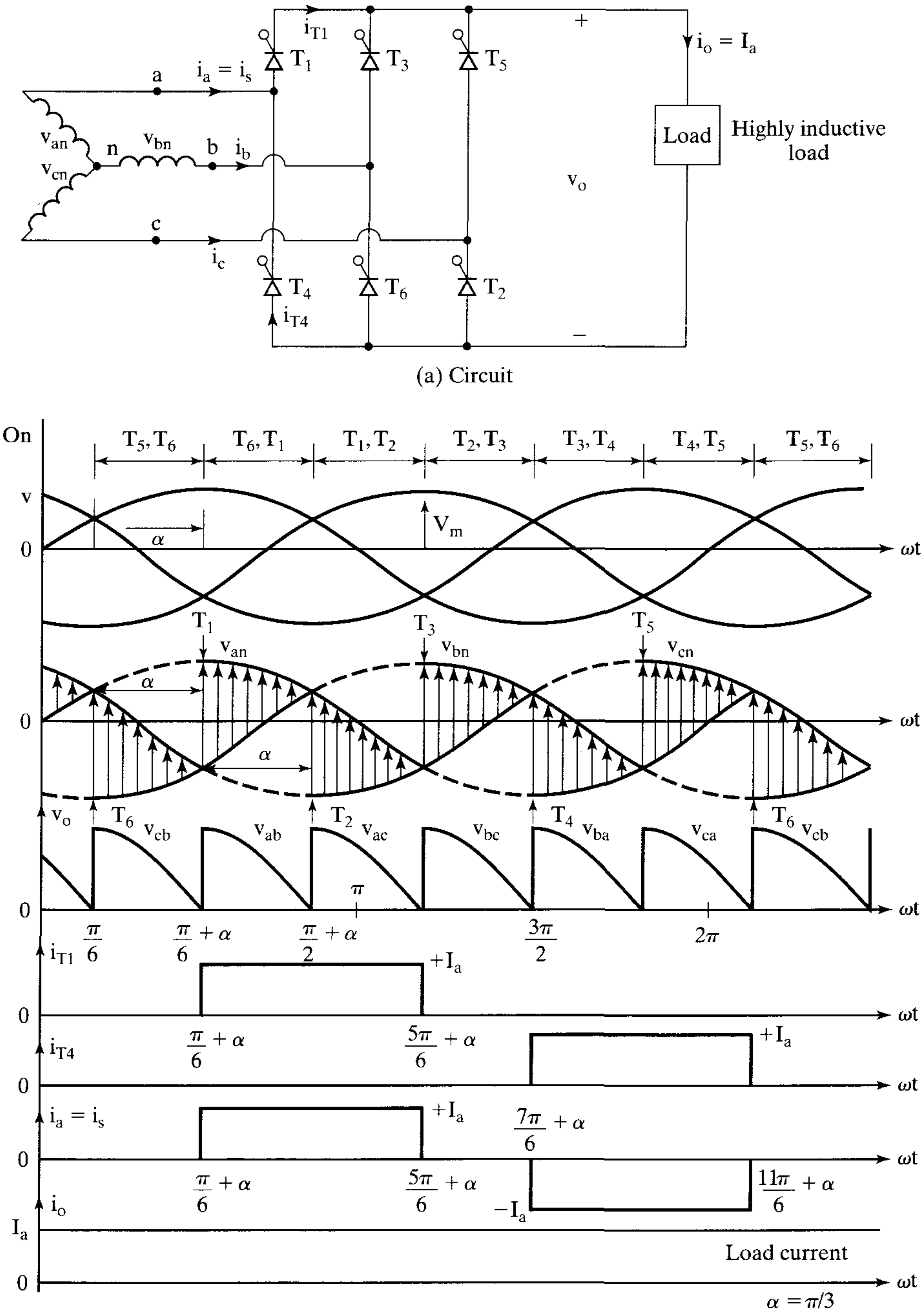
$$v_{ab} = v_{an} - v_{bn} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{bc} = v_{bn} - v_{cn} = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{ca} = v_{cn} - v_{an} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The average output voltage is found from

$$V_{dc} = \frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/2+\alpha} v_{ab}\, d(\omega t) = \frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) d(\omega t) = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha \quad (2.55)$$



(b) Waveforms

Figure 2.17: Three-phase full converter

The maximum average output voltage for delay angle, $\alpha = 0$, is

$$V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

and the normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha \tag{2.56}$$

The rms value of the output voltage is found from

$$V_{rms} = \left[\frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/2+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right) d(\omega t)\right]^{1/2} = \sqrt{3}V_m\left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right)^{1/2} \tag{2.57}$$

Figure 2.17(b) shows the waveforms for $\alpha = \pi/3$. For $\alpha > \pi/3$, the instantaneous output voltage v_0 has a negative part. Because the current through thyristors cannot be negative, the load current is always positive. Thus, with a resistive load, the instantaneous load voltage cannot be negative, and the full converter behaves as a semiconverter.

Gating sequence.

The gating sequence is as follows:

1. Generate a pulse signal at the positive zero crossing of the phase voltage v_{an}. Delay the pulse by the desired angle $\alpha + \pi/6$ and apply it to the gate and cathode terminals of T_1 through a gate-isolating circuit.
2. Generate five more pulses each delayed by $\pi/6$ from each other for gating T_2, T_3,..., T_6 respectively, through gate isolating circuits.

