UNIVERSITY OF ANBAR

COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING DEPARTMENT

Power Electronic

Fourth Class

Chapter 04

DC to DC Convertor

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CHAPTER 4

DC-DC CONVERTERS

Introduction to DC-DC Converter

DC-DC converters are power electronics circuits that convert a DC voltage to a different DC voltage level, often providing a regulated output. In other word converting the unregulated DC input to a controlled DC output with a desired voltage level. DC converters are widely used for traction control in electric automobiles, trolley cars, forklift trucks and mines haulers. They provide smooth acceleration control, high efficiency and fast dynamic response. It can be used in regenerative braking of dc motor to return energy back into supply, and this feature results in energy savings for transportation system with frequent stops. The circuits described in this chapter are classified as switched-mode DC-DC converters, also called switching power supplies or switchers. The chapter will describe some basic DC-DC converter circuits.

4.1 A Basics Linear Voltage Regulators

One method of converting a DC voltage to a lower DC voltage is the simple circuit shown in Fig. 4.1.

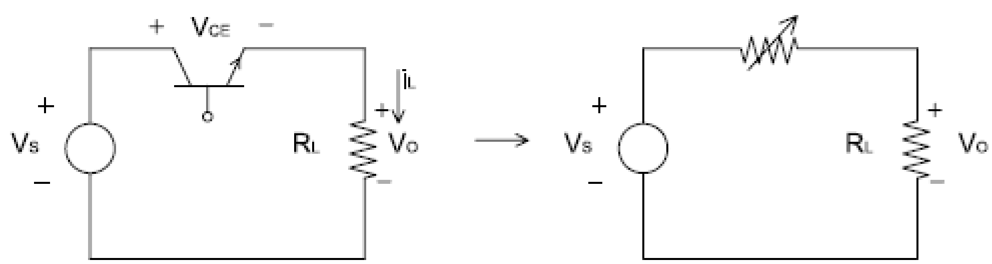

Figure 4.1: A basic linear regulators.

The output voltage is,

$$V_O = i_L R_L ,$$

The load current is controlled by the transistor. By adjusting the transistor base current, the output voltage may be controlled over a range of 0 to roughly V_S. The base current can be adjusted to compensate for variations in the supply voltage or the load, thus regulating the output. This type of circuit is called a linear DC-DC converter or a linear regulator because the transistor operates in the linear region, rather than in the saturation or cutoff region. The transistor, in effect, operates as a variable resistance.

While this may a simple way of converting a dc supply voltage to a lower dc voltage and regulating the output, the low efficiency of this circuit is a serious drawback for power applications. The power absorbed by the load is $V_o I_L$, and the power absorbed by the transistor is $V_{CE} I_L$, assuming a small base current. The power loss in the transistor makes this circuit inefficient. For example, if the output voltage is one-quarter of the input voltage, the load resistor absorbs one-quarter of the source power, which is efficiency of 25%. The transistor absorbs the other 75% of the power supplied. Lower output voltages result in even lower efficiencies.

4.2 A Basic Switching Converter

In a switching converter circuit, the transistor operates as an electronic switch by being completely on or completely off (saturation or cutoff for a BJT). This circuit also known as a DC chopper.

Assuming the switch is ideal in Fig. 4.2, the output is the same as the input when the switch is closed, and the output is zero when the switch is open. Periodic opening and closing of the switch result in the pulse output shown in Fig. 4.2(c).

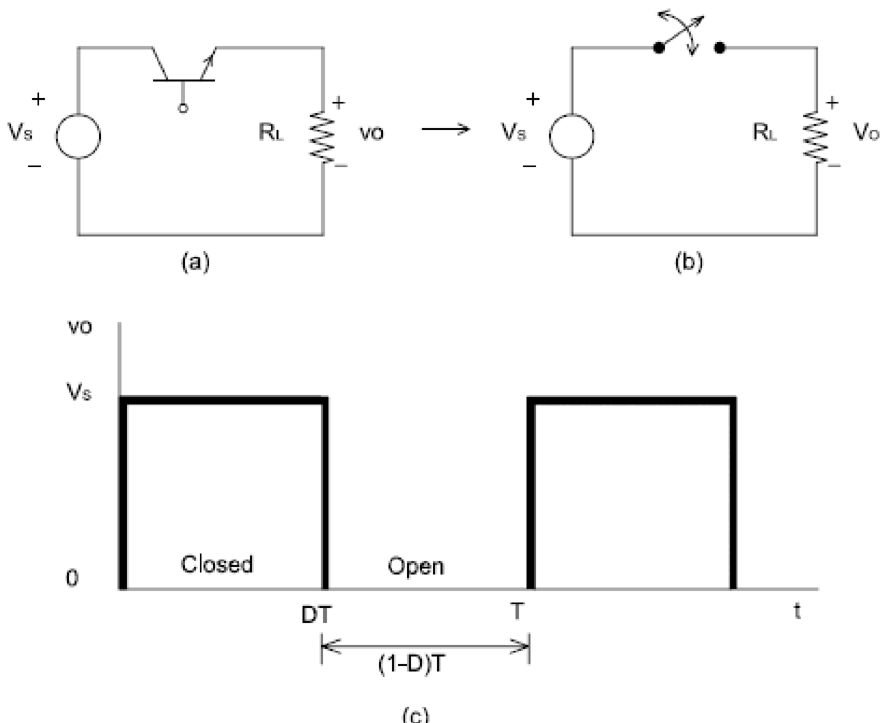

Figure 4.2 (a) A basic dc-dc switching converter (b) Switching equivalent (c) Output voltage.

The average or DC component of the output is;

$$V_O = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\int_0^{DT} V_S dt = V_S D \tag{4.1}$$

The DC component of the output is controlled by adjusting the duty ratio *D*, which is the fraction of the period that the switch is closed:

$$D = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} = t_{on}\, f \qquad (4.2)$$

Where, *f* is the switching frequency in hertz. The DC component of the output will be less than or equal to the input for this circuit. The power absorbed by the ideal switch is zero. When the switch is open, there is no current in it: when the switch is closed, there is no voltage across it. Therefore, all power is absorbed by the load, and the energy efficiency is 100%. Losses will occur in a real switch because the voltage across it will not be zero when it is on and the switch must pass through the linear region when making a transition from one state to the other.

4.3 The Buck Converter

Controlling the dc component of a pulse output of the type in Fig. 4.2(c) may be sufficient for some applications, but often the objective is to produce an output that is purely dc. One way of obtaining a do output from the circuit of Fig. 4.2a is to insert a low-pass filter after the switch. Figure 4.3(a) shows an inductor-capacitor (L-C) low-pass filter added to the basic converter. The diode provides a path for the inductor current when the switch is opened and is reverse biased when the switch is closed. This circuit is called a buck converter or a down converter because the output voltage is less than the input.

4.3.1 Voltage and Current Relationships

If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter. The input to the filter, v_x in Fig. 4.3(a), is V_s when the switch is closed and is zero when the switch is open, provided that the inductor current remains positive, keeping the diode on. If the switch is closed periodically at a duty ratio *D*, the average voltage at the filter input is $V_s D$, as seen by Equation 4.1.

This analysis assumes that the diode remains forward biased for the entire time that the switch is open, implying that the inductor current remains positive. An inductor current that remains positive throughout the switching period is known as *continuous current*. Conversely, discontinuous current is characterized by the inductor current returning to zero during each period.

