Laser Idea

In order for lasers to operate, they must have three basic conditions:

1- **Active media**

The base of the laser, a system with a large number of atoms, particles or ions that emit a spectrum, part of it is located in the visible range of electromagnetic radiation. The active medium is often referred to as a group of gas atoms, but atoms, molecules or ions of matter in their liquid or solid state also form an active medium in important types of lasers.

2- **Achieve inversion population**

It is a necessary condition to make the stimulated emission process active and since this condition cannot be achieved under normal conditions and therefore certain pumping methods are used according to special schemes appropriate to the energy levels of the active medium atoms.

3- **Feedback**

It is a necessary condition in order for the emitted radiation to receive its correct oscillation and thus to obtain a beam of high degree of directivity and coherence. Without this condition, the laser acts as a magnifier only for a narrow optical beam and loses the above features, which make it a special optical source.

This condition can be achieved by using a resonant cavity with a suitable design called a resonator. The first successful design of a resonant beam used in the visible range is the interference scale of Fabry-Perot. It consists of two parallel plane mirrors and the active medium is placed between them, as shown in Fig. 1 If one of the mirrors is partially transparent, a useful part of the directed beam of stimulated radiation that parallel of the mirror axial represent the product of the laser.

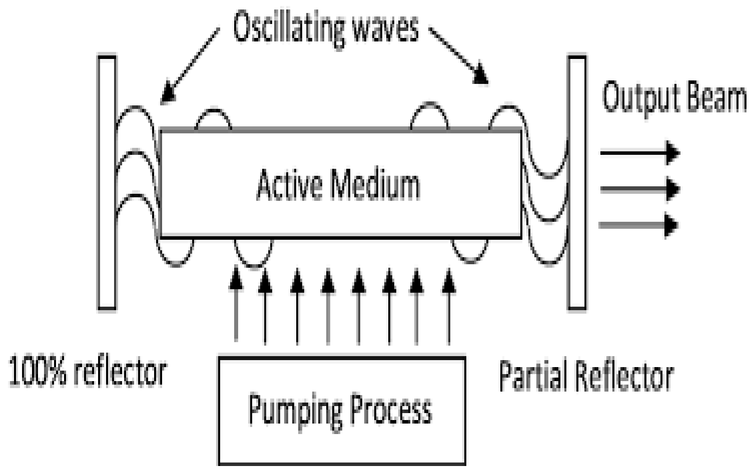


Fig. 1: Diagram of laser resonator.

When the above three conditions are achieved, there remains an important condition for the operation of the laser and is called the threshold condition. The requirements of this condition must be achieve to begin the process of magnification in the active medium and then the oscillation process in the resonator. This is due to the fact that lasers are a non-ideal device that contains a lot of causes of loss and waste. They are generally low efficiency devices when compared to other practical devices. To discuss the aspects of these factors and the condition that must be achieved, it is necessary to enter into some accounts about the amount of gain and loss in the medium and in the design of the resonator.

Inversion Population and Threshold

In the previous chapters, we discussed the gain and the gain factor in the stimulated emission process and the amount of this gain is small so, it is necessary to work to reduce all causes of loss in the laser. One of these causes is the absorption of the mirrors of the resonator. To reduce such loss, usually use high-reflection insulating coatings to coat the mirrors in many layers instead of metallic paint. These layers are deposited sequentially on the glass base material and because of the difference in phase at the contact point of any two layers, all the radiation is reflected in one phase and interference constructively.

The total loss in the laser is due to different factors and although the variation in its values because there was different types of laser, but they seem to be common and most important:

a- Transmittance at the mirrors of the resonator.

The transmittance of one of them is fabricated and represents the escape of the laser product. Mirrors are made in a manner that achieves optimal reflectivity but the loss of such mirrors is also due to absorption as well as diffraction losses.

b- Loss in the active medium of the laser.

Due to other transitions unrelated to laser transition, which occurs as a result of absorption of the medium to a wide range of pumping energy.

A Good Laser Medium:

A good laser medium that is easy to get population inversion by:

a. The pump rate of the upper laser level should be higher than that of the lower laser level.
b. The decay rate (lifetime) of the upper laser level should be higher (slower) than that of the lower laser level.
c. For some pairs of energy levels in certain materials, the spontaneous lifetime can be of the order of microseconds to a few milliseconds. We call this a metastable state.
d. We need to add more and more atoms to the upper metastable state, and hold them long enough to store energy, and allow the production of great numbers of stimulated photons.

Gain Factor

To simplify calculations, let us merge all the loss factors except for those that cause transmittance in mirrors with one loss factor equivalent to the amount (γ) which reduce the gain factor (G) to the value $(G-\gamma)$. And to calculate the amount of change in radiation intensity as a result of one trip within the resonator. If we assume a medium that fills the gap between the two mirrors (M_1, M_2) Which have a reflective (R_1, R_2) respectively, and they are at a distance of (l) between them, there for:

$$I = I_\circ \, e^{-\alpha l}$$

or

$$I = I_\circ \, e^{(G-\gamma) l}$$

This intensity of radiation after reflective from the mirror (M_2) will be:

$I = R_2 I_\circ \, e^{(G-\gamma)l}$

After a full trip, the intensity will become:

$I = R_1 R_2 I_\circ \, e^{(G-\gamma)2l}$

Thus, the gain value after full trip (Γ) represents the ratio between the final and the initial intensity, that is:

$$\Gamma = \frac{I}{I_\circ} = R_1 R_2 e^{(G-\gamma)2l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1-2)$$

If the gain is greater than one at the required frequency, the amplification will grow as well as oscillation. While in case the gain is less than one, the oscillation fades, so the threshold condition can be written as follows:

$$R_1 R_2 e^{(G_{th}-\gamma)2l} = 1 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2-2)$$

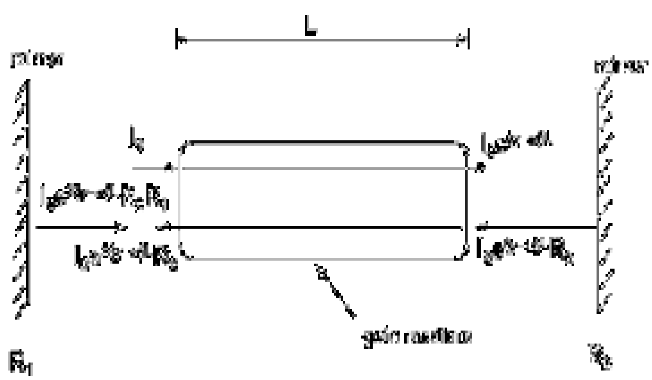


Figure 2: Gain and loss during a cavity round trip.

Where G_{th} represent the threshold gain factor and it is equivalent to: H.W

$$G_{th} = \gamma + \frac{1}{2l} \ln\left(\frac{1}{R_1 R_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3-2)$$

The first term of equation (3-2) represents the loss in the active medium (loss of size) and the second term represents the loss in the design of the resonator (R_1, R_2, l).

In terms of inversion population, expressed as (N), can be written as follows:

$$N = \left(N_2 - N_1 \frac{g_2}{g_1}\right) = \frac{G}{\sigma} \dots \dots \dots \dots \dots \dots (4-2)$$

By using, the value of σ which is equal to:

$\sigma = \frac{B\, n\, h\, \nu\, g(\nu)}{c}$, we can reach to the following formula:

$$\left(N_2 - N_1 \frac{g_2}{g_1}\right) = \frac{G_{th}}{B_{21}\, n\, h\, \nu\, g(\nu)} \dots \dots \dots \dots \dots \dots (5-2)$$

At the threshold, inversion population has a critical value equal to:

$$N_{th} = \left(N_2 - N_1 \frac{g_2}{g_1}\right)_c = \frac{G_{th}}{B_{21}\, n\, h\, \nu\, g(\nu)} \dots \dots \dots \dots \dots \dots (6-2)$$

Example

Helium Neon laser operates in threshold condition . Reflection coefficients of the mirrors are: 0.999, and 0.97. Length of the laser is 50 [cm]. Active medium gain is 1.02.

Calculate:

1. The loss factor M.
2. The loss coefficient (α).

Solution :

Since the laser operates in threshold condition, $G_L = 1$.
Using this value in the loop gain: $G_L = 1 = R_1R_2G_A{}^2M$
1. **The loss factor M**:
$M = 1/(R_1R_2G_A{}^2) = 1/(0.999*0.97*1.02^2) = 0.9919$
As expected, $M < 1$. Since $G_L > 1$, this laser operates above threshold.
2. **The loss coefficient (α)** is calculated from the loss factor:
$M = \exp(-2\alpha L)$: $\ln M = -2\alpha L$
$\alpha = \ln M/(-2L) = \ln(0.9919)/(-100) = 8.13*10^{-5}$ [cm^{-1}]

H.W

Calculate the required value of the inversion population to obtain a coefficient of gain ($1\ m^{-1}$) for the Nd-YAG laser ($1.06\ \mu m$) when the upper-life time ($\tau_2 = 230\ \mu S$), $\Delta\nu = 3 \times 10^{13} Hz$ and with 1.83 as a medium refractive index.

Pumping Schemes

This topic is concerned with the study of the inversion population of the laser medium, i.e. a study of how to pump an active medium from a source so that this process creates an abnormal situation to populate the energy levels associated with a specific emission spectrum. Achieving of inversion population in excess of the critical value of two levels within this scheme results in amplification of the radiation by stimulated emission.

