UNIVERSITY OF ANBAR

COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING DEPARTMENT

Power Electronic

Fourth Class

Chapter 03

DC to AC Convertor (Invertors)

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CHAPTER 3

INVERTERS: CONVERTING DC to AC

3.1 Introduction

Inverters are widely used in industrial applications such as for induction motor drives, traction, induction heating, standby power supplies and uninterruptible ac power supplies. An inverter performs the inverse process of a rectifier. It converts DC power into AC power at a desired output voltage or current and frequency. The input source of the inverters can be battery, fuel cell, solar cell or other types of dc source. The output voltage may be non-sinusoidal but can be made close to sinusoidal waveform. The general block diagram of an inverter is shown in Figure 3.1.

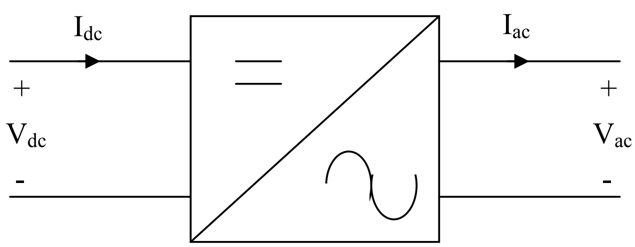
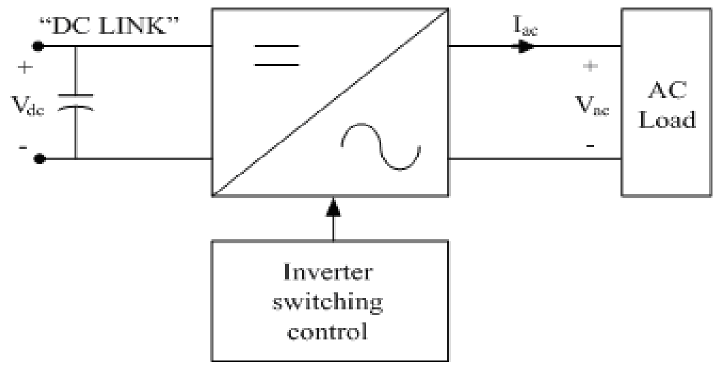
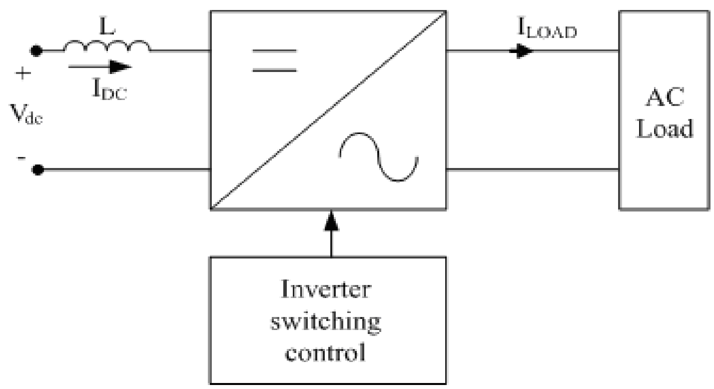


Figure 3.1 General block diagram of inverter

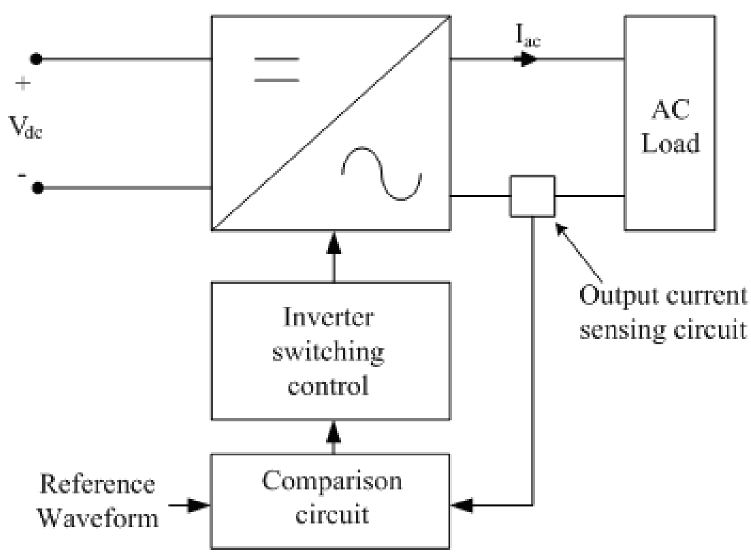
The inverters can be classified into three categories there are voltage source inverters, current source inverters and current regulated inverters (hysteresis-type). The voltage source inverter is the most commonly used type of inverter. The AC that it provides on the output side functions as a voltage source. The input DC voltage may be from the rectified output of an AC power supply or an independent source such as battery, which is called a 'DC link' inverter. On the other hand, the current source inverter, the output side is functions as AC current source. This type is also has a DC link inverter but its functions like a DC current source. Figure 3.2(a) and Figure 3.2(b) show the block diagram of voltage source inverter and current source inverter respectively.



(a) Voltage source inverter



(b) Current source inverter



(c) Current regulated inverter

Figure 3.2 Types of inverter

The current regulated inverters are becoming popular especially for speed control of AC motors. The input DC is the same as in conventional voltage source inverter. In this category, there is a current sensing circuit that senses the actual value of the current at every instant. The sensed value is compared against the value dictated by the reference waveform, and the switching inside the inverter is altered as necessary to correct any error. In this way, the output current waveform is made to conform as accurately as possible to the input reference waveform. The basic block diagram of current regulated inverter is depicted in Figure 3.2(c).

3.2 Basic Principles

To illustrate the basic concept of inverters generating an AC output waveform, the basic circuit for the single-phase full-bridge converter produces square-wave output voltage is shown in Figure 3.3. An AC output is synthesized from a DC input by closing and opening the switches in an appropriate sequence. The output voltage V_o can be $+V_{dc}$ and $-V_{dc}$ depending on which switches are closed. Figure 3.4 shows the equivalent circuits of switch combination in opened and closed position and its square-wave output waveforms.

When the switch S1 and S2 are closed, and at the same time switch S3 and S4 are opened, the output voltage waveform is $+V_{dc}$ between the interval of t_1 to t_2. Whereas, when the switch S3 and S4 are closed, and at the same time switch S1 and S2 are opened, the output voltage is $-V_{dc}$ in the interval of t_3 to t_4. The periodic switching of the load voltage between $+V_{dc}$ and $-V_{dc}$ produces a square-wave voltage across the load. In order to find the sinusoid output waveform, the square-wave waveform should be filtered.

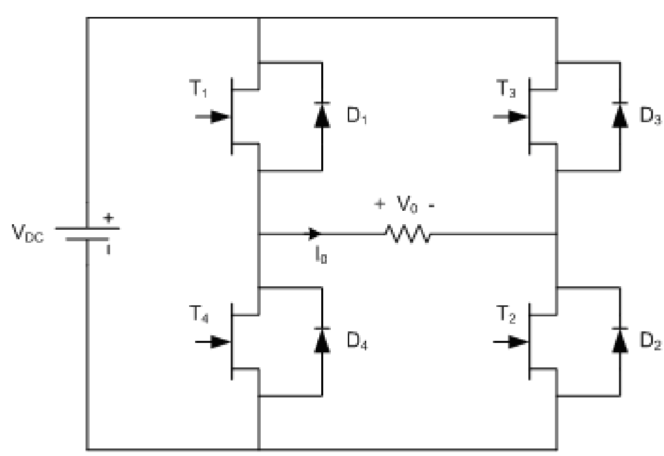
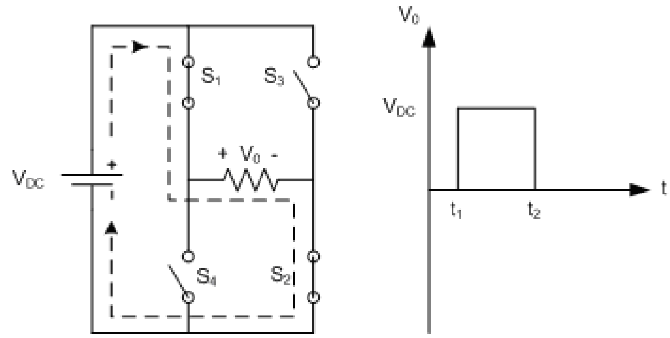
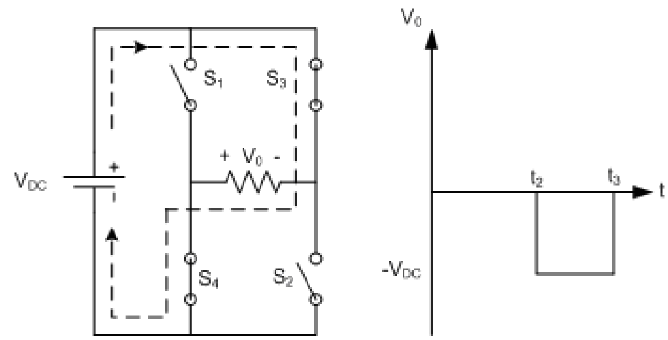


Figure 3.3 The schematic of single-phase full-bridge inverter circuit



(a)



(b)

Figure 3.4 The equivalent circuit of an inverter and the output voltage; (a) S1 and S2 closed, S3 and S4 opened, (b) S3 and S4 closed, S1 and S2 opened

Notice that, the switch S1 and S4 should not be closed at the same time, similarly for switch S2 and S3. These because if that condition are occurs, a short circuit would exist across the dc source. In practice, the switches are not turn on and off instantaneously. There are switching transition times in the control of the switches.