Another way of analyzing the operation of the buck convener of Fig. 4.3(a) is to examine the inductor voltage and current. This analysis method will prove useful for designing the filter and for analyzing circuits that are presented later in this chapter.

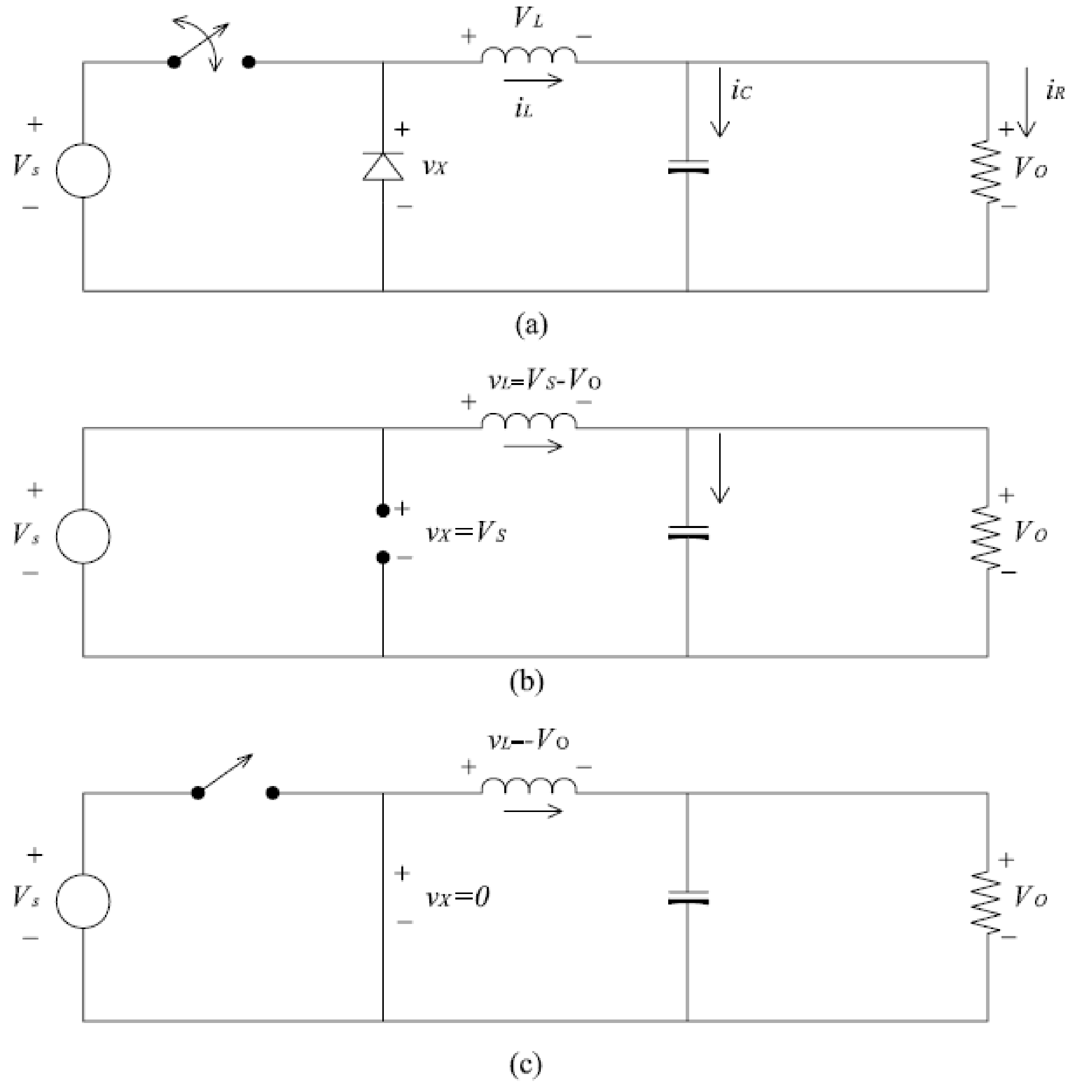

Figure 4.3 (a) Buck dc-dc converter
(b) Equivalent for switch closed
(c) Equivalent for switch open.

The buck converters (and dc-dc converters in general) have the following properties when operating in the steady state:

1. The inductor current is periodic:

$$i_L(t+T) = i_L(t). \tag{4.3}$$

2. The average inductor voltage is zero:

$$V_L = \frac{1}{T}\int_{t}^{1+T} v_L(\lambda)\,d\lambda = 0 \tag{4.4}$$

3. The average capacitor current is zero:

$$I_C = \frac{1}{T}\int_{t}^{1+T} i_C(\lambda)d\lambda = 0 \qquad (4.5)$$

4. The power supplied by the source is the same as the power delivered to the load. For nonideal components, the source also supplies the losses:

$$P_s = P_o \text{ (ideal)}$$
$$P_s = P_o + \text{losses (nonideal)}. \qquad (4.6)$$

Analysis of the buck converter of Fig. 4.3(a) begins by making these assumptions:

1. The circuit is operating in the steady state.
2. The inductor current is continuous (always positive).
3. The capacitor is very large, and the output voltage is held constant at voltage V_o. This restriction will be relaxed later to show the effects of finite capacitance.
4. The switching period is T; the switch is closed for time DT and opens for time $(I -D)T$.
5. The components are ideal

.

The key to the analysis for determining the output V_o is to examine the inductor current and inductor voltage first for the switch closed and then for the switch open. The net change in inductor current over one period must be zero for steady-state operation. The average inductor voltage is zero.

Analysis for the switch closed. When the switch is closed in the buck converter circuit of Fig. 4.3a the diode is reverses biased and Fig. 4.3b is an equivalent circuit. The voltage across the inductor is

$$v_L = V_S - V_O = L\frac{di_L}{dt},$$

Rearranging,

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L} \quad \text{(switch closed).}$$

Since the derivative of the current is a positive constant, the current increases linearly, as shown in Fig. 4.4b. The change in current while the switch is closed is computed by modifying the preceding equation:

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L} \right) DT \quad (4.7)$$

Analysis for the switch open. When the switch is open, the diode becomes forward biased to carry the inductor current, and the equivalent circuit of Fig. 4.3(c) applies. The voltage across the inductor when the switch is open is

$$v_L = -V_o = L\frac{di_L}{dt}$$

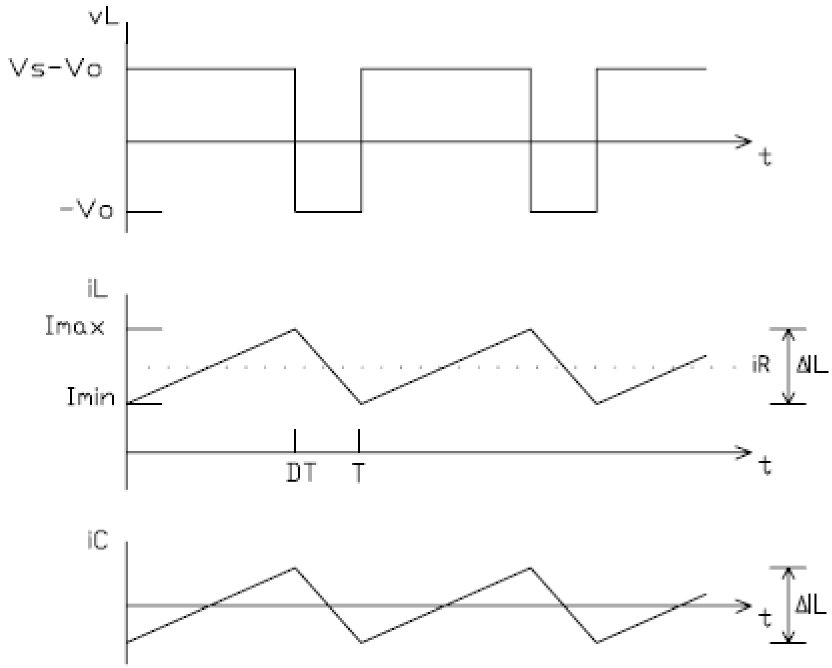

Figure 4.4 Buck converter waveforms, (a) Inductor Voltage. (b) Inductor current, (c) Capacitor current.