It can be seen from the above that this goal cannot be achieved by using a two-level atomic energy system (a two-level pumping plan), because using high-intensity electromagnetic radiation and suitable frequency, for example, the pumping process quickly generates saturation, then the population of two levels will be equalize and the medium becomes transparent to the radiation used.

Now, can more than two levels of energy be selected from among the large number of energy levels of the atomic system of the effective medium? The answer of this question is yes, this can be done by choosing three or four energy levels. Therefore, we are talking about a three-level laser or a four level laser of energy, depending on the number of levels used for the implementation of the pumping process and the achievement of inversion population to the energy level associated with the stimulated emission.

Different Pumping Schemes in LASER

To achieve population inversion, there are three schemes:

1. Two level pumping Scheme.
2. Three level pumping Scheme.
3. Three level pumping Scheme.

Two level pumping Scheme:

This scheme contains only two energy levels i.e., ground state and excited state. The atom absorbs the photon energy and jumps to excited state from ground state. In this case, it is difficult to achieve stimulated emission.

This is because, the electron spontaneously returns to ground state (time life 10^{-8} sec) by the emission of radiation and passing photon have practically no time to stimulated excited atom. Figure.3 shows such a scheme.

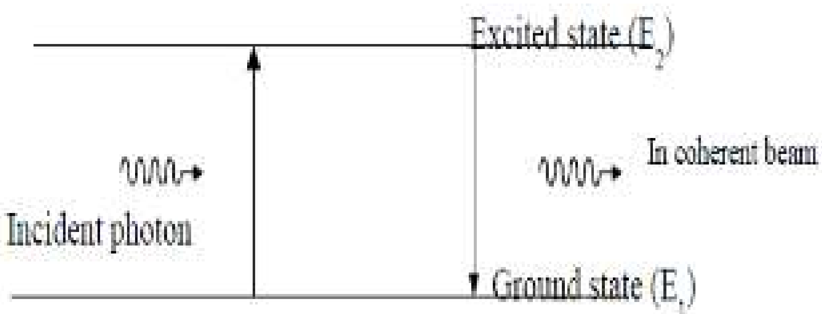


Figure 3: Two level pumping Scheme.

Therefore, only spontaneous emission takes place in two level pumping scheme. Also due to same reason, the density of atoms in excited state (N_2) is always less than density of atoms in ground-excited state (N_1). As the density atoms in ground state larger than in excited state, the photons have maximum chance undergo induced absorption than the stimulated emission. For this reason, it is difficult to achieve population inversion.

Hence, it required create a situation that the electron should stay some more time in the excited state to get effective stimulation. Introducing another one energy state in between the ground state and excited may solve this problem. This intermediate state is called as metastable state.

Metastable state: Metastable state is the energy state, which lies between ground state, and excited state, transition takes place to this state from excited state without emission of radiation. This state is more stable than the excited state and electron stay in this state for about 10^{-3} to 10^{-2} sec. and this time is sufficient undergo stimulation emission. During this time, the atom is stimulated by passing photon and atom returns to ground state by the emission two coherent radiations i.e. laser. The three level pumping schemes contains metastable state.

Three level pumping scheme:

This scheme contains three energy levels i.e., ground state, excited state and a metastable state in between ground state and excited state, as shown in Figure.4. In this scheme, the atom in ground state (E_1) absorbs photon energy and jumps to excited state (E_3). Within short interval (spontaneously) atom return to meta-stable state (E_2) in which it remain comparatively more time (10^{-3} to 10^{-2}). This time is sufficient to achieve population inversion and to construct the amplification of laser radiation.

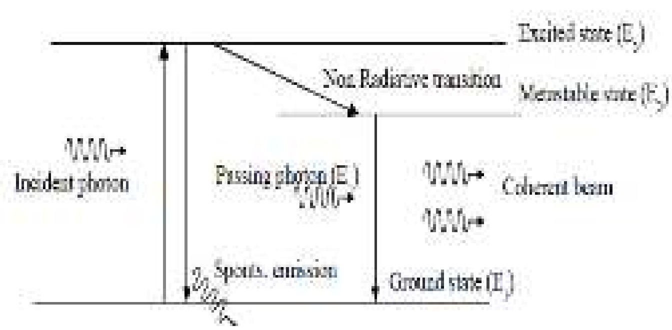


Figure 4: Three level pumping Scheme.

The time during which atom was in metastable state, some more atoms undergo induced absorption and transit to excited state, i.e. to metastable state. Therefor the population of atoms in ground state decreases and in turn, population of atoms in excited or metastable increases or number of atoms in metastable state exceeds the number atoms in ground state this is the condition of population inversion.

The atom in metastable state returns to ground state by the emission two coherent laser rays. The two coherent rays stimulate other two excited atoms and become four and four rays will become eight by stimulating four this process is continues and construct amplified coherent radiation, i.e. laser radiation.

Example of laser produced in this scheme is Ruby laser and Nd:YAG (Neodymium doped Yttrium Aluminum Garnet), which is a solid state laser. The population inversion is achieved in this laser is by Optical Pumping.

Limitation of this scheme is that, it is also not gets rid of from the spontaneous emission. Although the stimulated emission is achieved effectively, it suffers from number of spontaneous emission. Due to this reason, number of stimulated rays cause induced absorption instead of stimulating other excited atoms, hence laser action will decay and finally ceases. Hence, the laser is in the form of discontinuous pulse. Large amount of incident energy is needed to create effective population inversion.

Four Level Pumping Scheme:

This scheme contains four energy levels i.e., ground state, excited state, metastable state and intermediate state in between ground state and metastable state, as shown in Figure.5.

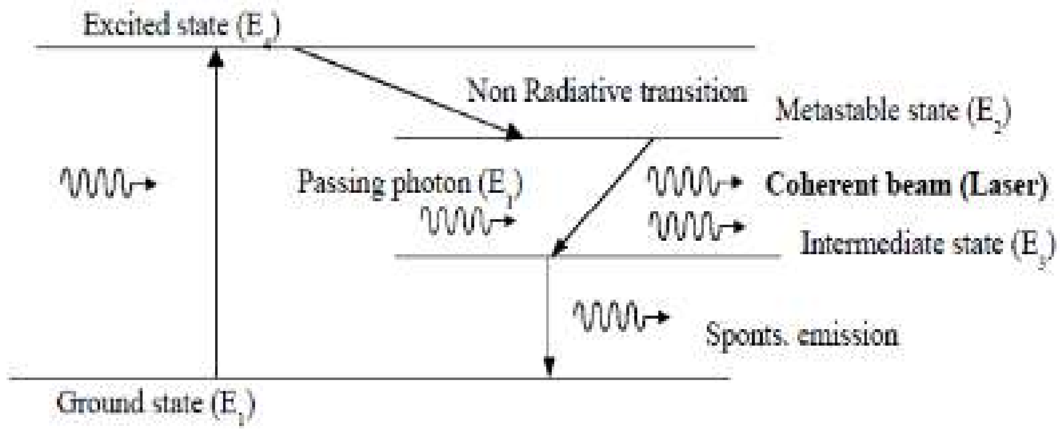


Figure. 5: Four level pumping Scheme.

In this scheme, the atom in ground state (E_1) absorbs the photon energy and jumps to excited state (E_4). Within short interval (Spontaneously) atom return to metastable state in which it remain comparatively more time (10^{-3} to 10^{-2}). In this time population, inversion is achieved not between metastable state and ground state but it is between metastable state (E_3) and intermediate state. The intermediate energy state (E_2) practically empty and metastable state (E_3) is completely filled, hence population inversion achieved effectively and transition takes place from E_3 to E_2 by the emission of coherent radiation i.e. laser radiation. Finally, the atom goes to ground state from state E_2 by the emission of in coherent radiation i.e. spontaneous emission. The example of four level pumping scheme is Helium-Neon laser and Carbon di-oxide laser or gas lasers. The population inversion is achieved in this laser is by electrical discharge method.

The advantage of this method is that as the electrons passing three levels to ground state, effective population inversion is achieved. In addition, number of spontaneous emissions is minimum; the energy required for pumping also less compared that in three level scheme. This laser gives the continuous pulses.

Question:

Why do use the four-level scheme as long as we have a three-level scheme capable of achieving population inversion and successful laser emission?

Answer:

1- The limitation of a three-level pumping scheme, as previously mentioned.
2- In the four-level pumping scheme, population inversion is more easily carried out than using the three-level pumping scheme. The energy required for pumping in the first case is less. According to Boltzmann's distribution, almost all atoms are present before pumping in the ground state. If we assume that (N_t) is the atoms density of the medium, this number will be in the ground state. When using a three-level pumping scheme, we begin to raise the atoms from this level $(E_\circ)$ to the level (E_2). Then these falls quickly to the level (E_1), so the level (E_2) is almost empty. In this case, we must raise the half of the total number of atoms $\left(\frac{1}{2}N_t\right)$ to the level (E_1) through the level (E_2) to first equilibrate its population with the population of ground level $(E_\circ)$, after this, the arrival of any additional atom to level (E_1) refers to population inversion. While, When using a four-level pumping scheme and since the level (E_1) is empty at the beginning, any atom that transits to the level (E_2) through the level (E_3) will be achieve population inversion.

It is therefore more appropriate to choose the medium that operates with a four-level pumping system to choose the medium that operates with a three-level pumping system. Of course, the use of pumping schemes of more than four levels is also possible.