Example 2.9: Finding the Performances of a Three-Phase Full-Wave Controlled Converter

Repeat Example 2.8 for the three-phase full converter in Figure 2.17(a).

Solution of Example 2.9

The phase voltage $V_s = 208/\sqrt{3} = 120.1$ V, $V_m = \sqrt{2}\ V_s = 169.83$, $V_n = 0.5$, and R = 10 Ω. The maximum output voltage $V_{dm} = 3\sqrt{3}V_m/\pi = 3\sqrt{3} \times 169.83/\pi = 280.9$V. The average output voltage $V_{dc} = 0.5 \times 280.9 = 140.45$ V.

a. From Eq. (2.56), 0.5 = cos a, and the delay angle a = 60°.
b. The average output current $I_{dc} = V_{dc}/R = 140.45/10 = 14.05$ A. From Eq. (2.57),

$$V_{rms} = \sqrt{3}\times 169.83\left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos\left(2\times 60^\circ\right)\right]^{1/2} = 159.29\,\text{V}$$

   and the rms current $I_{rms} = 159.29/10 = 15.93$ A.
c. The average current of a thyristor $I_A = I_{dc}/3 = 14.05/3 = 4.68$ A, and the rms current of a thyristor $I_R = I_{rms}\sqrt{(2/6)} = 15.93\,\sqrt{(2/6)} = 9.2$ A.
d. From Eq. (2.15) the rectification efficiency is

$$\eta = \frac{V_{dc} I_{dc}}{V_{rms} I_{rms}} = \frac{140.45 \times 14.05}{159.29 \times 15.93} = 0.778 \quad or \quad 77.8\%$$

e. The rms input line current $I_s = I_{rms}\sqrt{(4/6)} = 13$ A and the input VAR rating $VI = 3V_sI_s = 3 \times 120.1 \times 13 = 4683.9$ VA. From Eq. (2.8), $TUF = V_{dc}I_{dc}/VI = 140.45 \times 14.05/4683.9 = 0.421$.

f. The output power $P_0 = I_{rms}^2 R = 15.93^2 \times 10 = 2537.6$ W. The $PF = P_0/Vl = 2537.6/4683.9 = 0.542$ (lagging).

2.12 Three-Phase Full-wave Controlled Converter with *RL* Load

From Figure 2.17(b) the output voltage is

$$v_0 = v_{ab} = \sqrt{2}V_{ab} \sin\left(\omega t + \frac{\pi}{6}\right) \quad \text{for } \frac{\pi}{6} + \alpha \le \omega t \le \frac{\pi}{2} + \alpha$$

$$= \sqrt{2}V_{ab} \sin \omega t' \quad \text{for } \frac{\pi}{3} + \alpha \le \omega t' \le \frac{2\pi}{3} + \alpha$$

where $\omega t' = \omega t + \pi/6$, and V_{ab} is the line-to-line (rms) input voltage. Choosing v_{ab} as the time reference voltage, the load current i_L can be found from

$$L\frac{di_L}{dt} + Ri_L + E = \sqrt{2}V_{ab} \sin \omega t' \quad \text{for} \quad \frac{\pi}{3} + \alpha \le \omega t' \le \frac{2\pi}{3} + \alpha$$

whose solution from Eq. (2.46)

$$i_L = \frac{\sqrt{2}V_{ab}}{Z} \sin(\omega t' - \theta) - \frac{E}{R} + \left[I_{L1} + \frac{E}{R} - \frac{\sqrt{2}V_{ab}}{Z} \sin\left(\frac{\pi}{3} + \alpha - \theta\right) \right] e^{(R/L)((\pi/3+\alpha)/\omega - t')} \quad (2.58)$$

where $Z = [R^2 + (\omega L)^2]^{1/2}$ and $\theta = \tan^{-1}(\omega L/R)$. Under a steady-state condition, $i_L(\omega t' = 2\pi/3 + \alpha) = i_L(\omega t' = \pi/3 + \alpha) = I_{L1}$. Applying this condition to Eq. (2.58), the value of I_{L1} is

$$I_{L1} = \frac{\sqrt{2}V_{ab}}{Z} \frac{\sin(2\pi/3 + \alpha - \theta) - \sin(\pi/3 + \alpha - \theta)e^{-(R/L)(\pi/3\alpha)}}{1 - e^{-(R/L)(\pi/3\omega)}} - \frac{E}{R} \quad \text{for } I_{L1} \ge 0 \quad (2.59)$$

Discontinuous load current.

By setting $I_{L1} = 0$ in Eq. (2.58), dividing by $\sqrt{2}V_s/Z$ and substituting $R/Z = \cos\theta$ and $\omega L/R = \tan\theta$, the critical value of voltage ratio $x = E/\sqrt{2}V_{ab}$ is

$$x = \left[\frac{\sin\left(\frac{2\pi}{3} + \alpha - \theta\right) - \sin\left(\frac{\pi}{3} + \alpha - \theta\right) e^{-\left(\frac{\pi}{3\tan(\theta)}\right)}}{1 - e^{-\left(\frac{\pi}{3\tan(\theta)}\right)}} \right] \cos\theta \quad (2.60)$$

which can be solved for the critical value of $\alpha = \alpha_c$ for known values of x and θ. For $\alpha \geq \alpha_c$, $I_{L1} = 0$. The load current that is described by Eq. (2.58) flows only during the period $\alpha \leq \omega t \leq \beta$. At $\omega t = \beta$, the load current falls to zero again.