Rearranging,

$$\frac{di_L}{dt} = \frac{-V_o}{L}, \text{ (switch open).}$$

The derivative of current in the inductor is a negative constant and the current decreases linearly, as shown in Fig. 4.4(b). The change in inductor current when the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$(\Delta i_L)_{open} = -\left(\frac{V_O}{L}\right)(1-D)T \qquad (4.8)$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{closed} + (\Delta i_L)_{open} = 0.$$

Using Eqs. 4.7 and 4.8,

$$\left(\frac{V_s - V_o}{L}\right)DT - \left(\frac{V_o}{L}\right)(1-D)T = 0$$

Solving for Vo,

$$V_O = V_S D \qquad (4.9)$$

which is the same result as Eq. 4.1. *The buck converter produces an output* which is less *than or equal to the input.*

An alternative derivation of the output voltage is base on the inductor voltage, as shown in Fig. 4.4(a). Since the average inductor voltage is zero for periodic operation.

$$V_L = (V_s - V_o)DT + (-V_o)(1 - D)T = 0.$$

Solving the preceding equation for V_o yields the same result as Eq. 4.9, $V_o, = V_s D$.

Note that the output voltage depends only on the input and the duty ratio *D*. If the input voltage fluctuates, the output voltage can be regulated by adjusting the duty ratio appropriately. A feedback loop is required to sample the output voltage, compare it to a reference, and set the duty ratio of the switch accordingly.

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R} \tag{4.10}$$

Since the change in inductor current is known from Eqs. 4.7 and 4.8, the maximum and minimum values of the inductor current are computed as

$$I_{max} = I_L + \frac{\Delta i_L}{2}$$
$$= \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left[\frac{1}{R} + \frac{(1-D)}{2Lf}\right] \tag{4.11}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2}$$
$$= \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = Vo\left[\frac{1}{R} - \frac{(1-D)}{2Lf}\right] \tag{4.12}$$

where $f = 1/T$ is the switching frequency in hertz.

For the preceding analysis to be valid, continuous current in the inductor must he verified. An easy check for continuous current is to calculate the minimum inductor current from Eq. 4.12. Since the minimum value of inductor current must be positive for continuous current, a negative minimum calculated from Eq. 4.12 is not allowable due to the diode and indicates discontinuous current. The circuit will operate for discontinuous inductor current, but the preceding analysis is not valid. Discontinuous current operation is discussed later in this chapter.

Equation 4.12 can be used to determine the combination of L and f that will result in continuous current. Since $I_{min} = 0$ is the boundary between continuous and discontinuous current,

$$I_{min} = 0 = V_o\left[\frac{1}{R} - \frac{(1-D)}{2Lf}\right] \tag{4.13}$$

If the desired switching frequency is established,

$$L_{min} = \frac{(1-D)R}{2f} \tag{4.14}$$

where L_{min}, is the minimum inductance required for continuous current.

4.3.2 Output Voltage Ripple

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

Shown in Fig. 4.5(a),
While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$$Q = CV_o$$

$$\Delta Q = C\Delta V_o$$

$$\Delta V_o = \frac{\Delta Q}{C}$$

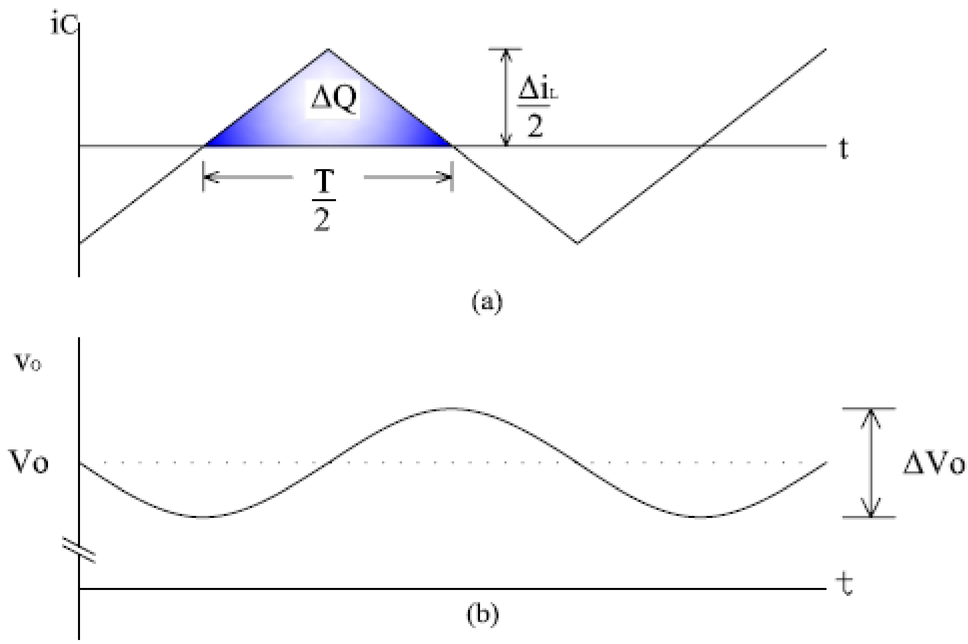

Figure 4.5 Buck converter waveforms
(a) Capacitor current, (b) Capacitor ripple voltage.

The change in charge, ΔQ, is the area of the triangle above the time axis:

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

Resulting in

$$\Delta V_o = \frac{T\Delta i_L}{8C}$$

Using Eq. 4.8 for Δi_L,

$$\Delta V_o = \frac{T}{8C}\frac{V_o}{L}(1-D)T = \frac{V_o(1-D)}{8LCf^2} \tag{4.15}$$

In this equation, ΔV_o is the peak-to-peak ripple voltage at the output, as shown in Fig. 4.5(b). It is also useful to express the ripple as a fraction of the output voltage:

$$\mathbf{\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}} \tag{4.16}$$

If the ripple is not large, the assumption of a constant output is reasonable and the preceding analysis is essentially valid. Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor:

$$P_s = P_o$$
$$V_s I_s = V_o I_o \tag{4.17}$$

$$or \quad \frac{V_o}{V_s} = \frac{I_s}{I_o}$$

Note that the preceding relationship is similar to the voltage-current relationship for transformer in ac applications. Therefore, the buck converter circuit is equivalent to a dc transformer.