Rate Equations and Population Inversion

Two Level system:

As we saw in the previous lecture, the rate equations for level transitions in a two-level system are:

$$\dot{N}_1 = A_{21}N_2 - B_{12}\rho(\nu)N_1 + B_{21}\rho(\nu)N_2 \ldots\ldots\ldots\ldots\ldots\ldots.\ldots..(2-1)$$

$$\dot{N}_2 = -A_{21}N_2 + B_{12}\rho(\nu)N_1 - B_{21}\rho(\nu)N_2 \ldots\ldots\ldots\ldots\ldots\ldots...(2-2)$$

We will consider N_1 and N_2 to be the number of atoms per volume in each of the two states. The energy per volume in the light field with frequency ν is $\rho(\nu)$. These equations show us how the number of atoms in each state will increase or decrease, depending on the light field and the current number of atoms in each state. Notice that $\dot{N}_1 + \dot{N}_2 = 0$, so $N_1 + N_2$ is constant. We will call this constant N_T, the total number of atoms per volume in the system.

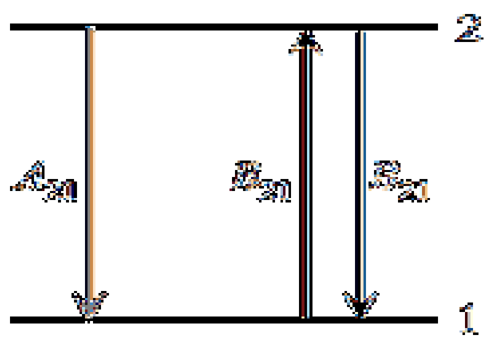


Figure 6: Two-level system.

In our last lecture, we showed that because the rate of stimulated absorption and emission are identical, we could not achieve a population inversion $(N_2 > N_1)$ merely by supplying many photons at the transition frequency. We will need a different manner of increasing N_2 if we want to make a laser amplifier. The simplest way of doing this called a three-level scheme. Figure 7 shows such a scheme, which involves a third energy level.

Three Level system:

As depicted in Fig. 7, some external `pump' promotes electrons to an excited state (level 3), whereupon the electrons quickly decay to level 2. For simplicity, we will consider the decay rate Γ_{32} to be infinitely fast.

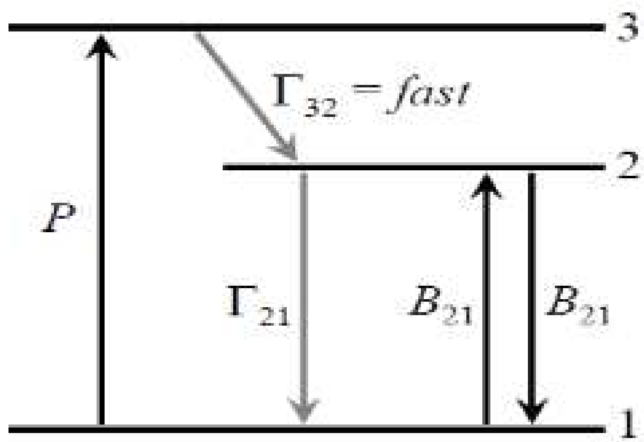


Figure 7: Three-level system to achieve population inversion.

We have changed letters from A_{32} to Γ_{32} to emphasize that these transitions can take place in other ways besides the spontaneous emission of a photon. For example, the transition could be assisted by collisions or vibrations. The pump could be, for example, an input flux of photons with energy $h\nu_{31}$. The rate equations for the three-level scheme are:

$$\dot{N}_1 = -PN_1 + \Gamma_{21}N_2 + B_{21}\rho(\nu)(N_2 - N_1) \dots\dots (3level) \dots\dots\dots (2-3)$$

$$\dot{N}_2 = PN_1 - \Gamma_{21}N_2 - B_{21}\rho(\nu)(N_1 - N_2) \dots\dots\dots\dots\dots\dots\dots\dots\dots (2-4)$$

The only difference from Eq. 1 and 2 is the introduction of the pump rate P. We do not include an equation for $\dot{N}_3$ since there never builds up any population in state 3, by virtue of the fast transition directly into state 2. Notice again that we have $\dot{N}_1 + \dot{N}_2 = 0$, so $N_1 + N_2 = const. = N_T$. We now consider the steady state solution to Eq. 3. This means that $\dot{N}_1 = \dot{N}_2 = 0$.

The solution is:

$PN_1 - \Gamma_{21}N_2 - B_{21}\rho(\nu)(N_2 - N_1) = 0 \ldots\ldots..(2-5)$ *(3 level, steady state)*

We can simplify this a little by assuming a small signal scenario where the light $\rho(\nu)$ is very weak, meaning we can neglect the term involving it in the previous equation. In that case, we have:

$PN_1 - \Gamma_{21}N_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2-6)$

And the solution is:

$N_2 = \frac{P}{\Gamma_{21}} N_1 \qquad (2-7)$ *(3 level, steady state, small signal)*

We would like to express N_2 in terms of the total number of atoms per volume, so we put $N_1 + N_2 = N_T$ in Eq. 5 to get:

$N_2 = \frac{P}{\Gamma_{21}+P} N_T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2-8)$

$N_1 = \frac{\Gamma_{21}}{\Gamma_{21}+P} N_T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2-9)$

$N_2 - N_1 = \frac{P-\Gamma_{21}}{P+\Gamma_{21}} N_T \ldots\ldots\ldots\ldots\ldots\ldots(2-10)$ *(3 level, steady state, small signal)*

Where the last equation is called the population inversion. If we want to have laser amplification, we need more atoms in the excited state than in the ground state, or $N_2 > N_1$. We see that, this will be the case if $P > \Gamma_{21}$. The pumping must be strong enough to overcome the decay rate Γ_{21} in order to put more than half of the atoms in state 2 and to achieve laser amplification. As a side note, the injected power necessary for a given pump rate is given by:

$\frac{Power}{V} = h\,\nu_{31} P N_1 \qquad (2-11)$ *(3 level, steady state, small signal)*

Four Level System:

Finally, we discuss a four-level scheme as depicted in Figure. 8. The appropriate equations are:

$$\dot{N}_0 = -PN_0 + \Gamma_{10}N_1 \ldots \ldots \ldots (2-11)$$

$$\dot{N}_1 = -\Gamma_{10}N_1 + \Gamma_{21}N_2 + B_{21}\rho(\nu)(N_2 - N_1) \ldots \ldots \ldots (2-12)$$

$$\dot{N}_2 = PN_0 - \Gamma_{21}N_2 - B_{21}\rho(\nu)(N_2 - N_1) \ldots \ldots \ldots (2-13)$$

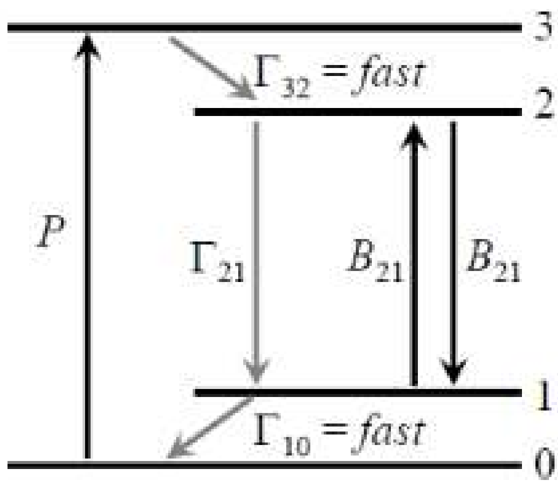


Figure 8: Four level scheme to achieve population inversion.

As before, we have $\dot{N}_0 + \dot{N}_1 + \dot{N}_2 = 0$, which implies $N_0 + N_1 + N_2 = cons.$

Again we will pursue a steady state solution $\dot{N}_0 = \dot{N}_1 = \dot{N}_2 = 0$, We will assume small signal so that terms with $\rho(\nu)$ can be neglected. **It is left as an exercise to show that**

$$N_0 = \frac{\Gamma_{10}\Gamma_{21}}{\Gamma_{10}\Gamma_{21}+\Gamma_{10}P+\Gamma_{21}P} N_T \ldots \ldots \ldots (2-14)$$

$$N_1 = \frac{\Gamma_{21}P}{\Gamma_{10}\Gamma_{21}+\Gamma_{10}P+\Gamma_{21}P} N_T \ldots \ldots \ldots (2-15)$$

$$N_2 = \frac{\Gamma_{10}P}{\Gamma_{10}\Gamma_{21}+\Gamma_{10}P+\Gamma_{21}P} N_T \ldots \ldots \ldots (2-16)$$

The population inversion equation is:

$$N_2 - N_1 = \frac{P(\Gamma_{10}-\Gamma_{21})}{\Gamma_{10}\Gamma_{21}+\Gamma_{10}P+\Gamma_{21}P} N_T \dots .(2-17)$$ *(4 level, steady state, small signal)*

We see that a population inversion ($N_2 > N_1$) occurs if $\Gamma_{10} > \Gamma_{21}$. In contrast with the three-level scheme, the four-level inversion condition does not depend on the pump power P. Even a very week pump can achieve a population inversion in a four-level scheme, which is a big advantage. The only essential ingredient is that electrons need to fall out of level 1 (via Γ_{10}) faster than they fall into level 1 (via Γ_{21}). This is determined by the material, not external conditions.