Let consider the output voltage of the inverter is filtered and V_0 can be assumed as sinusoid. Since the inverter supplies an inductive load such as ac motor, the output current I_0 will lag the output voltage V_0 as shown in Figure 3.5. The output waveform of Figure 3.5 shows that in the interval 1, V_0 and I_0 are positive, whereas in interval 3 V_0 and I_0 are both negative. Therefore, during interval 1 and 3, the instantaneous power flow $P_0 = V_0 I_0$ is from dc side to ac side. In the other hand, V_0 and I_0 are in opposite signs during interval 2 and 4, therefore P_0 flows from ac side to dc side of the inverter.

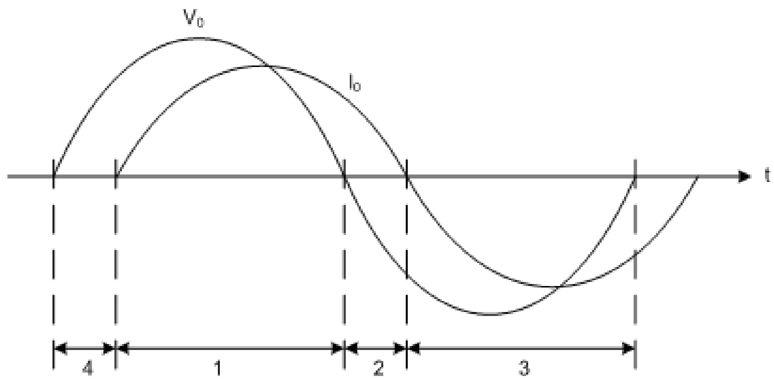


Figure 3.5 Filtered output voltage of the inverter

3.3 Single-phase Half-bridge Square-wave Inverter

The half-bridge inverter is constructed with two equal capacitors connected in series across the dc input source and their junction is at a midpoint or centre point G. The number of switches or called as '*inverter leg*' for half-bridge inverter is reduced to one leg which only consists of two switches. The voltage across of each capacitor has a value of $V_{DC}/2$. When the switch S1 is closed, the load voltage is $V_{DC}/2$ and when S2 is closed, the load voltage is $-V_{DC}/2$. The basic single-phase half-bridge inverter circuit and its output waveform are shown in Figure 3.6 below.

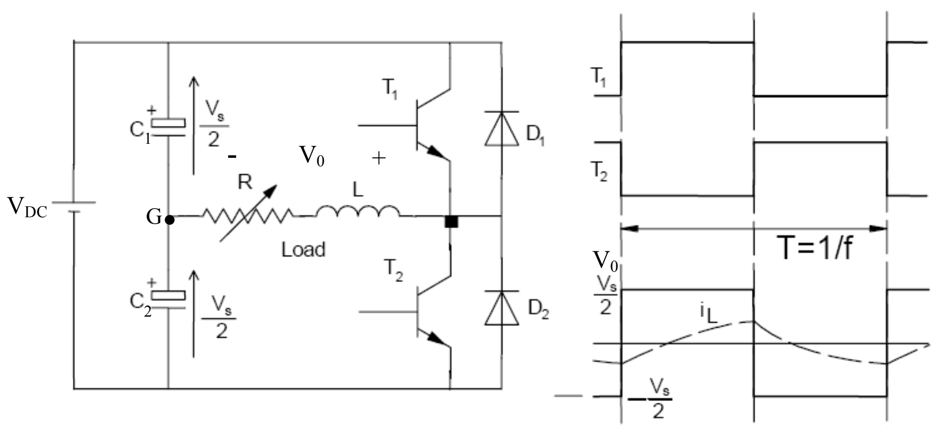


Figure 3.6 The basic single-phase half-bridge inverter circuit

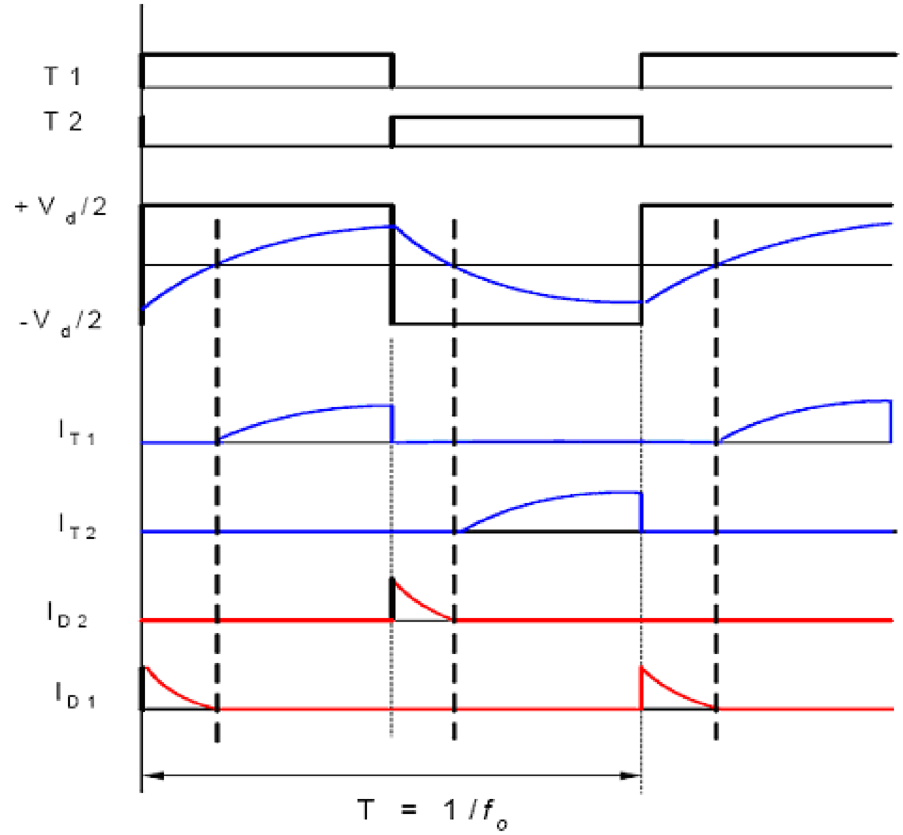


Figure 3.7 Waveforms of output voltage and switches with resistive and inductive load.

The load voltage is a square-wave of amplitude *VDC/2* and the load current waveform is exponentially rising and falling waveform determined by the load impedance. For a resistive load the current waveform follows the voltage waveform while for an inductive load the current waveform lags the voltage waveform by an angle which is, approximately the load power factor angle. Figure 3.7 shows the waveforms for the output voltage and the switches current with R-L load.

The total RMS value of the load output voltage,

$$V_O = \sqrt{\left\{\frac{2}{T}\int_0^{T/2}\left(\frac{V_{DC}}{2}\right)^2\right\}dt} = \frac{V_{DC}}{2} \tag{3.1}$$

And the instantaneous output voltage is

$$v_O = \sum_{n=1,3,5,\ldots} \frac{2V_{DC}}{n\pi}\sin n\omega t \tag{3.2}$$
$$= 0 \qquad \text{for n} = 2,4,....$$

The fundamental rms output voltage is

$$V_{O1} = \frac{\sqrt{2}V_{DC}}{\pi} = 0.45V_{DC} \tag{3.3}$$

In the case of RL load, the instantaneous load current i_o can be determined by dividing the instantaneous output voltage with the load impedance Z = R +jωL. Thus,

$$i_o = \sum_{n=1,3,5,..}^{\infty} \frac{2V_{DC}}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n) \tag{3.4}$$

where $\theta_n = \tan^{-1}(n\omega L / R)$

The fundamental output power is

$$\begin{aligned} P_{o1} &= V_{o1} I_{o1} \cos\theta_1 \\ &= I_{o1}^2 R \\ &= \left[\frac{2V_{DC}}{\sqrt{2}\pi\sqrt{R^2 + (\omega L)^2}} \right] \end{aligned} \tag{3.5}$$

The total harmonic distortion (THD), which is a measure of closeness in shape between a waveform and its fundamental components, is defined as

$$THD = \frac{1}{V_{O1}} \sqrt{\left\{ \sum_{n=3,5,7,....}^{\infty} V_n^2 \right\}} \tag{3.6}$$

Example 3.1

The single-phase half-bridge inverter has a resistive load of R = 2.4Ω and the DC input voltage is 48V. Determine:

(a) the rms output voltage at the fundamental frequency
(b) the output power
(c) the average and peak current of each transistor.
(d) The THD

Solution

V_{DC} = 48V and R = 2.4Ω

(a) The fundamental rms output voltage, $V_{o1} = 0.45V_{DC} = 0.45x48 = 21.6V$
(b) For single-phase half-bridge inverter, the output voltage $V_o = V_{DC}/2$
Thus, the output power,

$$\begin{aligned} P_o &= V_o^2 / R \\ &= \frac{(48/2)^2}{2.4} \\ &= 240W \end{aligned}$$

(c) The transistor current $I_p = 24/2.4 = 10$ A
Because each of the transistor conducts for a 50% duty cycle, the average current of each transistor is $I_Q = 10/2 = 5$ A.