Example 4.1 Buck Converter

The buck dc-dc converter of Fig. 4.3(a) has the following parameters:

$V_s = 50$V
$D = 0.4$
$L = 400\mu$H
C $=100\mu$F
$F = 20$ kHz
$R = 20\ \Omega$

Assuming ideal components, calculate;

(a) The output voltage V_o
(b) The maximum and minimum inductor current
(c) The output voltage ripple

Solution

(a) The inductor current is assumed to be continuous, and the output voltage is computed front Eq. 4.9:

$$V_o = V_s D = (50)(0.4) = 20 \text{ V}.$$

(b) Maximum and minimum inductor current are computed from eqs. 4.11 and 4.12:

$$I_{max} = Vo\left[\frac{1}{R} + \frac{1-D}{2Lf}\right]$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}\,20(10)^{3}}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75A$$

$$I_{min} = Vo\left[\frac{1}{R} - \frac{1-D}{2Lf}\right]$$

$$= 1 - \frac{1.5}{2} = 0.25A$$

The average inductor current is 1A, and $\Delta iL = 1.5A.$ Note that the minimum inductor current is positive, verifying that the assumption of continuous current was valid.

(c) The output voltage, ripple is computed from Eq. 4.16:

$$\frac{\Delta Vo}{Vo} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20000)^2}$$

$$= 0.00469 = 0.469\%.$$

Since the output ripple is sufficiently small, the assumption of a constant output voltage was reasonable.

4.4 Design Considerations

Most buck converters are designed for continuous-current operation. The choice of switch-in- frequency and inductance to give continuous current is given by Eq. 4.13, and the output ripple is described by Eq. 4.16. Note that as the switching frequency increases, the minimum size of the inductor to produce continuous current and the minimum size of the capacitor to limit output ripple both decrease. Therefore, high switching frequencies are desirable to reduce the size of both the inductor and the capacitor.

The trade-off for high switching frequencies is increased power loss in the switches, which is discussed later in this chapter. Increased power loss for the switches decreases the converter's efficiency, and the larger heat sink required for the transistor switch offsets the reduction in size of the inductor and capacitor. Typical switching frequencies are in the 20-kHz to 50-kHz range, although frequencies in the hundreds of kilohertz are not uncommon. As switching devices improve, switching frequencies will increase.

The inductor wire must be rated at the rms current, and the core should not saturate for peak inductor current. The capacitor must be selected to limit the output ripple to the design specifications, to withstand peak output voltage, and to carry the required rms current.

The switch and diode must withstand maximum voltage stress when off and maximum current when on. The temperature ratings must not be exceeded, possibly requiring a heat sink.

Example 4.2 Buck Converter Design

Design a buck convener to produce an output voltage of 18 V across a 10-Ω load resistor. The output voltage ripple must not exceed 0.5%. The dc supply is 48 V. Design for continuous inductor current. Specify the duty ratio, the sizes of the inductor and capacitor, the peak voltage rating of each device, and the rms current in the inductor and capacitor.

Solution

The duty ratio for continuous-current operation is determined from Eq. 4.9:

$$D = \frac{V_o}{V_s} = \frac{18}{48} = 0.375$$

The switching frequency and inductor must be selected for continuous-current operation. Let the switching frequency arbitrarily be 40 kHz, which is well above the audio range and is low enough to keep switching losses small. The minimum inductor size is determined from Eq. 4.14:

$$L_{min} = \frac{(1-D)R}{2f} = \frac{(1-0.375)10}{2(40000)} = 78\mu H$$

Let the inductor be 25% larger than the minimum to ensure that inductor current is continuous:

$$L = 1.25L_{min} = (1.25)\ (78\mu\text{H}) = 97.5\ \mu\text{H}.$$

Average inductor current and the change in current are determined from Eqs. 4.10 and 4.7

$$I_L = \frac{V_o}{R} = \frac{18}{10} = 1.8A$$

$$\Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT = \left(\frac{48-18}{97.5(10)^{-6}}\right)(0.375)\left(\frac{1}{40000}\right) = 2.88A$$

The maximum and minimum inductor currents are determined from Eqs. 4.11 and 4.12.

$$I_{max} = I_L + \frac{\Delta i_L}{2} = 1.8 + 1.44 = 3.24A$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = 1.8 - 1.44 = 0.36A$$

For the offset triangular wave,

$$I_{IL,rms} = \sqrt{I_L{}^2 + \left(\frac{\Delta i_L/2}{\sqrt{3}}\right)^2} = \sqrt{(1.8)^2 + \left(\frac{1.44}{\sqrt{3}}\right)^2} = 1.98A$$

The capacitor is selected using Eq. 4.16:

$$C = \frac{1-D}{8L\left(\frac{\Delta V_o}{V_o}\right)f^2} = \frac{1-0.375}{8(97.5)(10)^{-6}(.005)(40000)^2} = 100\mu F$$

Peak capacitor current is $\Delta iL/2 = 1.44A$, and rms capacitor current for the triangular waveform is $1.44/\sqrt{3} = 0.83A$. .

The maximum voltage across the switch and diode is Vs or 48 V. The inductor voltage when the switch is closed is Vs - Vo = 48 -18 = 30 V. The inductor voltage when the switch is open is Vo = 18 V. Therefore, the inductor must withstand 30 V. The capacitor must be rated for the 18 V output.

4.5 The Boost Converter

The boost converter is shown in Fig. 4.6. This is another switching converter that operates by periodically opening and closing an electronic switch. It is called a boost converter because the output voltage is larger than the input.

4.5.1 Voltage and Current Relationships

The analysis assumes the following:

1. Steady-state conditions exist.
2. The switching period is *T*. and the switch is closed for time *DT* and open for *(1-D)T*.
3. The inductor current is continuous (always positive).
4. The capacitor is very large, and the output voltage is held constant at voltage V_o.
5. The components are ideal.