Population inversion and pumping threshold condition

From the equation of small signal gain one can conclude that the population inversion required for reaching the lasing threshold:

$$G_{th}(\nu) = A_{21}\frac{\lambda^2}{8\pi}\left(N_2 - N_1 \frac{g_2}{g_1}\right) g(\nu) \dots\dots\dots\dots\dots\dots\dots\dots\dots..(2-18)$$

$$\left(N_2 - N_1 \frac{g_2}{g_1}\right) = \frac{8\pi\, G_{th}(\nu)}{A_{21}\,\lambda^2\, g(\nu)} \dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots\dots..(2-19)$$

At threshold the population inversion:

$$\Delta N_{th} = \frac{8\pi\, G_{th}(\nu)\,\tau_{21}}{\lambda^2\, g(\nu)} \dots\dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots\dots\dots\dots\dots (2-20)$$

Note that the lasing threshold will be readily when $g(\nu)$ is maximum at $\nu = \nu_o$ corresponding to the center of the natural linewidth.

$$g(\nu_o) = \frac{1}{\Delta\nu}$$

$$\Delta N_{th} = \frac{8\pi\, G_{th}(\nu)\,\tau_{21}}{\lambda^2}\Delta\nu \dots\dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots\dots\dots\dots (2-21)$$

Pumping Power Required to Reach Threshold Condition

To find the power required for a four-level laser system to reach the threshold we will use the rate equations.

First, we assume that $E_1 \ll KT$ so the thermal population of the energy level 1 is negligible. Second, we assume that the population of the ground state does not change during lasing action. R_1 and R_2 are the rate of pumping then the rate equation for the population for the change in N_2 and N_1, as showm in Figure. 9.

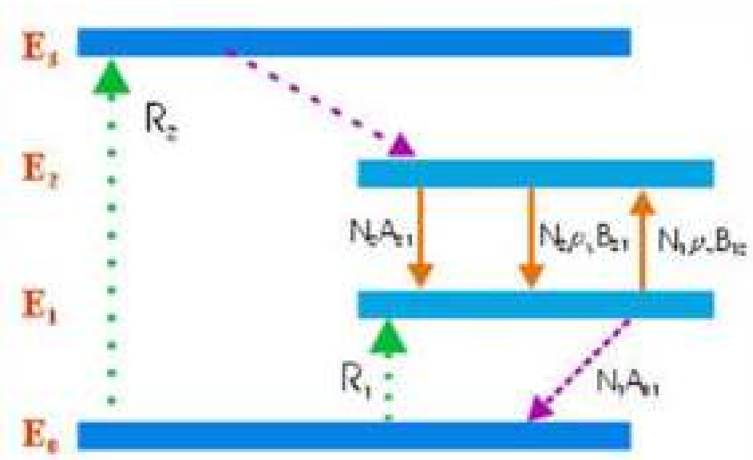


Figure 9: Four level scheme to achieve population inversion.

$$\frac{dN_2}{dt} = R_2 - N_2A_{21} - \rho_\nu B_{21}(N_2 - N_1) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (2-22)$$

$$\frac{dN_1}{dt} = R_1 + \rho_\nu B_{21}(N_2 - N_1) + N_2A_{21} - N_1A_{10} \dots\dots\dots\dots\dots\dots.. (2-23)$$

In steady state condition $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$ (with assumed that $g_1 = g_1$ and $R_1 = 0$).

By solving the above two rate equations we get:

$$N_1 = \frac{R_2}{A_{21}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2-24)$$

$$N_2 = R_2\left[1+\frac{\rho_\nu B_{21}}{A_{10}}\right](A_{21}+\rho_\nu B_{21})^{-1} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2-25)$$

Hence:

$$N_2 - N_1 = R_2\left(\frac{1-\frac{A_{21}}{A_{10}}}{A_{21}+\rho_\nu B_{21}}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (2-26)$$

For population inversion must be the upper lasing level has a longer spontaneous emission life-time than the lower level, that's mean: ($A_{21} < A_{10}$ or $\tau_{21} > \tau_{10}$). In most laser $\tau_{21} \gg \tau_{10}$ and hence, $1 - \frac{A_{21}}{A_{10}} \approx 1$.

At threshold: the radiation density ρ_ν is very small and we can assume that $\rho_\nu = 0$, (If the pumping power is not sufficient to reach the threshold, the amount of population inversion produced is not sufficient to support the amplification in the medium. In this case, the energy density of the radiation ρ_ν can be neglect because the absorption energy will be as a spontaneous emission). Then:

$$(N_2 - N_1)_{th} = \Delta N_{th} = R_{th}\left(\frac{1-\frac{A_{21}}{A_{10}}}{A_{21}}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . . (2-27)$$

Or

$$\Delta N_{th} = \frac{R_{th}}{A_{21}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . \dots\dots . . (2-28)$$

The energy we need to pumping one atom to level 2 equals to E_3 and to raise the number N_{th} of the atoms in volume unit we need to the power P_{th}, given as:

$$P_{th} = \frac{E_3\, N_{th}}{\tau_2}$$

Now, if we substitutes about N_{th} from Eq. 6, get:

$$P_{th} = \frac{8\pi\, \nu^2\, G_{th} n^2\, E_3\, \Delta\nu}{c^2} \quad \ldots (2-29)$$

In steady state: in this situation, the gain becomes equal to the losses then:

$$(N_2 - N_1)_{ss} = (N_2 - N_1)_{th}$$

If pumping power P increases from the quantity P_{th}, this can increasing the value of $\rho_\nu B_{21}$. So that the amount N_{th} remains constant and from the Eq. 26, we can write:

$$\Delta N_{th} = R\left(\frac{1 - \frac{A_{21}}{A_{10}}}{A_{21} + \rho_\nu B_{21}}\right) \quad \ldots (2-30)$$

From equations 27 and 30, get:

$$\frac{R_{th}}{A_{21}} = \frac{R}{A_{21} + \rho_\nu B_{21}}$$

Or

$$R_{th} = \frac{A_{21}\, R}{A_{21} + \rho_\nu B_{21}} \quad \ldots (2-31)$$

Hence, the radiation density ρ_ν will became:

$$\rho_\nu = \frac{A_{21}}{B_{21}}\left(\frac{R}{R_{th}} - 1\right) \quad \ldots (2-32)$$

This mean that the power output is directly proportional to the pumping power within the laser cavity.

Also, Since the output of the laser is directly proportional to the ρ_ν in the medium and also with R_2, the laser output power can be expressed W as follows:

$$W = W_o\left(\frac{P}{P_{th}} - 1\right) \quad \ldots (2-33)$$

Where W_o is constant.

If we study the relation between pumping power, gain and laser power as a function to the time, as shown in Figure. 10, we will conclude these facts:

At time t_1 the excitation mechanism is activated. As a result, the active medium gain and loop gain increase.

At time t_2 the active medium gain is equal to the threshold gain, and the round trip gain is equal to 1. Lasing starts, and output power of the laser start to increase.

At time t_3 the input power reaches its steady state (constant input power). The active medium gain is a little above threshold, and the round trip gain is a little above "1". Output power from the laser continues to rise.

At time t_4 when it reaches its steady state value. Then the active medium gain is equal to the threshold gain, and the gain is equal to "1".

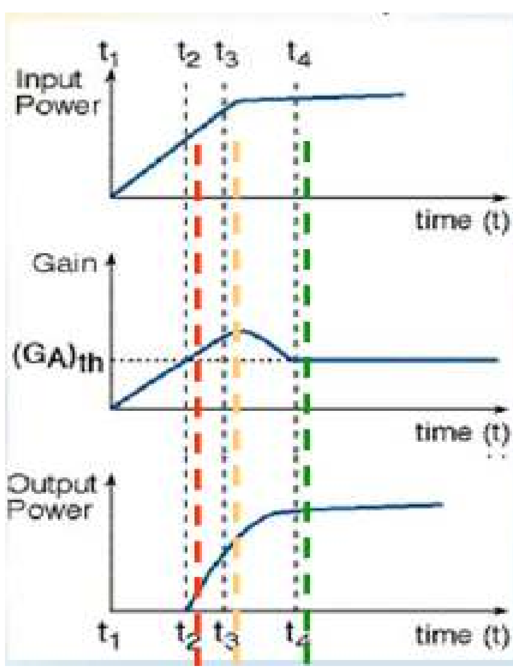


Figure. 10: pumping power, gain and laser power as a function to the time.

H.W

Calculate the ratio between the laser power in the four-levels and the three-levels laser system if the population inversion is 10^{16} cm^{-3} and the number of atoms in upper level is 2×10^{12} cm^{-3}.

Laser Pumping

Pumping is the process by which atoms are raised from a lower to a higher energy level to achieve a population inversion. There are various techniques for laser pumping, and the two that are commonly used for industrial lasers are:

1- Optical pumping.
2- Electrical pumping.
3- Other mechanisms such as pumping by chemical reactions, electron beams and so on.

In this lecture, we outline the basic principles of these pumping techniques. For each technique, we start with a discussion on the common methods used in pumping and outline their advantages and disadvantages.

- **Optical Pumping**

This form of pumping involves the excitation of the active medium using an intense source of light. Two types of light sources are normally used, and these are:

a) An Arc lamp or a Flash Lamp.
b) Diode Laser Pumping.

a) An Arc lamp or a Flash Lamp

Optical pumping is employed for those lasers that have a transparent active medium. Solid-state and liquid-dye lasers are typical examples. The most commonly used pump sources are the flash lamp in the case of pulsed and the arc lamp in the case of continuous-wave solid-state lasers.

Flash lamps are pulsed sources of light and are widely used for the pumping of pulsed solid-state lasers. These are available in a wide range of arc lengths (from a few centimeters to as large as more than a meter, although arc length of 5–10 cm is common), cavity diameter (typically in the range of 3–20 mm), wall thickness (typically 1–2 mm) and shape (linear, helical). Figures 1.22 and 1.23 depict the constructional features of typical linear (Figure. 11) and helical (Figure.12) flash lamps.