(d) $$THD = \frac{1}{V_{O1}}\sqrt{V_o^2 - V_{o1}^2}$$
$$= \frac{1}{0.45xV_{DC}}\sqrt{\left(\frac{V_{DC}}{2}\right)^2 - \left(0.45V_{DC}\right)^2}$$
$$= 48.34\%$$

3.4 Single-phase Full-bridge Square-wave inverter

A single-phase full-bridge inverter circuit is built from two half-bridge leg which consists of four choppers as depicted in Figure 3.8. The switching in the second leg is delayed by 180 degrees from the first leg. With the same dc input voltage, the maximum output voltage of this inverter is twice that of half-bridge inverter. When transistor T1 and T2 are closed simultaneously, the input voltage V_{DC} appears across the load. If transistors T3 and T4 are closed, the voltage across the load is reversed that is - V_{DC}. Figure 3.9 illustrates the output waveforms for the output voltage and the switches current with R-L load.

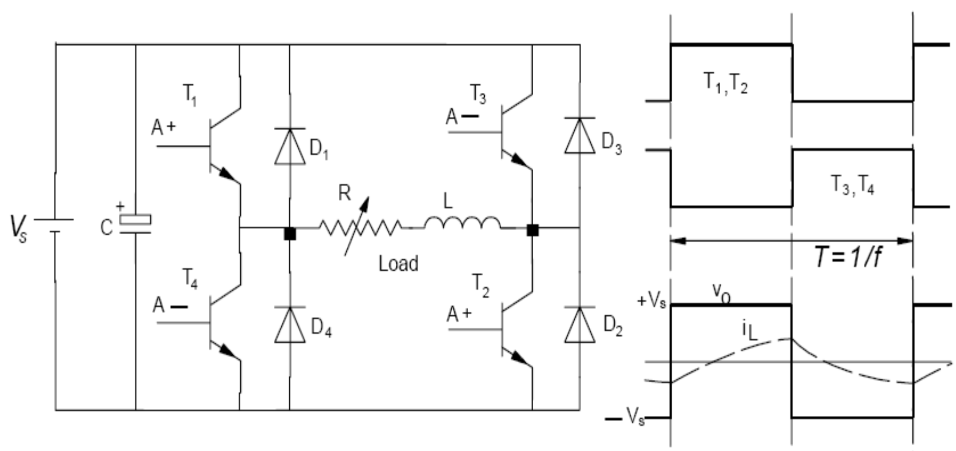


Figure 3.8 Single-phase full-bridge inverter with R-L load

The output RMS voltage

$$V_O = \sqrt{\left\{\frac{2}{T}\int V_{DC}^2 dt\right\}} = V_{DC} \tag{3.7}$$

And the instantaneous output voltage in a Fourier series is

$$v_O = \sum_{n=1,3,5,\dots} \frac{4V_{DC}}{n\pi} \sin n\omega t \tag{3.8}$$

The fundamental RMS output voltage,

$$V_1 = \frac{4V_{DC}}{\sqrt{2}\pi} = 0.9V_{DC} \tag{3.9}$$

In the case of RL load, the instantaneous load current is

$$i_o = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_{DC}}{n\pi\sqrt{R^2 - (n\omega L)^2}} \sin(n\omega t - \theta_n) \tag{3.10}$$

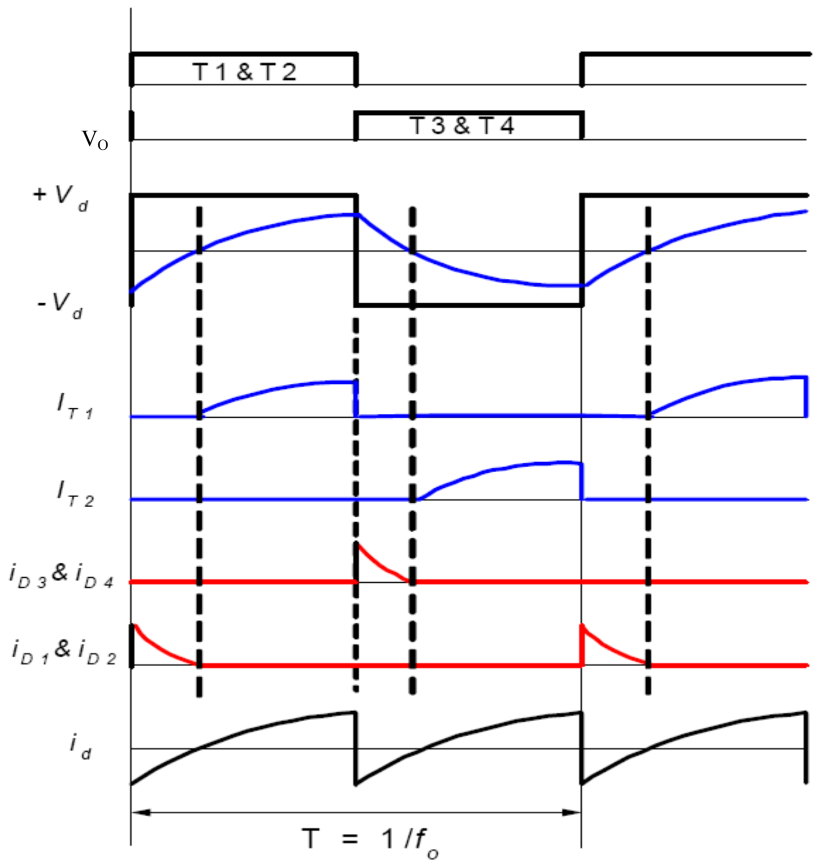


Figure 3.9 Waveforms of the output voltage and currents of single-phase full-bridge inverter

Example 3.2

A single-phase full-bridge inverter with VDC = 230 and consist of RLC in series. If R = 1.2Ω, ωL = 8 Ω and 1/ωC = 7 Ω, find:

(a) The amplitude of fundamental rms output current, i_{o1}

(b) The fundamental component of output current in function of time.

(c) The power delivered to the load due to the fundamental component.

Solution

(a) The fundamental output voltage $V_{o1} = \dfrac{4V_{DC}}{\pi\sqrt{2}} = \dfrac{4x230}{\pi\sqrt{2}} = 207.1V$

The rms value of fundamental current is

$$I_{o1} = \frac{V_{o1}}{Z_1} = \frac{V_{o1}}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$= \frac{207.1}{\sqrt{(1.2)^2 + (8-7)^2}}$$

$$= \frac{207.1}{1.562}$$

$$= 132.586A$$

(b) The fundamental component of output current in function of time is

$$i_{o1} = \sqrt{2}I_{o1}\sin(\omega t - \theta_1)$$

Where $\theta_1 = \tan^{-1}\dfrac{X_L - X_C}{R} = \tan^{-1}\dfrac{8-7}{1.2} = 39.80^\circ$

Therefore, $i_{o1} = 187.5\sin(\omega t - 39.80^\circ)$

(d) Power delivered to the load

$$P_o = I_{o1}^2 R$$

$$= (132.586)^2 x1.2$$

$$= 21094.85W$$

Example 3.3

A single-phase full-bridge inverter has an RLC load with R = 10Ω, L = 31.5mH and C = 112μF. The inverter frequency is 60Hz and the DC input voltage is 220V. Determine:

(a) Express the instantaneous load current in Fourier series.

(b) Calculate the rms load current at the fundamental frequency.

(c) the THD of load current

(d) Power absorbed by the load and fundamental power.