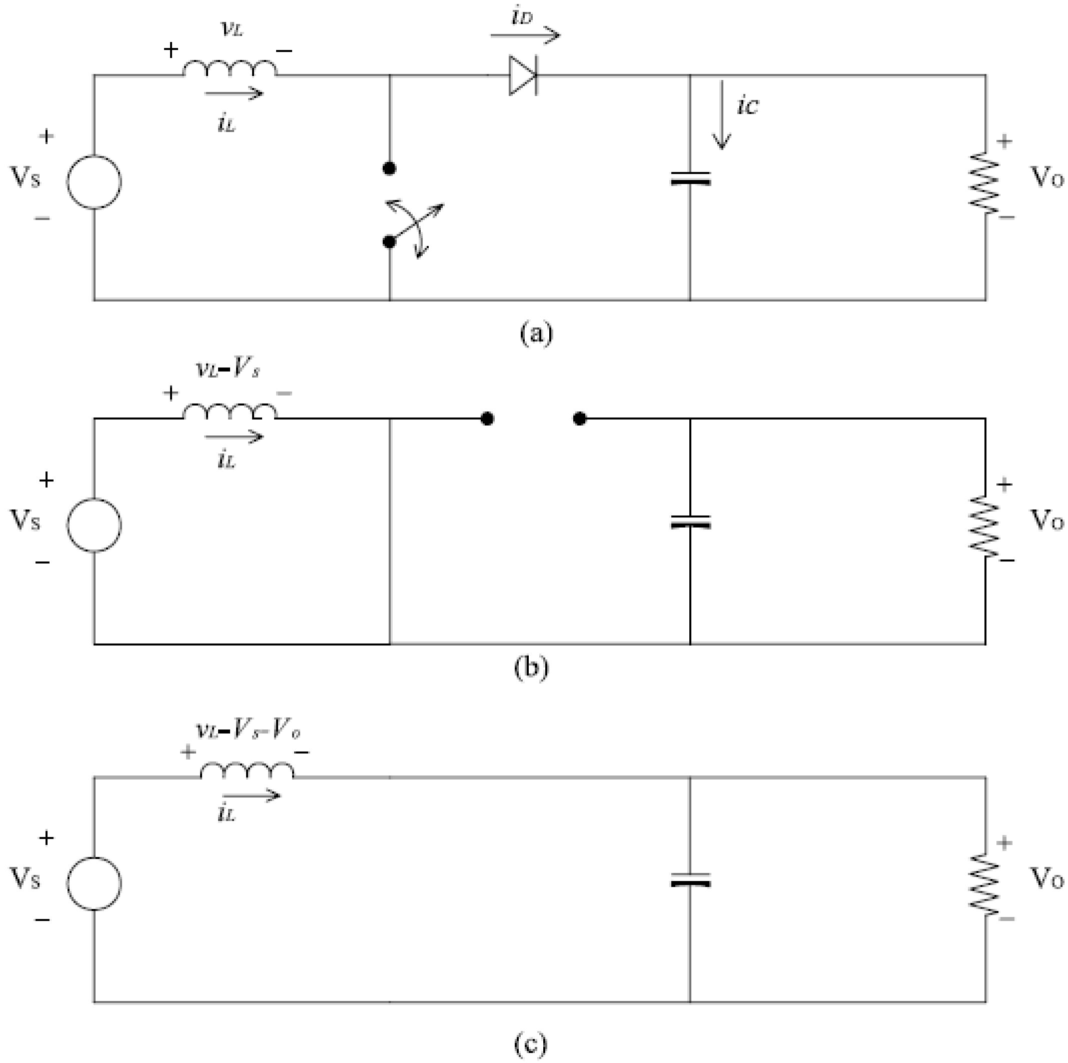

Figure 4.6 The boost converter: (a) Circuit. (b) Equivalent for the switch closed. (c) Equivalent for the switch open.

The analysis proceeds by examining the inductor voltage and current for the switch closed and again for the switch open.

Analysis for the switch closed. When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$v_L = V_S = L\frac{di_L}{dt} \quad or \quad \frac{di_L}{dt} = \frac{V_S}{L} \tag{4.18}$$

The rate of change of current is n constant, so the current increases-linearly while the switch is closed, as shown in pig. 4.7(b). The change in inductor current is computed from

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_S}{L}$$

Solving for $Jr_{l,}$ for the switch closed.

$$(\Delta i_L)_{closed} = \frac{V_S DT}{L} \tag{4.19}$$

Analysis for the switch open. When the switch is opened, the inductor current cannot change instantly, so the diode becomes forward biased to provide a path for inductor current. Assuming that the output voltage V_o is a constant, the voltage across the inductor is

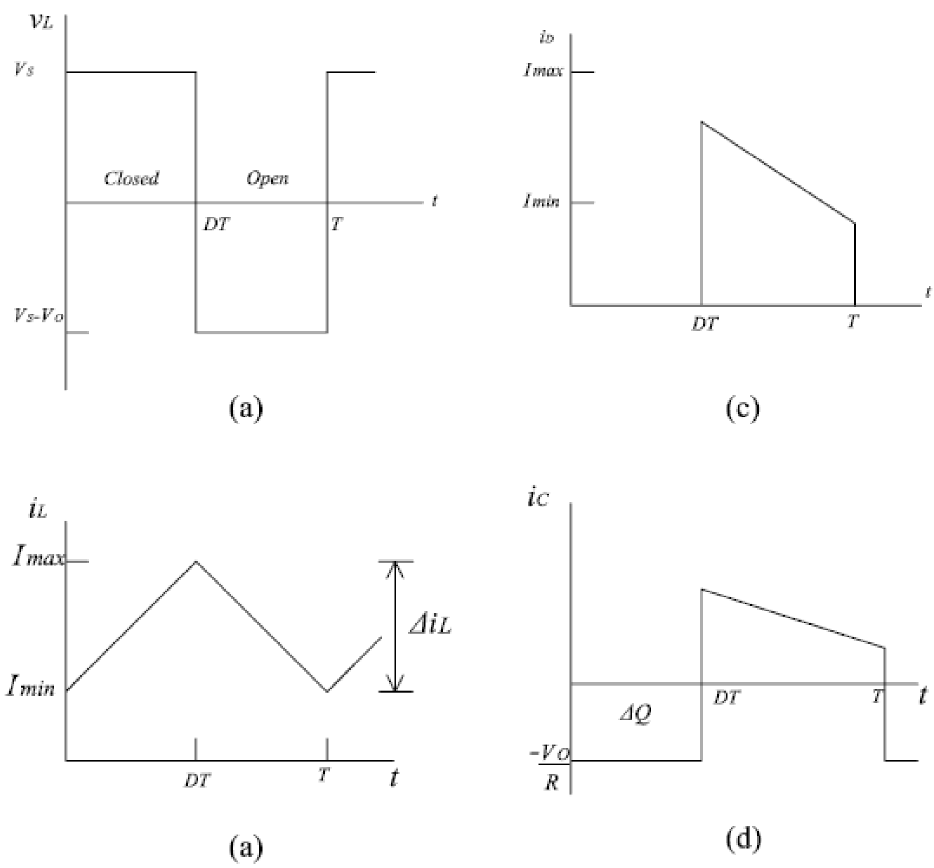

Figure 4.7 Boost converter waveforms. (a) Inductor voltage. (b) Inductor current. (c) Diode current. (d) Capacitor current.

$$v_L = V_S - V_O = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_S - V_O}{L}$$

The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_S - V_O}{L}$$

Solving for Δi_L ,

$$(\Delta i_L)_{open} = \frac{(V_S - V_O)(1-D)T}{L} \qquad (4.20)$$

For steady- state operation, the net change in inductor current must be zero. Using Eqs. 4.19 and 4.20,

$$(\Delta i_L)_{closed} + (\Delta i_L)_{open} = 0$$

$$\frac{V_S DT}{L} + \frac{(V_S - V_O)(1-D)T}{L} = 0$$

Solving for V_o,

$$V_S(D + 1 - D) - V_O(1-D) = 0$$

$$V_O = \frac{V_S}{1-D} \qquad (4.21)$$

Also, the average inductor voltage must be zero for periodic operation. Expressing the average inductor voltage over one switching period,

$$V_L = V_S D + (V_S - V_O)(1 - D) = 0.$$

Solving for V_O yields the same result as Eq. 4.21.

Equation 4.21 shows that if the switch is always open and D is zero, the output is the same as the input. As the duty ratio is increased, the denominator of Eq. 4.21 becomes smaller, and the output becomes larger than the input. *The boost converter produces an output voltage that s greater than or equal to the input voltage*. However, the output cannot be less than the input, as was the case with the buck converter.

As the duty ratio of the switch approaches one, the output goes to infinity according to Eq. 4.21. However, Eq. 4.21 is based on ideal components. Real components which include losses will prevent such an occurrence, as is shown later in this chapter. Figure 4.7 shows the voltage and current waveforms for the boost converter.