Flash lamps for pumping solid-state lasers are usually filled with a noble gas such as xenon or krypton at a pressure of 300–400 torr. Two electrodes are sealed in the envelope that is usually made of quartz. An electrical discharge created between the electrodes leads to a very high value of pulsed current, which further produces an intense flash. The electrical energy to be discharged through the lamp is stored in an energy storage capacitor/capacitor bank.

Figure 11: Linear flash lamps.

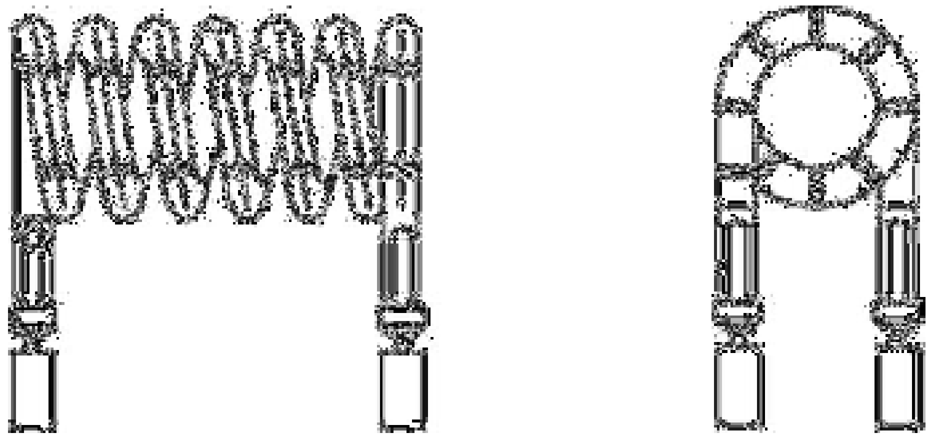


Figure 12: Helical flash lamp.

Xenon-filled lamps produce higher radiative output for a given electrical input as compared to krypton-filled lamps. Krypton however offers a better spectral match, more so with Nd:YAG. That is, the emission spectrum of a krypton flash lamp is better matched to the absorption spectrum of Nd:YAG.

Emission spectra in the case of xenon- and krypton-filled lamps are depicted by Figures 13 and 14, respectively. The absorption spectrum of a Nd:YAG laser is given in Figure 15.

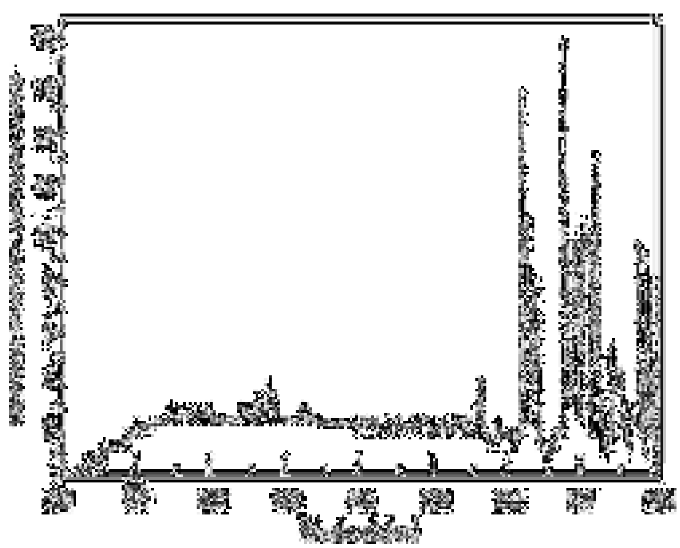


Figure 13: Emission spectrum of xenon-filled flash lamp.

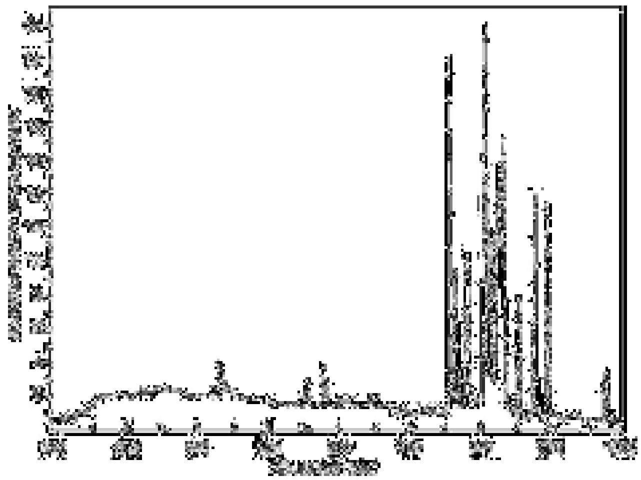


Figure 14: Emission spectrum of Krypton-filled flash lamp.

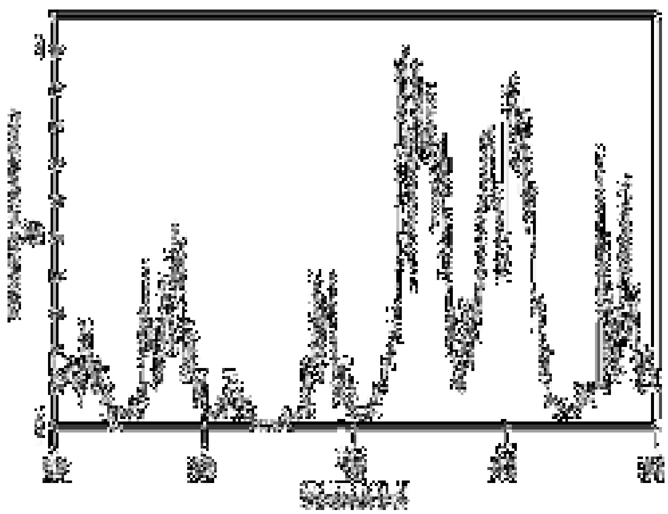


Figure 15: Absorption spectrum of Nd:YAG.

Major electrical parameters include the flash lamp impedance parameter, maximum average power, maximum peak current, minimum trigger voltage and explosion energy. Impedance characteristics of a flash lamp are extremely important as they determine the energy transfer efficiency from energy storage capacitor, where it is stored, to the flash lamp.

Arc lamps are used for CW pumping of solid-state lasers. Arc lamps suitable for solid-state laser pumping are linear lamps (Figure 16), which are very much like linear flash lamps except for electrode design. As evident from

Figure 16, arc lamps use pointed cathodes rather than the rounded cathodes used in flash lamps. Arc lamps are filled with xenon or krypton at a pressure of 1–3 atmospheres. Krypton-filled linear arc lamps are more common because of their relatively better spectral match to the Nd:YAG absorption band. Bore (cavity) diameters of 4–7mm and arc lengths in the range of 50–150 mm are common. However, the efficiency with which pump output is usefully transferred to excite the lasing species is definitely lower in the case of the broadband optical pumping provided by flash lamps and arc lamps.

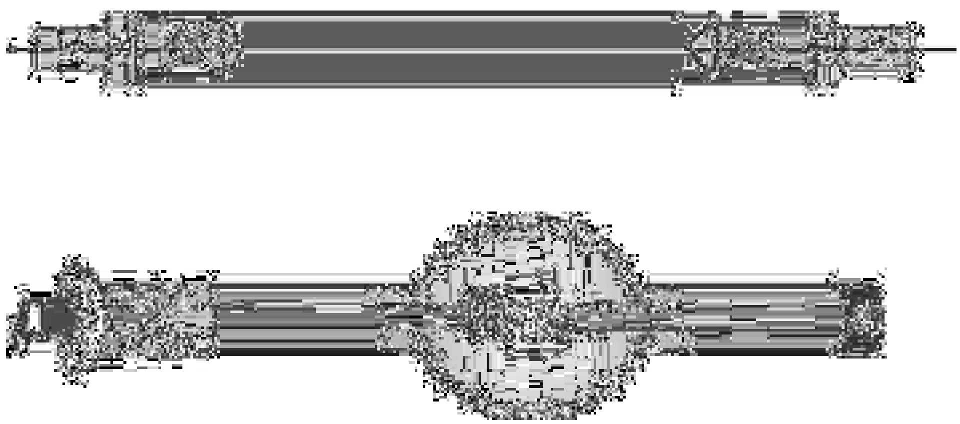


Figure 16: Construction of linear arc lamp.

Optical pumping at a single wavelength in a laser with an absorption level corresponding to that wavelength in the pump band achieves a relatively higher pumping efficiency, which leads to higher overall laser efficiency.

b) Diode Laser Pumping

The efficiency of optical pumping is further enhanced when another laser is used for pumping. Since the pumping laser infuses energy at a specific wavelength, very little of its energy is wasted. However, for this to be true, the output wavelength range of the pumping diode laser must be nearly or the same as the absorption bandwidth of the active medium being pumped. Even though any laser can essentially be used, diode lasers (see Chapter 8) have been found very convenient and developed specifically for pumping other lasers, usually solid-state lasers. They are used mainly because:

1. They have a relatively high efficiency, which results in a reduced electrical power consumption for a given desired output power.
2. They can generate high output power when stacked.
3. They are available in many wavelengths.
4. They require reduced maintenance. The working life of diode-pumped lasers is generally more than 12,000 h, while that for flash lamp lasers is in the range of 600 – 1,000 h.
5. They are much smaller in size, making them more portable.