(e) The average DC supply current and

(f) the rms and peak supply current of each transistor

Solution

V_{DC} = 220, f = 60Hz, R=10 Ω, L=31.5mH, C=112 μF

ω = 2π x 60 = 377 rad/s

The inductive reactance fot *n*th harmonic voltage is

$X_L = jn\omega L = j377 \text{ x } 31.5\text{mH} = j11.87n\ \Omega$

The capacitive reactance for the *n*th harmonic voltage is

$$X_C = -\frac{j}{n\omega C} = -\frac{j}{377n \times 112\mu F} = -\frac{j23.68}{n}\Omega$$

The impedance for the nth harmonic voltage is

$$|Z| = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}$$

$$= \sqrt{\left[10^2 + \left(11.87n - \frac{23.68}{n}\right)^2\right]}$$

The load impedance angle for *n*th harmonic voltage is

$$\theta_n = \tan^{-1}\frac{11.78n - 23.68/n}{10} = \tan^{-1}\left(1.187n - \frac{2.368}{n}\right)$$

(a) The instantaneous output voltage can be expressed as

$$v_o(t) = 280.1\sin(377t) + 93.4\sin(3x377t) + 56.02\sin(5x377t)$$
$$+40.02\sin(7x377t) + 31.12\sin(9x377t) + ...$$

Dividing the output voltage by the load impedance and considering the delay due to the load impedance angles, the instantaneous load current is

$$i_o(t) = 18.1\sin(377t + 49.72^\circ) + 3.17\sin(3x377t - 70.17^\circ)$$
$$+\sin(5x377t - 79.63^\circ) + 0.5\sin(7x377t - 82.85^\circ)$$
$$+0.3\sin(9x377t - 84.52^\circ) + ...$$

(b) The peak of fundamental load current is I_o = 18.1 A

Therefore, the rms fundamental load current $I_{o1} = 18.1/\sqrt{2}$ = 12.8 A

(c) Consider to 9th harmonic, the peak load current

$I_{o,p} = \sqrt{(18.1^2 + 3.17^2 + 1.0^2 + 0.5^2 + 0.3^2)}$ =18.41 A

The rms harmonic load current is

$$I_h = \sqrt{I_{o,rms}^2 - I_{o1}^2} = \sqrt{\left(\frac{18.41}{\sqrt{2}}\right)^2 - (12.8)^2} = 2.3715A$$

Therefore, the THD of the load is

$$THD = \frac{I_h}{I_{o1}} = \frac{2.3715}{12.8} = 18.53\%$$

(d) The rms load current is $I_{o,rms} = I_{o,p}/\sqrt{2} = 13.02A$

And the load power is

$$P_o = 13.02^2 x10 = 1695W$$

Henc, the fundamental output power is

$$P_{o1} = I_{o1}^2 R = (12.8)^2 x10 = 1638.4W$$

(e) The average supply current $I_{DC} = \frac{P_o}{V_{DC}} = \frac{1695}{220} = 7.7A$

(f) The peak transistor current $I_p \cong I_{o,p} = 18.41A.$

Hence, the maximum permissible rms current of each transistor is

$$I_{Q(\max)} = \frac{I_{o,p}}{\sqrt{2}} = \frac{I_p}{2} = \frac{18.41}{2} = 9.2A$$

3.5 Three-phase Inverter

Three phase bridge inverters can be viewed as extensions of the single-phase bridge circuit, as shown in figure 3.10. The switching signals for each inverter leg are displaced or delayed by 120^o with respect to the adjacent legs to obtain three-phase balanced voltage. The output line-line voltages are determined by the potential differences between the output terminals of each leg. With 120^o conduction, the switching pattern is T1T2 – T2T3 – T3T4 – T4T5 – T5T6 – T6T1 – T1T2 for the positive A-B-C sequence and the other way round for the negative (A-C-B) phase sequence.

Whenever an upper switch in an inverter leg connected with the positive DC rail is turned ON, the output terminal of the leg goes to potential +*VDC/2* with respect to the center-tap of the DC supply. Whenever a lower switch in an inverter leg connected with the negative DC rail is turned ON, the output terminal of that leg goes to potential -*VDC/2* with respect to the center-tap of the DC supply. Note that a center-tap of the DC supply *VDC* has been created by connecting two equal valued capacitors across it. The center-tap is assumed to be at zero potential or grounded. However, this contraption is artificial and really not essential; the center-tap may not exist in practice.

The switching signals for each phase leg, line-line voltage and line-to-neutral voltage are shown in Figure 3.11. As a result, the switching signals for each phase leg have 60^o of non overlap. Because of this, switches of a phase leg do not need any dead-time which is the time each switch waits before the other completely turns OFF. Note that only two switches conduct at one time.

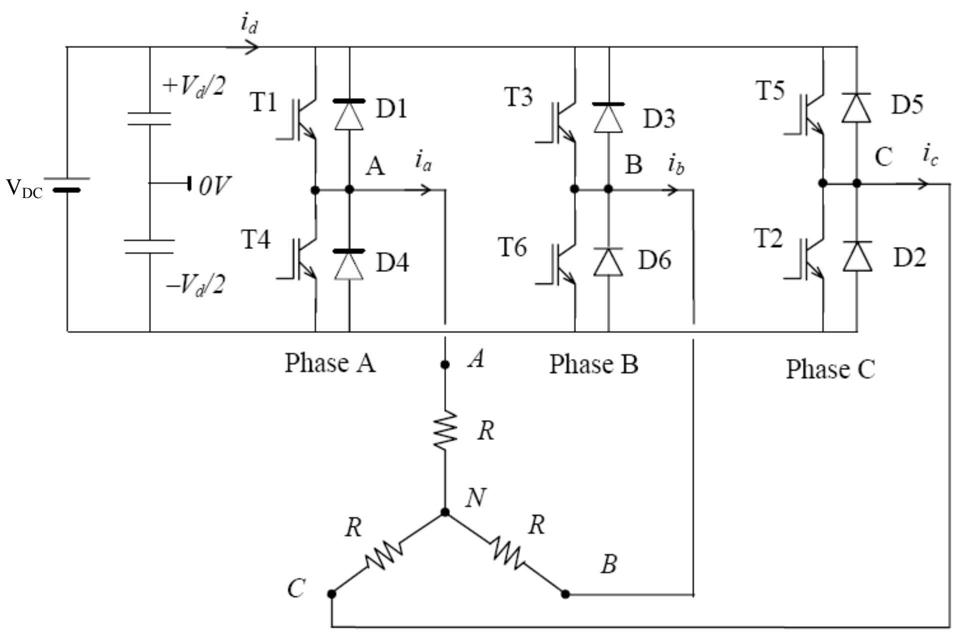


Figure 3.10 Three-phase inverter with star connected load

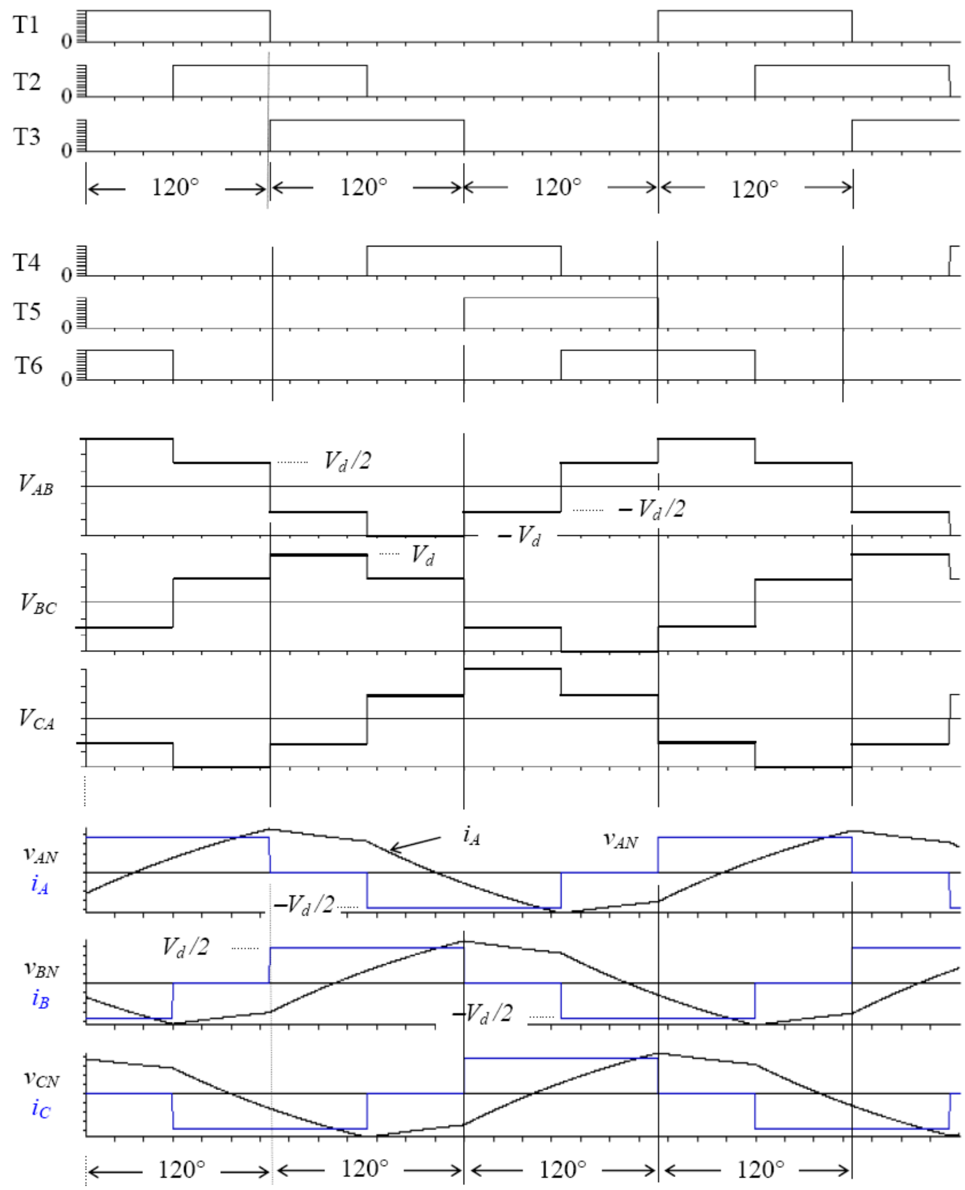


Figure 3.11 The waveforms of three-phase bridge inverter

3.6 Fourier Series and Harmonics Analysis

The Fourier series is the most practical way to analyze the load current and to compute power absorbed by a load. Fourier series is a tool to analyze the wave shapes of the output voltage and current in terms of Fourier series.