The average current in the inductor is determined by recognizing that the power supplied by the source must be the same as the power absorbed by the load resistor. Output power is

$$P_o = \frac{V_o^2}{R}$$

and input power is $V_s I_s = V_s I_L$. Equating input ad output powers and using Eq. 4.21,

$$V_s I_L = \frac{V_o^2}{R} = \frac{\left(\frac{V_s}{1-D}\right)^2}{R} = \frac{V_s^2}{(1-D)^2 R} \quad (4.22)$$

or,

$$I_L = \frac{V_s}{(1-D)^2 R} \quad (4.22)$$

Maximum and minimum inductor currents are determined by using the average value and he change in current from Eq. 4.19:

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \quad (4.23)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \quad (4.24)$$

Equation 4.21 was developed with the assumption that the inductor current is continuous, meaning that it is always positive. A condition necessary for continuous inductor current is for I_{min} to be positive. Therefore, the boundary between continuous and discontinuous inductor current is determined from

$$I_{\min} = 0 = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

or

$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is therefore

$$(Lf)_{\min} = \frac{D(1-D)^2 R}{2} \quad (4.25)$$

or

$$L_{\min} = \frac{D(1-D)^2 R}{2f} \tag{4.26}$$

Output Voltage Ripple

The preceding equations were developed on the assumption that the output voltage was a constant, implying an infinite capacitance. In practice, a finite capacitance will result in some fluctuation in output voltage, or ripple.

The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform, shown in Fig. 4.7(d). The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_O}{R}\right) DT = C\Delta V_O$$

An expression for ripple is then

$$\Delta V_O = \frac{V_O DT}{RC} = \frac{V_O D}{RCf}$$

or

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf} \tag{4.27}$$

where f is the switching frequency in hertz.

Example 4.3 Boost Converter Design

Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than 1%. The load is a resistance of 50Ω. Assume ideal components for this design.

Solution First, determine the duty ratio from Eq. 4.21:

$$D = 1 - \frac{V_S}{V_O} = 1 - \frac{12}{30} = 0.6$$

If the switching frequency is selected at 25 kHz to be above the audio range, then the minimum inductance for continuous current is determined from Eq. 4.26:

$$L_{\min} = \frac{D(1-D)^2 R}{2f} = \frac{0.6(1-0.6)^2 50}{2(25{,}000)} = 96\mu H$$

To provide a margin to ensure continuous current, let L = 120 μH. Note that L and f are selected somewhat arbitrarily and that other combinations will also give continuous current.

Using Eqs. 4.22 to 4.24,

$$I_L = \frac{V_S}{(1-D)^2 R} = \frac{12}{(1-0.6)^2 50} = 1.5A$$

$$\frac{\Delta i_L}{2} = \frac{V_S DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2A$$

$$I_{\max} = 1.5 + 1.2 = 2.7A$$

$$I_{\min} = 1.5 - 1.2 = 0.3A$$

Output ripple voltage is determined from Eq. 4.27:

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf} < 1\%$$

$$C > \frac{D}{Rf(\Delta V_O / V_O)} = \frac{0.6}{(50)(25)(10)^3(0.01)} = 48uF$$

4.6 The Buck- Boost Converter

Another basic switched- mode converter is the buck- boost converter shown in Fig. 4.8. The output of the buck- boost converter can be either higher or lower than the input voltage.

4.6.1 Voltage and Current Relationships

Assumptions made about the operation of the converter are as follows:

1. The circuit is operating in the steady state.
2. The inductor current is continuous.
3. The capacitor is large enough to assume a constant output voltage.
4. The switch is closed for time *DT* and open for *(1-D)T*.
5. The components are ideal.

Analysis for the switch closed. When the switch is closed, the voltage across the inductor is

$$v_L = V_S = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_S}{L}$$

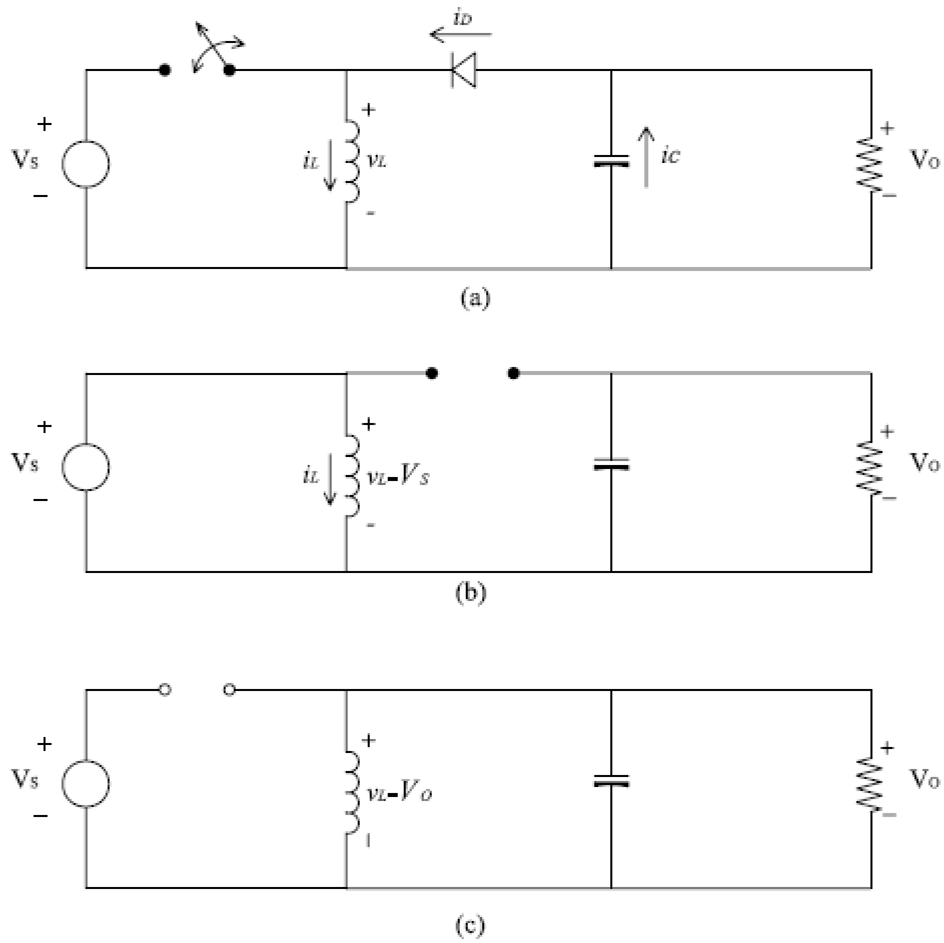

Figure 4.8 Buck- boost converter. (a) Circuit. (b) Equivalent for the switch closed. (c) Equivalent for the switch open.

The rate of change of inductor current is a constant, indicating a linearly increasing inductor current. The preceding equation can be expressed as

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_S}{L}$$

Solving for $\Delta\ i_L$ when the switch is closed,

$$(\Delta i_L)_{closed} = \frac{V_S DT}{L} \tag{4.28}$$

Analysis for the switch open. When the switch is open, the current in the inductor cannot change instantly, resulting in a forward-biased diode and current into the resistor and capacitor. In this condition, the voltage across the inductor is

$$v_L = V_O = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_O}{L}$$

Again, the rate of change of inductor current is constant, and the change in current is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_O}{L}$$

Solving for Δ i_L,

$$(\Delta i_L)_{open} = \frac{V_O(1-D)T}{L} \tag{4.29}$$

For steady- state operation, the net change in inductor current must be zero over one period. Using Eqs. 4.28 and 4.29,

$$(\Delta i_L)_{closed} + (\Delta i_L)_{open} = 0$$

$$\frac{V_S DT}{L} + \frac{V_O(1-D)T}{L} = 0$$

Solving for V_o,

$$V_O = -V_S\left[\frac{D}{1-D}\right] \tag{4.30}$$

The average inductor voltage is zero for periodic operation, resulting in

$$V_L - V_S D + V_O(1-D) - 0$$

Solving for V_o yields the same result as Eq. 4.30.