- **Electrical Pumping**

Pumping by electrical discharge is common in gas lasers. The excited electrons in the gas-discharge plasma transfer their energy to the lasing species either directly or indirectly through the atoms or molecules of another element. A helium-neon laser is a typical example of an indirect transfer of pump energy. The electrons first transfer the energy to helium atoms and then the excited helium atoms transfer the energy to neon atoms. A high voltage initially ionizes the gas and, once the discharge is struck, it can be sustained by a relatively much lower voltage and current. In a typical He-Ne laser, initiating voltage is of the order 8–10 kV while the sustaining voltage is around 1.5–2 kV.

- **Other Mechanisms**

Some of the other methods of pumping or creating population inversion, which are specific to certain types of lasers, include:

1- Excitation by combustion reaction as in gas dynamic CO2 lasers.
2- Chemical reaction as in chemical lasers such as hydrogen fluoride (HF) laser, deuterium fluoride (DF) laser and chemical oxygen iodine laser (COIL).
3- Electron acceleration as in free electron lasers.

In the case of a gas dynamic laser for example, a combustion reaction produces a high-temperature high-pressure mixture of CO2 and other gases required in a CO2 laser. This gas mixture is then rapidly expanded through a set of nozzles to a very low-pressure low-temperature condition. Although the temperature and pressure drop rapidly a large number of molecules still remain in the excited state, thus creating population inversion.

Pumping Efficiency

The efficiency of the pumping process is determined by a number of factors, including:

1. The conversion of electrical energy into optical energy by the pumping source in the frequency range of interest.
2. The transmission of the optical energy from the source into the rod (determined primarily by the design of the pumping configuration). The helical configuration is always lower in efficiency than linear pumping. However, the helical configuration results in a more uniform pumping process, making it attractive for high-power systems.
3. The absorption by the active medium.

4. The actual number of atoms resulting in lasing action as compared to the number of atoms excited by the optical energy absorbed by the rod. The efficiency of a lamp-based system is normally in the range of 1–3%. This is the fraction of the input electrical energy that is converted to useful output. The 97–99% of energy that is not used is removed in the form of heat using a chiller.

Using a diode laser for pumping increases the efficiency to the range of 30–40%.

Optical Resonators

The resonator is a resonant cavity, which is the source of feedback in Maser and laser devices. It is a design that is necessary to support the amplification occurring in the effective medium as a result of the stimulated emission as well as directing it and maintaining a single-wave formula for its emission. The most widely used laser resonators have either plane or spherical mirrors of rectangular (or, more often, circular) shape, separated by some distance L. Typically, L may range from a few centimeters to a few tens of centimeters, while the mirror dimensions range from a fraction of a centimeter to a few centimeters.

These resonances are characterized by the following:

1- The resonator dimensions are much greater than the laser wavelength.

2- Resonators are usually open, i.e. no lateral surfaces are used.

The resonator length is usually much greater than the laser wavelength because this wavelength usually ranges from a fraction of a micrometer to a few tens of micrometers. A laser cavity with length comparable to the wavelength would then generally have too low a gain to allow laser oscillation. Laser resonators are usually open because this drastically reduces the number of modes, which can oscillate with low loss.

In fact, it is seen that even a narrow linewidth laser such as a He-Ne laser would have a very large number of modes ($\approx 10^9$) if the resonator were closed. By contrast, on removing the lateral surfaces, the number of low-loss modes reduces to just a few (≈ 6 in the example). In these open resonators, in fact, only the very few modes corresponding to a superposition of waves traveling nearly parallel

to the resonator axis will have low enough losses to allow laser oscillation.

According to the previous discussion, it is seen that open resonators have inevitably some losses due to diffraction of the electromagnetic field, which leads to some fraction of the energy leaving the sides of the cavity (diffraction losses).

Types of Laser Resonators

Of the various possible resonators, we make particular mention of the following types:

a. ***Plane – Parallel (or Fabry–Perot) Resonator***.

This consists of two plane mirrors set parallel to one another. To a first approximation, the modes of this resonator can be thought of as the superposition of two plane e.m. waves propagating in opposite directions along the cavity axis, as shown schematically in Fig. 17a. Within this approximation, the resonant frequencies can be readily obtained by imposing the condition that the cavity length *L* must be an integral number of half-wavelengths, i.e. $L = n\lambda/2$, where n is a positive integer (round trip length= $2L = n\lambda$). This is a necessary condition for the electric field of the e.m. standing wave to be zero on the two mirrors. It then follows that the resonant frequencies are given by:

$$\nu = n(c/2L) \ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-34)$$

It is interesting to note that the same expression Eq. (5.1.2) can also be obtained by imposing the condition that the phase shift of a plane wave due to one round-trip through the cavity must equal an integral number times 2π i.e. 2kL=2n π. This condition is readily obtained by a self-consistency argument. If the frequency of the plane wave is equal to that of a cavity mode, the phase shift after one round trip must be zero (apart from an integral number of 2) since only in this case will the amplitudes at any arbitrary point, due to successive reflections, add up in phase so as to give an appreciable total field.

Note that, according to Eq. (2-33), the frequency difference between two consecutive modes, i.e. modes whose integers differ by one, is given by:

$$\Delta\nu = c/2L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots.\ldots (2-35)$$

This difference is called the frequency difference between two consecutive longitudinal modes with the word longitudinal used because the number n indicates the number of half-wavelengths of the mode along the laser resonator, i.e. longitudinally.

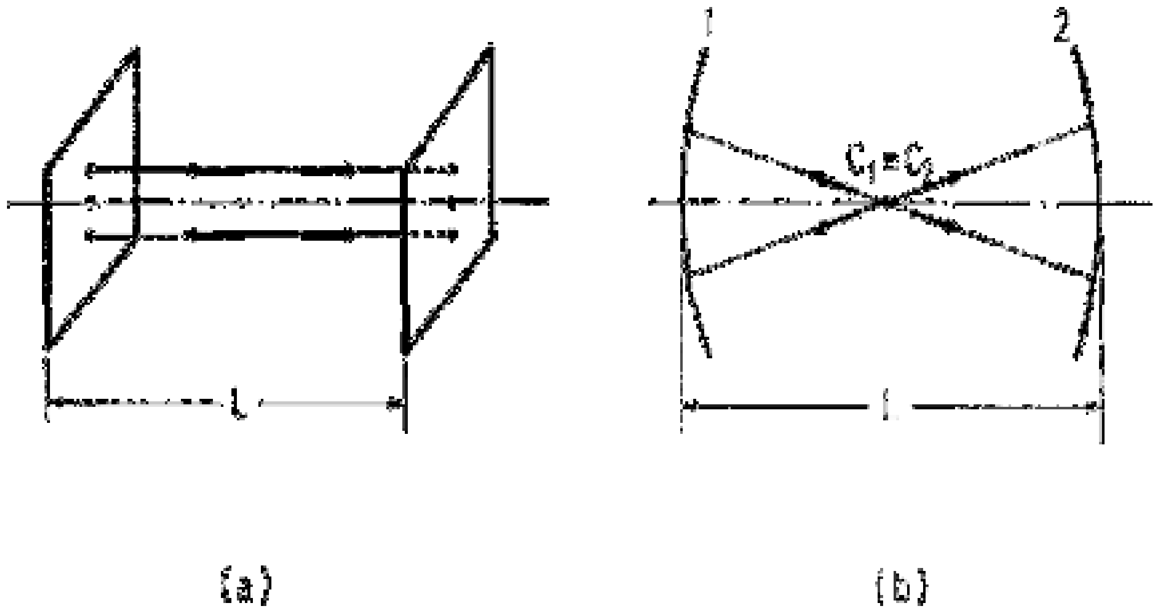


Figure 17: (a) Plane-parallel resonator; (b) concentric resonator.

b. ***Concentric (or Spherical) Resonator***.

This consists of two spherical mirrors having the same radius R and separated by a distance L such that the mirror centers of curvature C_1 and C_2 are coincident, i.e. $L = 2R$ (Fig. 17b). In this case, the modes are approximated by a superposition of two oppositely traveling spherical waves originating from the point C. The application of the above self-consistency argument again leads to Eq. (2-34) as the expression for the resonant frequencies and to Eq. (2-35) for the frequency difference between consecutive longitudinal modes.

c. ***Confocal Resonator***.

This consists of two spherical mirrors of the same radius of curvature R and separated by a distance L such that the mirror foci F_1 and F_2 are coincident, (Fig. 18). It then follows that the center of curvature C of one mirror lies on the surface of the second mirror i.e. $(L = R)$. From a geometrical-optics point of view, we can draw any number of closed optical paths of the type shown in Fig. 18 by changing the distance of the two parallel rays from the resonator axis $C_1\ C_1$.

Note also that the direction of the rays can be reversed in Fig. 18. This geometrical optics description, however, does not give any indication of what the mode configuration will be, and we shall see that in fact this configuration cannot be described either by a purely plane or a purely spherical wave. For the same reason, the resonant frequencies cannot be readily obtained from geometrical-optics considerations.