Fourier series

$$a_o = \frac{1}{\pi}\int_0^{2\pi} f(v)d\theta \tag{3.11}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\cos(n\theta)d\theta \tag{3.12}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\sin(n\theta)d\theta \tag{3.13}$$

Inverse Fourier

$$f(v) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos n\theta + bn \sin n\theta) \tag{3.14}$$

Where $\theta = \omega t$

If no DC component in the output, the output voltage and current are given in equation (3.15) and (3.16).

$$v_o(t) = \sum_{n=1}^{\infty} V_n \sin(n\omega t + \theta_n) \tag{3.15}$$

$$i_o(t) = \sum_{n=1}^{\infty} I_n \sin(n\omega t + \phi_n) \tag{3.16}$$

The rms current of the load can be determined by equation (3.17)

$$I_{rms} = \sqrt{\sum_{n=1}^{\infty} I_{n,rms}^2} = \sqrt{\sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2} \tag{3.17}$$

Where $I_n = \frac{V_n}{Z_n}$

And Z_n is the load impedance at harmonic number.

The total power absorbed in the load resistor can be determined by

$$P = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} I_{n,rms}^2 R \tag{3.18}$$

3.6.1 Total Harmonic Distortion

Since the objective of the inverter is to use a DC voltage source to supply a load that requiring AC voltage, hence the quality of the non-sinusoidal AC output voltage or current can be expressed in terms of THD. The harmonics needs to be considered is to ensure that the waveform quality must match to the utility supply which means of power quality issues. This is due to the harmonics may cause degradation of the equipments and needs to be de-rated.

The THD of the load voltage is expressed as,

$$THD_v = \frac{\sqrt{\sum_{n=2}^{\infty} (V_{n,rms})^2}}{V_{1,rms}} = \frac{\sqrt{V_{rms}^2 - V_{1,rms}^2}}{V_{1,rms}} \tag{3.19}$$

Where n is the harmonics number.

The current THD can be obtained by replacing the harmonic voltage with harmonic current,

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty} (I_{n,rms})^2}}{I_{1,rms}} \tag{3.20}$$

3.6.2 Harmonics of Square-wave Waveform

The harmonics of square-wave waveform as depicted in Figure 3.12 can be obtained as in equation (3.21), (3.22) and (3.23).

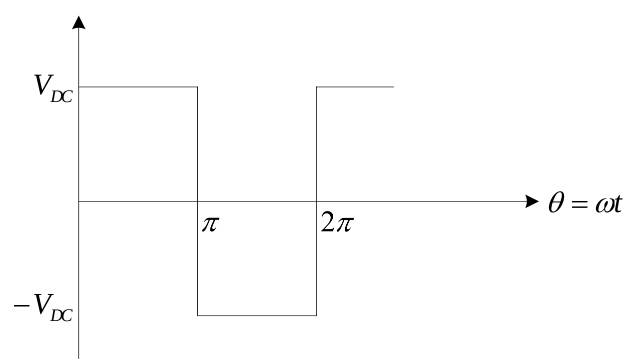


Figure 3.12 Square-wave waveform

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} V_{DC}\,d\theta + \int_{\pi}^{2\pi} -V_{DC}\right] = 0 \tag{3.21}$$

$$a_n = \frac{V_{DC}}{\pi}\left[\int_0^{\pi} \cos(n\theta)d\theta - \int_{\pi}^{2\pi} \cos(n\theta)d\theta\right] = 0 \tag{3.22}$$

$$\begin{aligned} b_n &= \frac{V_{DC}}{\pi}\left[\int_0^{\pi} \sin(n\theta)d\theta - \int_{\pi}^{2\pi} \sin(n\theta)d\theta\right] \\ &= \frac{V_{DC}}{n\pi}\left[-\cos(n\theta)\Big|_0^{\pi} + \cos(n\theta)\Big|_{\pi}^{2\pi}\right] \\ &= \frac{V_{DC}}{n\pi}\left[(\cos 0 - \cos n\theta) + (\cos 2n\pi - \cos n\pi)\right] \\ &= \frac{2\pi}{n\pi}\left[(1 - \cos n\pi)\right] \end{aligned} \tag{3.23}$$

3.6.3 Spectrum of Square-wave

When the harmonics number, n of a waveform is even number, the resultant of $\cos n\pi = 1$,

Therefore,

$b_n = 0$.

When n is odd number, $\cos n\pi = -1$

Hence,

$$b_n = \frac{4V_{DC}}{n\pi} \tag{3.24}$$

The spectrum of a square-wave waveform is illustrated in Figure 3.13.

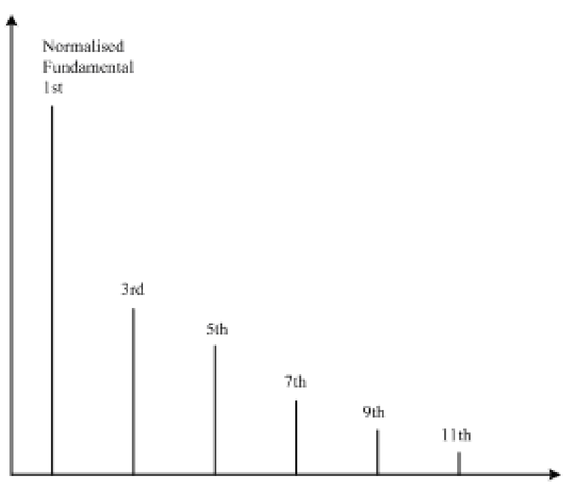


Figure 3.13 Spectrum of square-wave

Example 3.4

The full-bridge inverter with DC input voltage of 100V, load resistor and inductor of 10Ω and 25mH respectively and operated at 60 Hz frequency. Determine:

(a) The amplitude of the Fourier series terms for the square-wave load voltage.

(b) The amplitude of the Fourier series terms for load current.

(c) Power absorbed by the load.

(d) The THD of the load voltage and load current for square-wave inverter.

Solution

(a) The amplitude of each voltage terms is

$$V_n = \frac{4V_{DC}}{n\pi} = \frac{4(100)}{n\pi}$$

(b) The amplitude of each current term is determined by

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}} = \frac{4(100)/n\pi}{\sqrt{10^2 + [n(2\pi 60)(0.025)]^2}}$$

(c) The power at each frequency is

$$P_n = I_{n,rms}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R$$

The power absorbed by the load is computed by

$$P = \sum P_n = I_{1,rms}^2 R + I_{3,rms}^2 R + I_{5,rms}^2 R + I_{7,rms}^2 R + I_{9,rms}^2 R + ...$$
$$= 429.3 + 10 + 1.4 + 0.37 + 0.14 + ... \approx 441W$$

(e) The THD for voltage

$$THD_V = \frac{\sqrt{V_{rms}^2 - V_{1,rms}^2}}{V_{1,rms}} = \frac{\sqrt{V_{DC}^2 - (0.9V_{DC})^2}}{0.9V_{DC}} = 0.483 = 48.3\%$$

The THD of the current

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{1,rms}}$$

$$= \frac{\sqrt{\left(\frac{1.42}{\sqrt{2}}\right)^2 + \left(\frac{0.53}{\sqrt{2}}\right)^2 + \left(\frac{0.27}{\sqrt{2}}\right)^2 + \left(\frac{0.17}{\sqrt{2}}\right)^2}}{\left(\frac{9.27}{\sqrt{2}}\right)}$$

$$= 0.167 = 16.7\%$$

3.7 Pulse-Width Modulation (PWM)

Pulse-width modulation provides a way to decrease the total harmonics distortion (THD) of load current. A PWM inverter output with some filtering can generally meet THD requirements more easily than the square-wave switching scheme. There are several types of PWM techniques to control the inverter switching such as natural or sinusoidal sampling, regular sampling, optimized PWM, harmonic elimination or minimization PWM and space-vector modulation (SVM). In PWM, the amplitude of the output voltage can be controlled with the modulating waveforms.