Equation 4.30 shows that the output voltage has opposite polarity from the source voltage. *Output magnitude of the buck- boost converter can be less than the source greater than the source, depending on the duty ratio of the switch.* If *D*> 0.5, the output is larger than the input, and if *D*< 0.5, the output is smaller than the input. Therefore, this circuit combines the capabilities of the buck and boost converters. Polarity reversal on the output may be a disadvantage in some applications, however. Voltage and current waveforms are shown in Fig. 4.9.

Note that the source is never connected directly to the load in the buck-boost converter. Energy is stored in the inductor when the switch is closed and transferred to the load when the switch is open. Hence, the buck- boost converter is also referred to as an *indirect* converter.

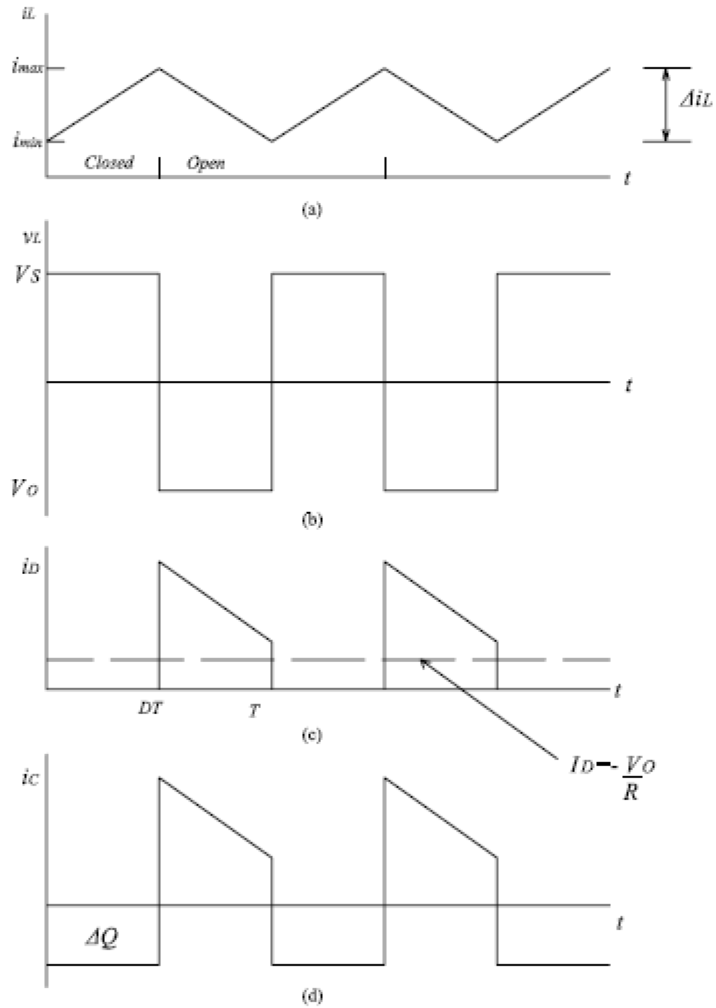

Figure 6.9 Buck- boost converter waveform. (a) Inductor current. (b) Inductor voltage. (c) Diode current. (d) Capacitor current.

Power absorbed by the load must be the same as that supplied by the source, where

$$P_O = \frac{V_O^{\ 2}}{R}$$

$$P_S = V_S I_S$$

$$\frac{V_O^{\ 2}}{R} = V_S I_S$$

Average source current is related to average inductor current by

$$I_S = I_L D$$

resulting in

$$\frac{V_O{}^2}{R} = V_S I_L D$$

Substituting for V_o using Eq. 4.30 and solving for I_L,

$$I_L = \frac{V_O{}^2}{V_S RD} = \frac{P_O}{V_S D} = \frac{V_S D}{R(1-D)^2} \quad (4.31)$$

Maximum and minimum inductor currents are determined using Eqs. 4.28 and 4.31:

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_S D}{R(1-D)^2} + \frac{V_S DT}{2L} \quad (4.32)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_S D}{R(1-D)^2} + \frac{V_S DT}{2L} \quad (4.33)$$

For continuous current, the inductor current must remain positive. To determine the boundary between continuous and discontinuous current, I_{min} is set to zero in Eq. 4.33, resulting in

$$(Lf)_{\min} = \frac{(1-D)^2 R}{2} \quad (4.34)$$

or

$$L_{\min} = \frac{(1-D)^2 R}{2f} \quad (4.35)$$

where f is the switching frequency in hertz.

Output Voltage Ripple

The output voltage ripple for the buck- boost converter is computed from the capacitor current waveform of Fig. 4.9(d):

$$|\Delta Q| = \left(\frac{V_O}{R}\right) DT = C\Delta V_O$$

Solving for ΔV_o,

$$V_O = \frac{V_O DT}{RC} = \frac{V_O D}{RCf}$$

Or

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf} \tag{4.36}$$

Example 4.4 Buck- boost Converter

The buck- boost circuit of Fig- 4.8 has these parameters:

V_s= 24V

D= 0.4

R= 5Ω

L= 100 μH

C= 400 μF

f= 20 kHz

Determine the output voltage, inductor current, and output ripple.

Solution Output voltage is determined from Eq. 4.30:

$$V_O = -V_S\left(\frac{D}{1-D}\right) = -24\left(\frac{0.4}{1-0.4}\right) = -16V$$

Inductor current is described by Eqs. 4.31 to 4.33:

I_L= 5.33 A

i_{max}= 7.73 A

I_{min}= 2.93 A

Continuous current is verified by I_{min}> 0. Output ripple is determined from Eq. 4.36:

$$\frac{\Delta V_O}{V_O} = \frac{D}{RCf} = \frac{0.4}{(5)(400)(10)^{-6}\,20(10)^3} = 0.01 = 1\%$$

4.7 The Ćuk Converter

The Ćuk switching topology is shown in Fig. 4.10(a). Output voltage magnitude can be either larger or smaller than the input, and there is a polarity reversal on the output.

The inductor on the input acts as a filter for the dc supply, to prevent large harmonic content. Unlike the previous converter topologies, where energy transfer is associated with the inductor, energy transfer for the Ćuk converter depends on the capacitor C_1.