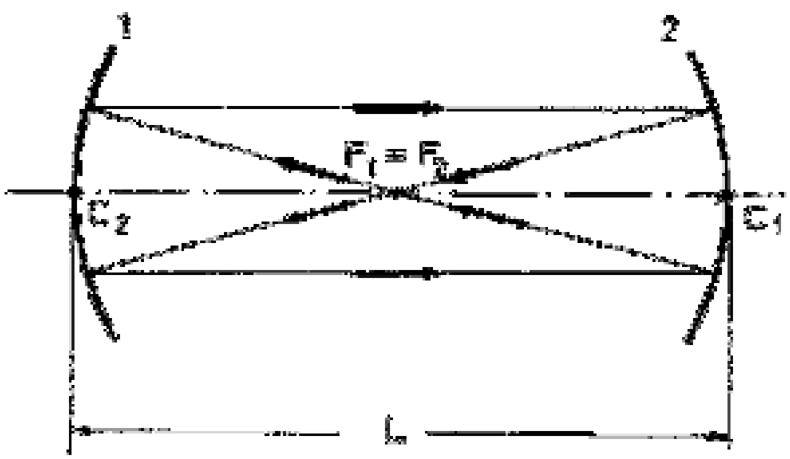


Figure 18: Confocal resonator.

Resonators formed by two spherical mirrors of the same radius of curvature R and separated by a distance L such that $R < L < 2R$ (i.e. somewhere between the confocal and concentric conditions) are also often used. In addition, we can have $L > R$. For these cases, it is not generally possible to use a ray description in which a ray retraces itself after one or a few passes.

All of these resonators can be considered as particular examples of a general resonator consisting of two either concave $(R > 0)$ or convex $(R < 0)$ spherical mirrors, of different radius of curvature, spaced by some arbitrary distance R. These various resonators can be divided into two categories, namely, ***stable resonators*** and **unstable resonators**. A resonator will be described as unstable when an arbitrary ray, in bouncing back and forth between the two mirrors, will diverge indefinitely away from the resonator axis. An obvious example of an unstable resonator is shown in Fig. 19. Conversely, a resonator for which the ray remains bounded will be described as a stable resonator.

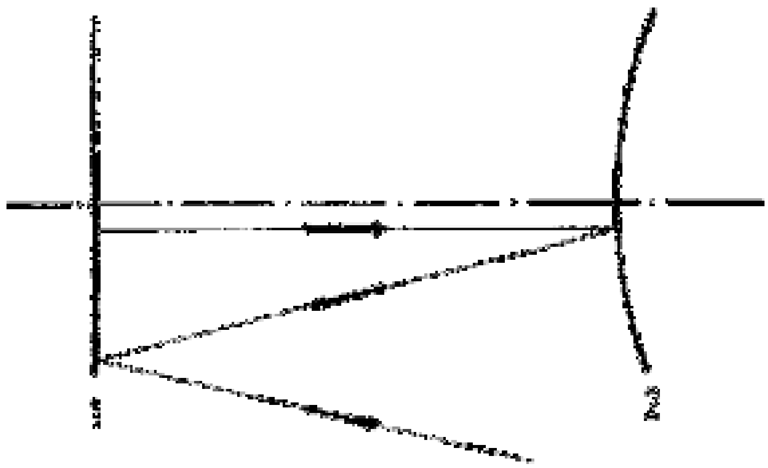


Figure 19: Example of an unstable resonator.

A particularly important class of laser resonator is the ring resonator where the path of the optical rays is arranged in a ring configuration (Fig. 20a) or in a more complicated configuration such as the folded configuration of Fig. 20b.

In both cases the resonance frequencies can be obtained by imposing the condition that the total phase shift along the ring path of Fig. 20a or along the closed-loop path of Fig. 20b (continuous paths) be equal to an integral number of 2π. We then readily obtain the expression for the resonance frequencies as:

$$\nu = \frac{nc}{L_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-36)$$

Where L_p is the perimeter of the ring or the length of the closed-loop path of Fig. 20b, and n is an integer. Note that the arrows of the continuous paths of

Fig. 20 can in general be reversed which means that e.g. in Fig. 20a the beam can propagate either clockwise or anticlockwise. Thus, in general, a standing wave pattern will be formed in a ring resonator.

One can see, however, that, if a unidirectional device is used, allowing the passage of e.g. only the right to left beam in Fig. 20a (optical diode), then only the clockwise propagating beam can exist in the cavity. So the concepts of a cavity mode and cavity resonance frequency are not confined to standing-wave configurations. Note also that ring resonators can be either of the stable (such as in Fig. 20) or unstable configuration, Fig. 19.

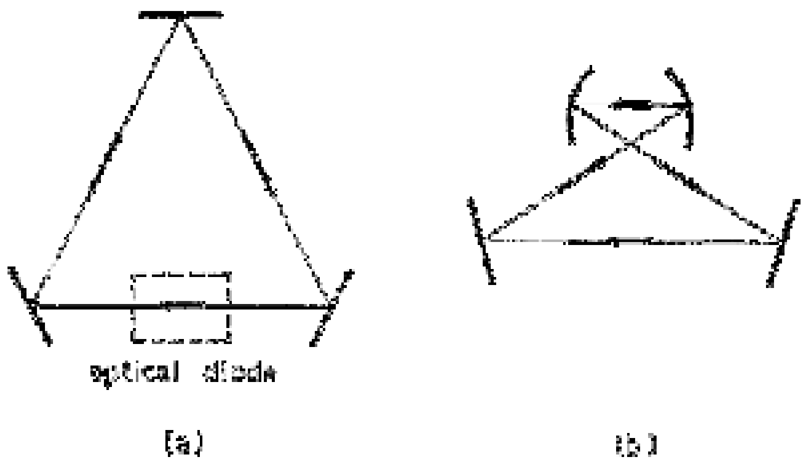


Figure 20: (a) Simplest three-mirror ring resonator. (b) Folded ring resonator.

Example:

Number of modes in closed and open resonators. Consider a He-Ne laser oscillating at the wavelength of $\lambda = 633\ nm$, with a Doppler-broadened gain linewidth of $\Delta\nu_\circ = 1.7 \times 10^9$ Hz. Assume a resonator length $L = 50\ cm$ and consider first an open resonator. According to Eq. (2-35) the number of longitudinal modes, which fall within the laser linewidth is

$N_{open} = 2L\Delta\nu_\circ/c \approx 6$. Assume now that the resonator is closed by a cylindrical lateral surface with a cylinder diameter of $2a = 3\ mm$. the number of modes of this closed resonator which fall within the laser linewidth $\Delta\nu_\circ$ is $N_{closed} = 8\pi\nu^2 V\ \Delta\nu_\circ/c^3$, where $\nu = c/\lambda$ is the laser frequency and $V = \pi a^2 L$ is the resonator volume. From the previos expression and data, we readily obtain $N_{closed} = (2\ \pi a/\lambda^2)\ N_{open} \approx 1.2 \times 10^9$ mods.

Example 5.1. *Number of modes in closed and open resonators.* Consider a He-Ne laser oscillating at the wavelength of $\lambda = 633$ nm, with a Doppler-broadened gain linewidth of $\Delta\nu_0^* = 1.7 \times 10^9$ Hz. Assume a resonator length $L = 50$ cm and consider first an open resonator. According to Eq. (5.1.3) the number of longitudinal modes which fall within the laser linewidth is $N_{open} = 2L\Delta\nu_0^*/c \simeq 6$. Assume now that the resonator is closed by a cylindrical lateral surface with a cylinder diameter of $2a = 3$ mm. According to Eq. (2.2.16) the number of modes of this closed resonator which fall within the laser linewidth $\Delta\nu_0^*$ is $N_{closed} = 8\pi\nu^2 V\Delta\nu_0^*/c^3$, where $\nu = c/\lambda$ is the laser frequency and $V = \pi a^2 L$ is the resonator volume. From the previous expressions and data we readily obtain $N_{closed} = (2\pi a/\lambda)^2 N_{open} \simeq 1.2 \times 10^9$ modes.

Generally, the most common optical cavities are shown if Fig.21.