When using PWM several definitions should be stated:

(i) Amplitude Modulation Ratio, M_a

The amplitude modulation ratio or also called as modulation index is defined as the ratio of amplitudes of the reference signal to the carrier signal as given in equation (3.25)

$$M_a = \frac{V_{m,\,reference}}{V_{m,\,\mathrm{carrier}}} = \frac{V_{m,\,\sin e}}{V_{m,\,\mathrm{tri}}} \tag{3.25}$$

The M_a is related to the fundamental of sine wave output voltage magnitude. If $Ma \leq 1$, the amplitude of the fundamental frequency of the output voltage, V_1 is linearly proportional to M_a. If M_a is high, then the sine wave output is high and vice versa. This can be represent by

$$V_1 = M_a V_{DC} \tag{3.26}$$

(ii) Frequency Modulation Ratio, M_f

The frequency modulation ratio is defined as the ratio of the frequency of the carrier and reference signal as given in equation (3.27).

$$M_f = \frac{f_{carrier}}{f_{\mathrm{reference}}} = \frac{f_{tri}}{f_{\sin e}} \tag{3.27}$$

Mf is related to the harmonic frequency. When the carrier frequency increases the frequency at which the harmonics occur also will increase.

3.7.1 Bipolar Switching of PWM

The principle of sinusoidal bipolar PWM is illustrated in Figure 3.14. Figure 3.14(a) shows a sinusoidal reference signal and a triangular carrier signal while Figure 3.14(b) is the modulated output signal. When the instantaneous value of the reference signal is larger than the triangular carrier, the output is at $+V_{DC}$ and when the reference is less than the carrier the output is at $-V_{DC}$. The bipolar switching technique, the output alternates between plus and minus the DC supply voltage.

$$\begin{aligned} V_o &= +V_{DC} \quad \text{for} \quad V_{\sin e} > V_{tri} \\ V_o &= -V_{DC} \quad \text{for} \quad V_{\sin e} < V_{tri} \end{aligned} \tag{3.28}$$

If the bipolar scheme is implemented for the full-bridge inverter, the switching is determined by comparing the instantaneous reference and carrier signal as given by,

S_1 and S_2 ON when $V_{sine} > V_{tri}$ ($V_o = +V_{DC}$)

S_3 and S_4 ON when $V_{sine} < V_{tri}$ ($V_o = -V_{DC}$)

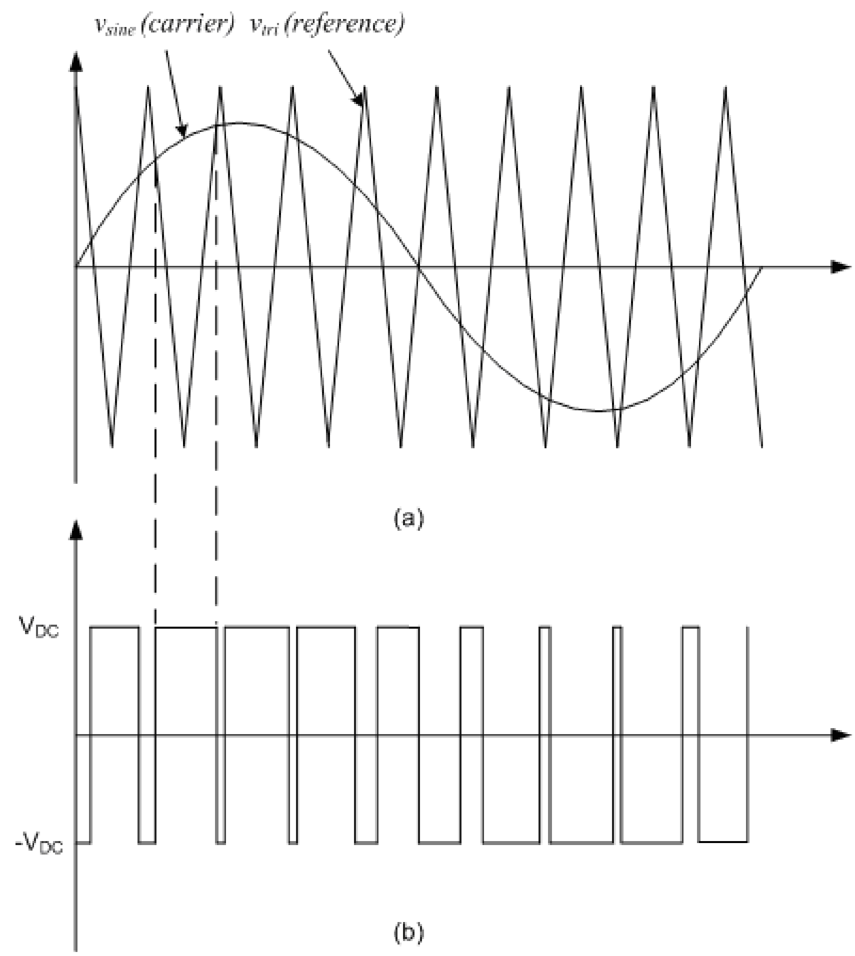


Figure 3.14 Bipolar pulse-width modulation (a) Sinusoidal reference and triangular carrier, (b) Output modulated signal

3.7.2 Unipolar Switching of PWM

The unipolar switching scheme of PWM, the output is switched from either higher to zero or low to zero, rather than between high and low as in bipolar schemes. The switch controls for unipolar switching scheme given as,

S_1 is ON when $V_{sine} > V_{tri}$

S_2 is ON when $-V_{sine} < V_{tri}$

S_3 is ON when $-V_{sine} > V_{tri}$

S_4 is ON when $V_{sine} < V_{tri}$

The switches are operating in a pair where if one pair is closed the other pair is opened. The full bridge unipolar PWM switching scheme is shown in Figure 3.15.

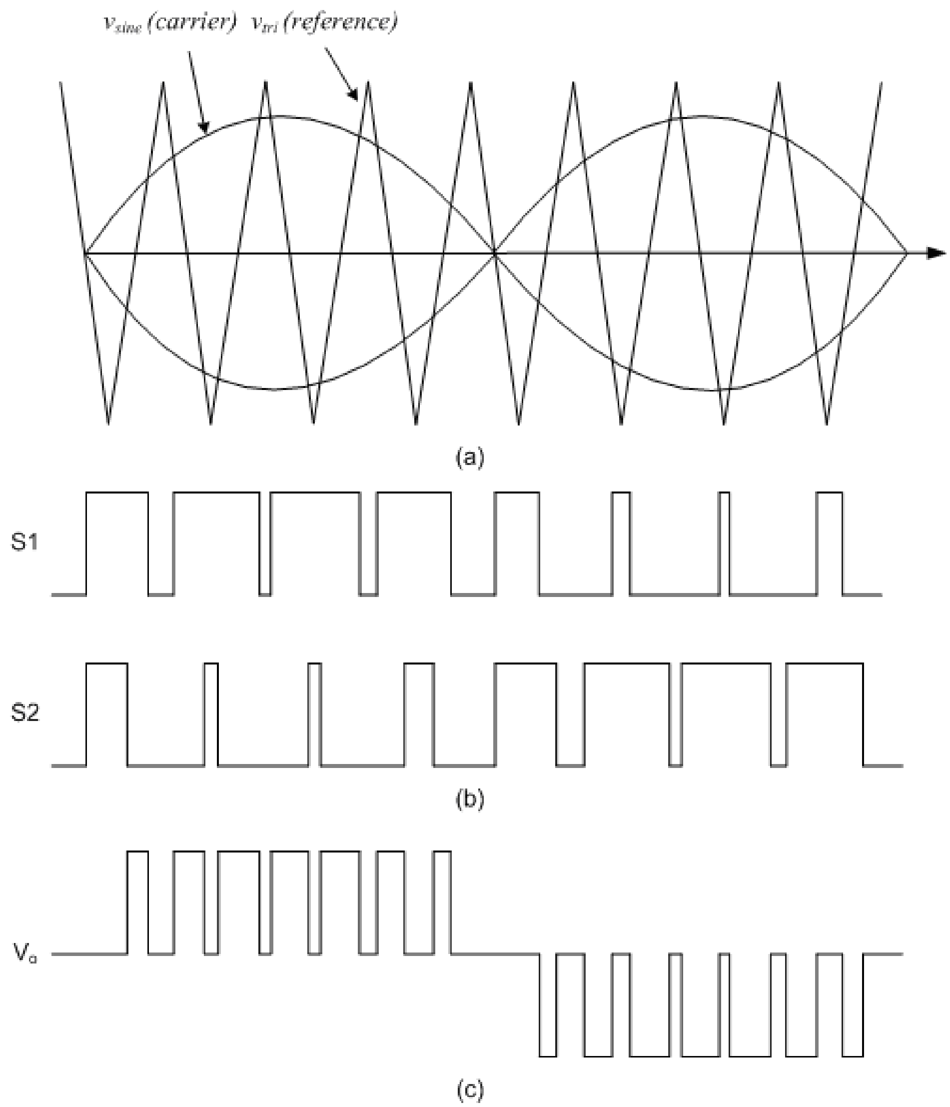


Figure 3.15 Full-bridge unipolar PWM scheme (a) Reference and carrier signal, (b) Voltage at S1 and S2, (c) Modulated output voltage