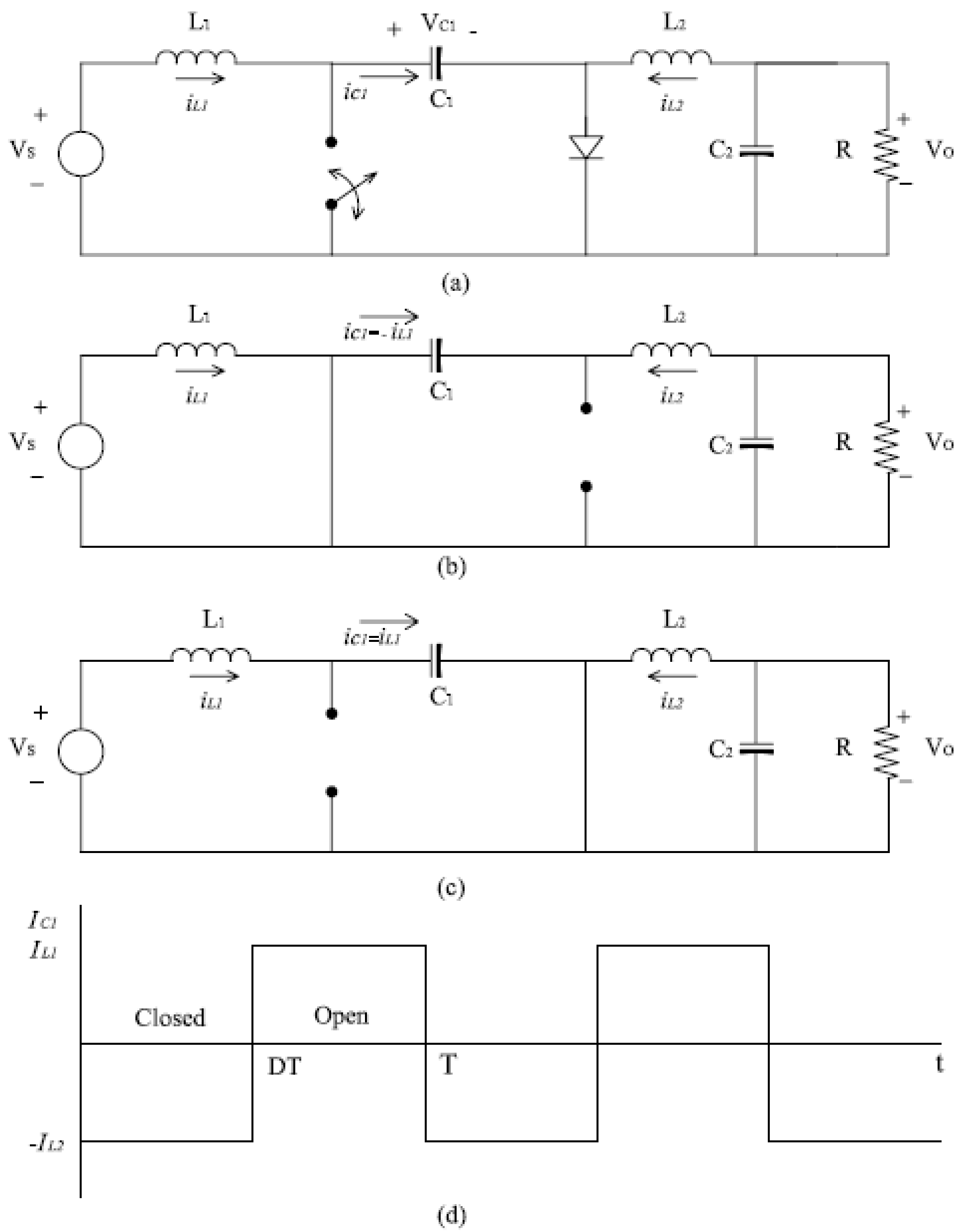

Figure 4.10 The Ćuk converter. (a) Circuit. (b) Equivalent for the switch closed. (c) Equivalent for the switch open. (d) Current in L_1 for a large inductance.

The analysis begins with these assumptions:

1. Both inductors are very large and the currents in them are constant.
2. Both capacitors are very large and the voltages across them are constant.
3. The circuit is operating in the steady state, meaning that voltage and current waveforms are periodic.
4. For a duty ratio of D, the switch is closed for time DT and open for (1-D)T.
5. The switch and the diode are ideal.

The average voltage across C_1 is computed from Kirchoff's voltage law around the outermost loop. The average voltage across the inductors is zero for steady- state operation, resulting in

$$V_{C1} = V_S - V_O$$

With the switch closed, the diode is off and the current in capacitor C_1 is

$$(i_{C1})_{closed} = -I_{L2} \tag{4.37}$$

With the switch open, the currents in L_1 and L_2 force the diode on. The current in capacitor C_1 is

$$(i_{C1})_{open} = I_{L1} \tag{4.38}$$

The power absorbed by the load is equal to the power supplied by the source:

$$-V_O I_{L2} = V_S I_{L1} \tag{4.39}$$

For periodic operation, the average capacitor current is zero. With the switch on for time DT and off for (1-D)T,

$$\left[(i_{C1})_{closed}\right]DT + \left[(i_{C1})_{open}\right](1-D)T = 0$$

Substituting using Eqs. 4.37 and 4.38,

$$-I_{L2}DT + I_{L1}(1-D) = 0$$

$$or \qquad \frac{I_{L1}}{I_{L2}} = \frac{D}{1-D} \tag{4.40}$$

Next, the average power supplied by the source must be the same as the average power absorbed by the load:

$$P_S = P_O$$

$$V_S I_{L1} = -V_O I_{L2}$$

$$\frac{I_{L1}}{I_{L2}} = \frac{-V_O}{V_S} \tag{4.41}$$

Combining Eqs. 4.40 and 4.41, the relationship between the output and input voltage is

$$\frac{V_O}{V_S} = -\left(\frac{D}{1-D}\right) \tag{4.42}$$

Note that the components on the output (L_2, C_2, and R) are in the same configuration as the buck converter and that the inductor current has the same form as for the buck converter. Therefore, the ripple, or variation, in output voltage is the same as for the buck converter:

$$\frac{\Delta V_O}{V_O} = \frac{1-D}{8L_2C_2f^2} \tag{4.43}$$

The ripple in C_1 can be estimated by computing the change in v_{C1} in theinterval when the switch is open and the currents i_{L1} and i_{C1} are the same. Assuming the current in L_1 to be constant at a value I_{L1},

$$\Delta v_{C1} \approx \frac{1}{C_1}\int_{DT}^{T} I_{L1} d(t) = \frac{I_{L1}}{C_1}(1-D)T = \frac{V_S}{RC_1 f}\left(\frac{D^2}{1-D}\right)$$

or

$$\Delta v_{C1} \approx \frac{V_O D}{RC_1 f} \tag{4.44}$$

The fluctuations in inductor currents can be computed by examining the inductor voltages while the switch is closed. The voltage across L_1 with the switch closed is

$$v_{L1} = V_S = L_1\frac{di_{L1}}{dt} \tag{4.45}$$

In the time interval DT when the switch is closed, the change in inductor current is

$$\frac{\Delta i_{L1}}{DT} = \frac{V_S}{L_1}$$

or

$$\Delta i_{L1} = \frac{V_S DT}{L_1} = \frac{V_S D}{L_1 f} \tag{4.46}$$

For inductor L_2, the voltage across it when the switch is closed is

$$v_{L2} = V_O + (V_S - V_O) = V_S = L_2\frac{di_{L2}}{dt} \tag{4.47}$$

The change in L_2 is then

$$\Delta i_{L2} = \frac{VsDT}{L_2} = \frac{V_S D}{L_2 f} \tag{4.48}$$

which is the same as Δi_{L1}.

For continuous current in the inductors, the average current must be greater than one half the change in current. Minimum inductor sizes for continuous current are

$$L_{1,\min} = \frac{(1-D)^2 R}{2Df}$$
$$L_{2,\min} = \frac{(1-D)R}{2f} \tag{4.49}$$