3.8 PWM Harmonics

3.8.1 Harmonics of Bipolar PWM

The Fourier series of the bipolar PWM output in Figure 3.14 is determined by examining each pulse. The triangular waveform is synchronized to its reference signal and the modulation index m_f is chosen as an odd integer. If assuming the PWM output is symmetry, the harmonics of each *k*th PWM pulse as in Figure 3.16, the Fourier coefficient can be expressed as

$$\begin{aligned} V_{nk} &= \frac{2}{\pi}\int_0^T v(t)\sin(n\omega t)d(\omega t) \\ &= \frac{2}{\pi}\left[\int_{\alpha k}^{\alpha k+\delta k} V_{DC}\sin(n\omega t)d(\omega t) + \int_{\alpha k+\delta k}^{\alpha k+1}(-V_{DC})\sin(n\omega t)d(\omega t)\right] \end{aligned} \tag{3.29}$$

Finally, the resultant of the integration is

$$V_{nk} = \frac{2V_{DC}}{n\pi}\left[\cos n\alpha_k + \cos n\alpha_{k+1} - 2\cos n(\alpha_k + \delta_k)\right] \tag{3.30}$$

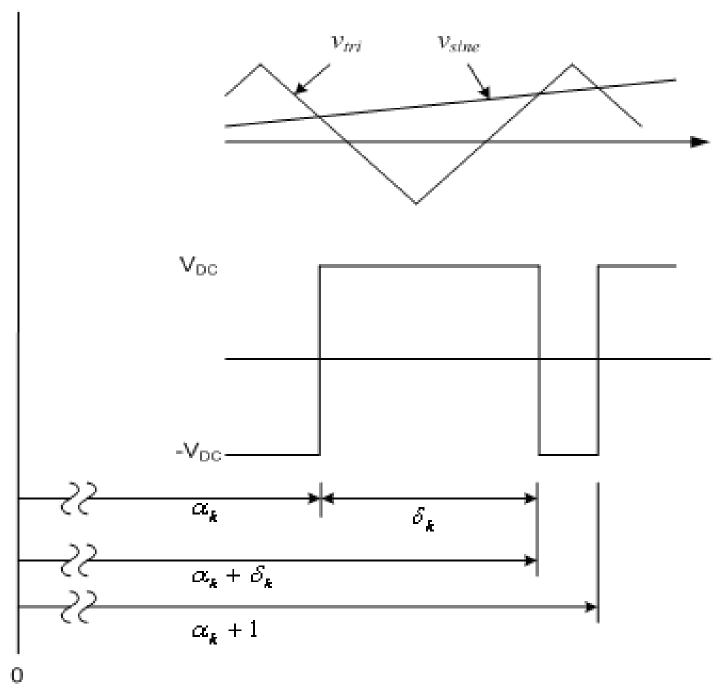


Figure 3.16 A PWM pulse for determining Fourier series of bipolar PWM

The Fourier coefficient for the PWM waveform is the sum of V_{nk} for the p pulses over one period.

$$V_n = \sum_{k=1}^{p} V_{nk} \tag{3.31}$$

The normalized frequency spectrum for bipolar switching for $m_a = 1$ is shown in Table 3.1.

Table 3.1 Normalized Fourier coefficient for bipolar PWM

	$m_a = 1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
n=1	1.00	0.90	0.80	0.70	0.60	0.50	o.40	0.30	0.20	0.10
$n = m_f$	0.60	0.71	0.82	0.92	1.01	1.08	1.15	1.20	1.24	1.27
$n = m_f \pm 2$	0.32	0.27	0.22	0.17	0.13	0.09	0.06	0.03	0.02	0.00

3.8.2 Amplitude and Harmonics Control

The output voltage of the full-bridge inverter can be controlled by adjusting the interval of α on each side of the pulse as zero. Figure 3.17(a) shows the interval of α when the output is zero at each side of the pulse while Figure 3.17(b) indicates the inverter switching sequence.

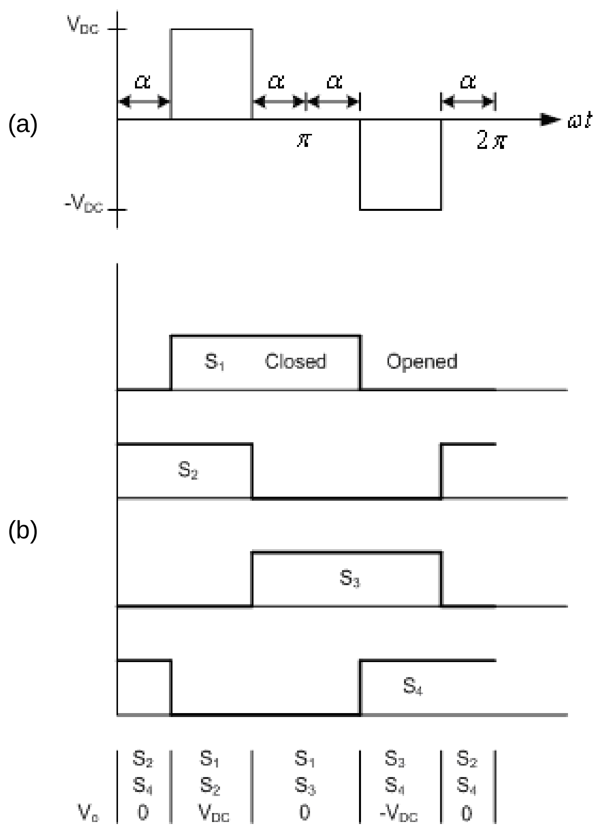


Figure 3.17 Inverter output for the amplitude and harmonic control

The rms value of the voltage waveform is

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{DC}^2 d(\omega t)} = V_{DC}\sqrt{1-\frac{2\alpha}{\pi}} \tag{3.32}$$

The Fourier series of the waveform is expressed as

$$v_O(t) = \sum_{n,odd} V_n \sin(n\omega t) \tag{3.33}$$

The amplitude of half-wave symmetry is

$$V_n = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} V_{DC}\sin(n\omega t)d(\omega t) = \left(\frac{4V_{DC}}{n\pi}\right)\cos(n\alpha) \tag{3.34}$$

The amplitude of the fundamental frequency is controllable by adjusting the angle of α.

$$V_1 = \left(\frac{4V_{DC}}{\pi}\right)\cos\alpha \tag{3.35}$$

The *n*th harmonic can be eliminated by proper choice of displacement angle α if

$\cos n\alpha = 0$

or

$$\alpha = \frac{90^\circ}{n} \tag{3.36}$$

Thus, the third harmonic can be eliminated if $\alpha = \frac{90^o}{3} = 30^o$. This is called as 'triplens' harmonics effect for *n*th harmonics.

Example 3.5

The inverter has a resistive load of 10Ω and inductive load of 25mH connected in series with the fundamental frequency current amplitude of 9.27A. The THD of the inverter is not more than 10%. If at the beginning of designing the inverter, the THD of the current is 16.7% which is does not meet the specification, find the voltage amplitude at the fundamental frequency, the required DC input supply and the new THD of the current.

Solution

The require voltage amplitude at the fundamental frequency

$$V_1 = I_1 Z_1 = I_1\sqrt{R^2 + (\omega L)^2} = (9.27)\left(\sqrt{(10)^2 + (2\pi 50x0.025)^2}\right) = 117.87V$$

The THD can be reduced by eliminate the third harmonic which is n=3

Therefore the required DC input voltage is

$$V_{DC} = \frac{V_1\pi}{4\cos\alpha} = \frac{(117.87)\pi}{4\cos(30^\circ)} = 107V$$

The THD of load current now is

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{1,rms}}$$

$$= \frac{\sqrt{\left(\frac{0.53}{\sqrt{2}}\right)^2 + \left(\frac{0.27}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2}}{\left(\frac{9.27}{\sqrt{2}}\right)}$$

$$= 0.066$$

$$= 6.6\%$$

which is has meet the specifications.

Example 3.6

The single-phase full-bridge inverter is used to produce a 60Hz voltage across a series R-L load using bipolar PWM. The DC input to the bridge is 100V, the amplitude modulation ratio is 0.8, and the frequency modulation ratio is 21. The load has resistance of R = 10Ω and inductance L = 20mH. Determine:

(a) The amplitude of the 60Hz component of the output voltage and load current.

(b) The power absorbed by the load resistor

(c) The THD of the load current

Solution

(a) Using Table 3.1, the amplitude of the 60Hz fundamental frequency is

$$V_1 = m_a V_{DC} = (0.8)(100) = 80V$$

The fundamental current amplitude is

$$I_1 = \frac{V_1}{Z_1} = \frac{80}{\sqrt{(10)^2 + [(2\pi 60)(0.02)]^2}} = 6.39A$$

(b) With mf = 21, the first harmonics are at n = 21, 19, and 23. Using Table 3.1,

$$V_{21} = (0.82)(100) = 82V$$
$$V_{19} = V_{23} = (0.22)(100) = 22V$$

Current at each harmonics is determine by

$$I_n = \frac{V_n}{Z_n}$$

and power at each frequency is determined by

$$P_n = (I_{n,rms})^2 R$$

The voltage amplitudes, current, and power at each frequencies is depicted in Table 3.2

Table 3.2 Voltage amplitudes, current, and power for each frequencies

n	f_n (Hz)	V_n (V)	Z_n (Ω)	I_n (A)	*In,rms* (A)	P_n (W)
1	60	80.0	12.5	6.39	4.52	204.0
19	1140	22.0	143.6	0.15	0.11	0.1
21	1260	81.8	158.7	0.52	0.36	1.3
23	1380	22.0	173.7	0.13	0.09	0.1

Therefore, the power absorbed by the load is

$$P = \sum P_n \approx 204.0 + 0.1 + 1.3 + 0.1 = 205.5W$$

(c) The THD of the load current is

$$THD_i = \frac{\sqrt{(0.11)^2 + (0.36)^2 + (0.09)^2}}{4.52}$$
$$= 0.087$$
$$= 8.7\%$$

3.9 Three-phase PWM Inverter

Three phase PWM inverters are controlled in the same way as a single-phase PWM inverter. Three sinusoidal modulating signals at the frequency of the desired output are compared with the triangular carrier waveform of suitably high frequency. The resulting switching signals from each comparator are used to drive the inverter switches of the corresponding leg. The switching signals for each inverter leg are complementary, and the switching signals for each switch span 120^o. These are shown in Figure 3.18

It should be noted from the above waveforms in Figure 3.19 that an identical amount of DC voltage exists in each line-neutral voltage V_{AN} and V_{BN} when these are measured with respect to the negative DC link voltage bus. The dc components are canceled when V_{AB} is obtained by subtracting V_{BN} from V_{AN}. It should also be noted that the V_{AB} waveform is 30^0 ahead of the control voltage (*ec A*) for phase A.

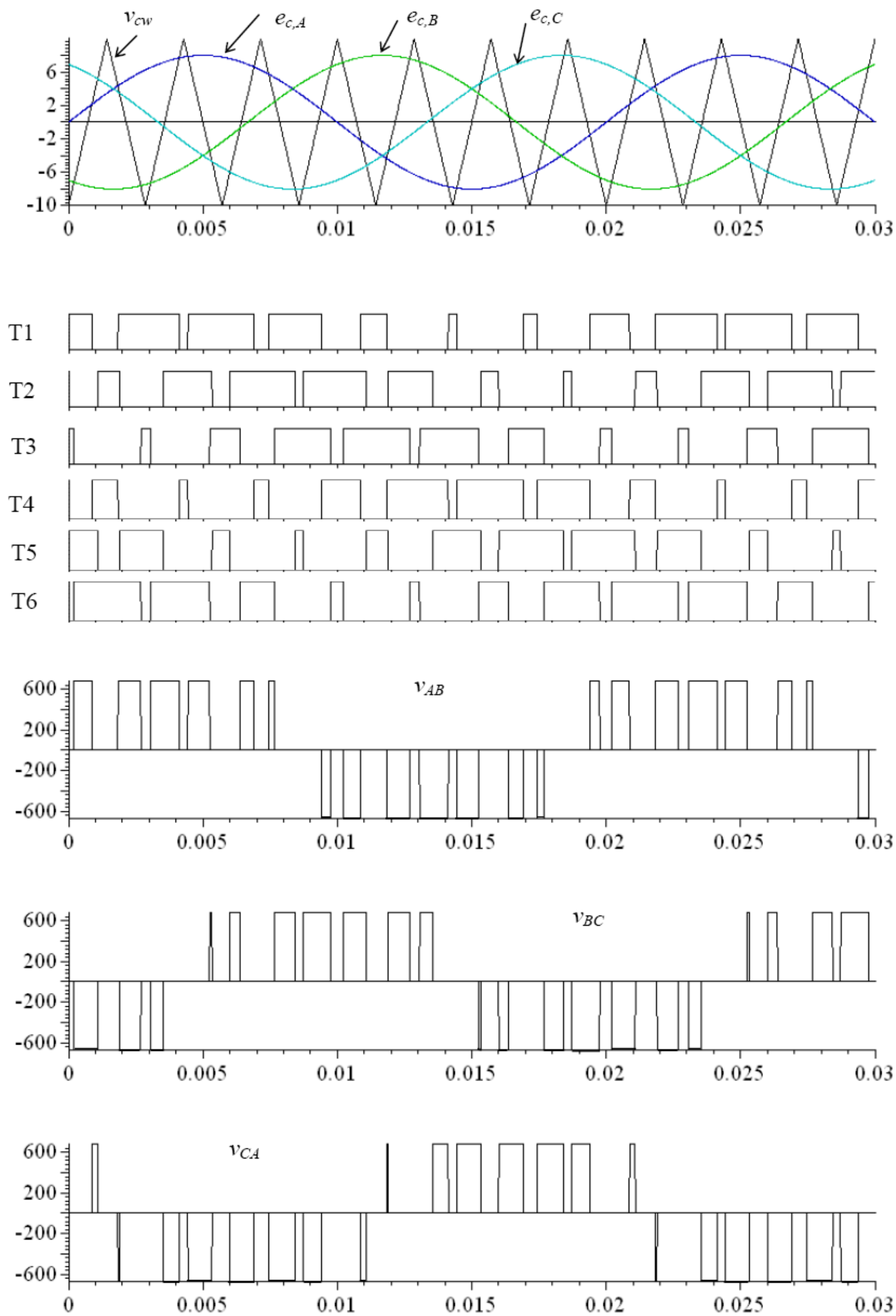


Figure 3.18 Three-phase inverter switching signal for each switch and the output line voltage

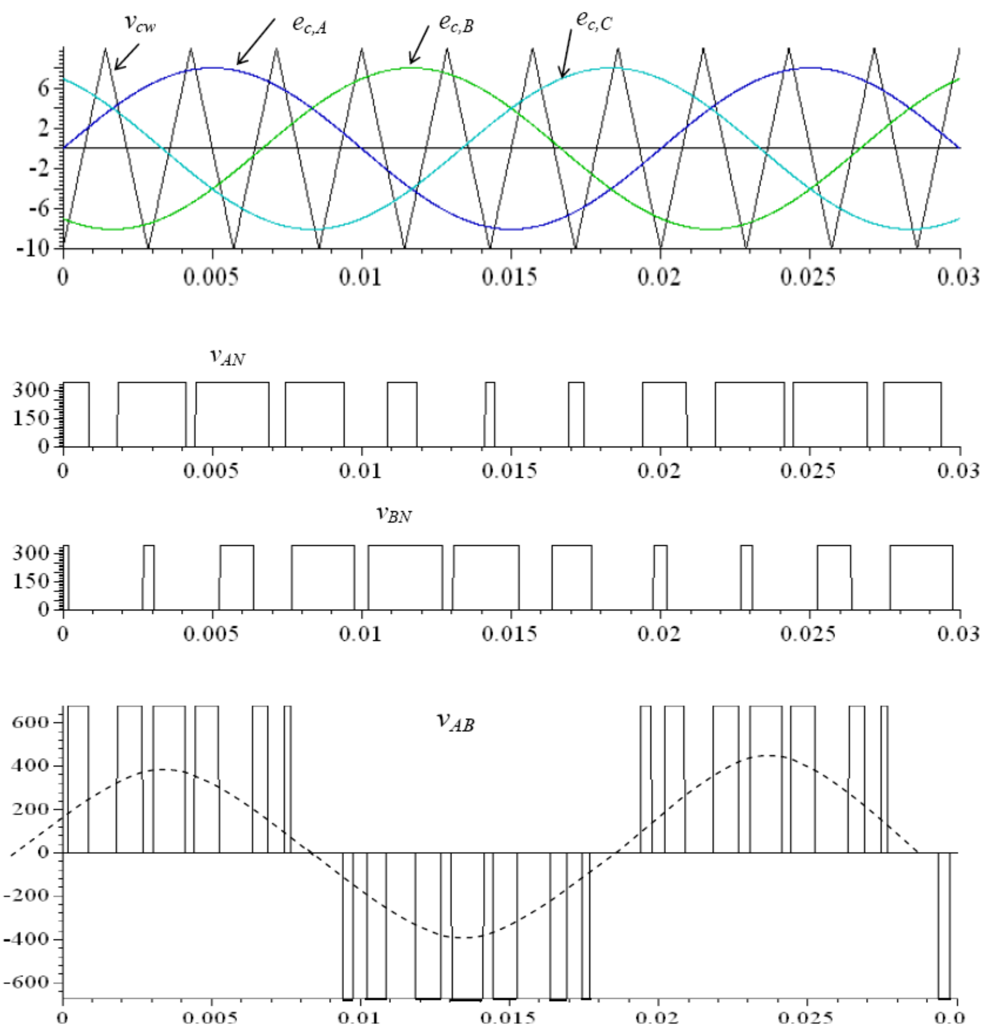


Figure 3.19 Phase voltage switching signal of three phase inverter