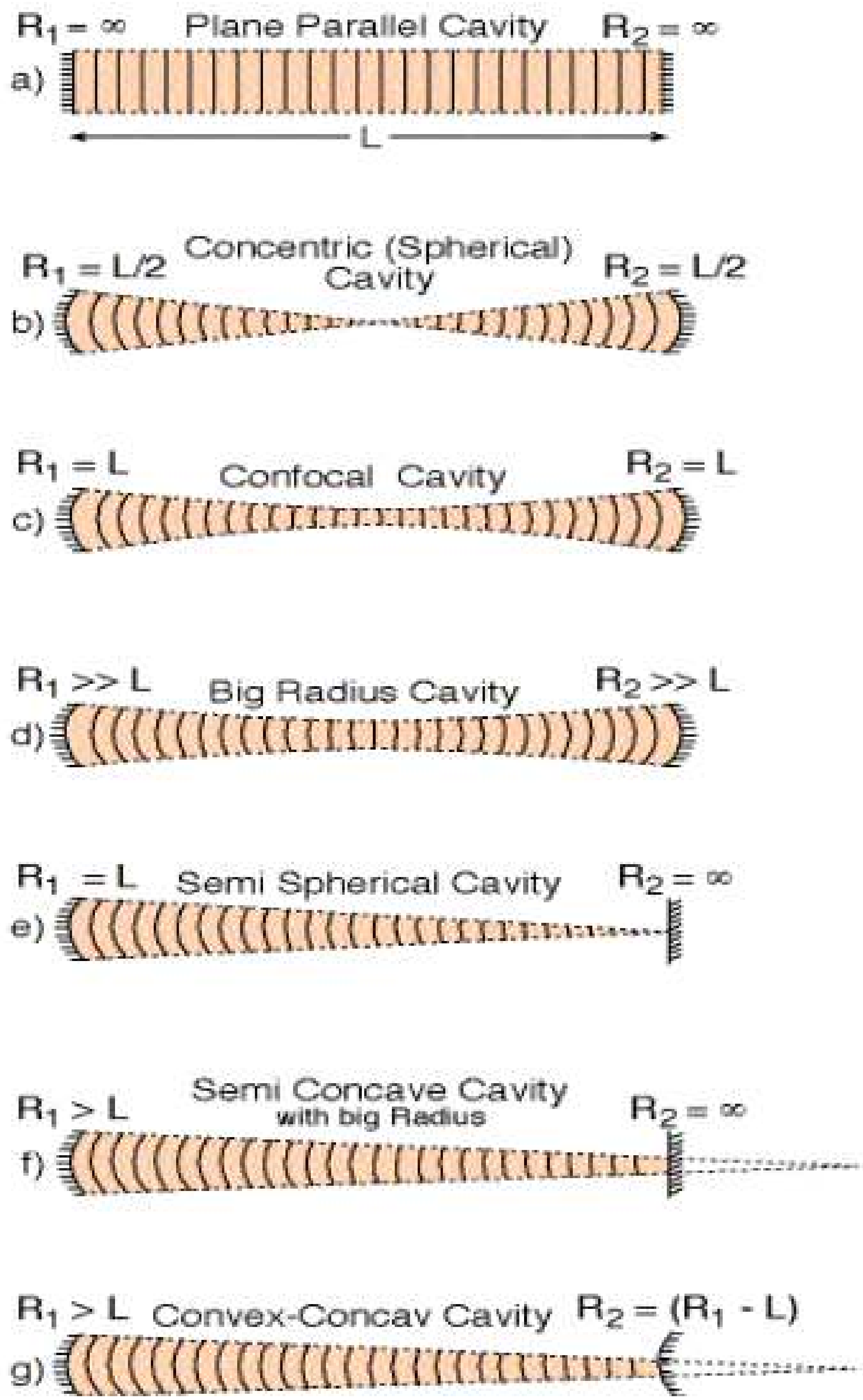


Figure 21: Variety of the optical cavities.

Basic Concepts

Optical Cavity -Laser Cavity: The region between the end mirrors of the laser.

Optical Axis: The imaginary line connecting the centers of the end mirrors, and perpendicular to them. The optical axis is in the middle of the optical cavity.

Aperture: The beam diameter-limiting factor inside the laser cavity. Usually the aperture is determined by the diameter of the active medium, but in some lasers a pinhole is inserted into the laser cavity to limit the diameter of the beam. An example is the limiting aperture for achieving single mode operation of the laser.

Losses inside Optical Cavity: Include all the radiation missing from the output of the laser (emitted through the output coupler), the gain of the active medium must overcome these losses.

Losses inside an optical cavity are caused by the following:

1. Misalignment of the laser mirrors:

   The cavity mirrors are not exactly aligned perpendicularto the laser axis, and parallel to each other (symmetric), the radiation inside the cavity will not be confined during its path between the mirrors.

2. Absorption, scattering and losses in optical elements:
   Since optical elements are not ideal, each interaction with optical element inside the cavity cause some losses.
3. Diffraction Losses:

Every time a laser beam pass through a limiting aperture it diffract. It is not always possible to increase the aperture for reducing the diffraction. As an example, such increase will allow lasing in higher transverse modes which are not desired.

Two parameters determine the structure of the optical cavity:

1. The volume of the laser mode inside the active medium.
2. The stability of the optical cavity.

Stable Resonators

A stable optical resonator consist of two mirrors with radii of curvature R_1 and R_2 separated by an optical distance, $L = n\ L_\circ$, where $L_\circ$ is the geometrical spacing between the mirrors and n the index of refraction inside the resonator. The range of L within which a resonator is stable is determined by the condition that a ray launched inside the resonator parallel to the optical axis remains inside the resonator after an infinite number of bounces.

By introducing the g-parameters of the resonator mirrors:

$$g_{i=}1 - \frac{L}{R_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-37)\ \ ,\ i=1,2$$

The condition for these stable resonators reads:

$$0 \leq g_1 g_2 \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-38)$$

It is convenient to visualize optical resonators in the g-diagram also referred to as the stability diagram, in which a resonator is determined by a point in the $g_1 g_2$ plane. The area of stable resonators is limited by the coordinate axes and the hyperbolas $g_2 = \mp g_i$. This is indicated by the hatched area of Fig. 22.

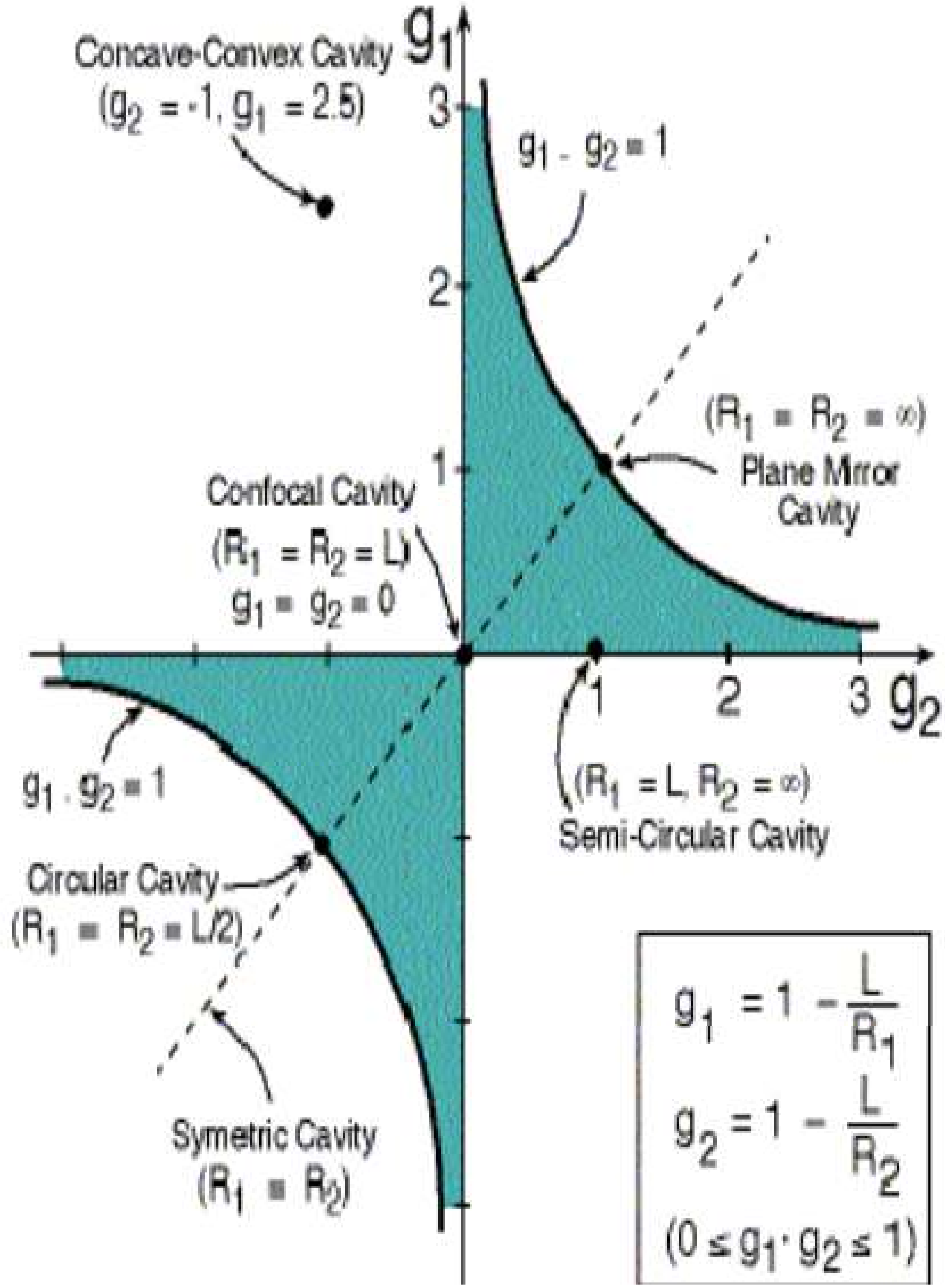


Figure 22: Stability Diagram of an Optical Cavity.

Example:

The laser cavity length is 1 m. At one end a concave mirror with radius of curvature of 1.5 m. At the other end a convex mirror with radius of curvature of 10 cm. Find if this cavity is stable.

Solution:

$R_1 = 1.5\, m.$

As common in optics, a convex mirror is marked with minus sign:

$R_2 = -0.1\, m$

$g_1 = 1 - L/R_1 = 1 - 1/1.5 = 0.333$

$g_2 = 1 - L/R_2 = 1 + 1/0.1 = 11$

The product: $g_1 g_2 = 11 \times 0.333 > 1$

The product is greater than 1, so the cavity is unstable.

Example:

Determine the stability of the following cavity resonators:

(i) $L = 1.5\, m,\ R_1 = 3\, m,\ R_2 = 2m.$

(ii) $L = 1\, m,\ R_1 = 0.5\, m,\ R_2 = 2m.$

(iii) $L = 1\, m,\ R_1 = 3\, m,\ R_2 = -2m.$

Solution:

The stability condition for a laser resonator cavity is given by the expression,

$$0 \le g_1 g_2 \le 1$$

$g_1 = 1 - L/R_1$ And $g_2 = 1 - L/R_2$

(i) $\left(1-\frac{1.5}{3}\right)\left(1-\frac{1.5}{2}\right) = (1-0.5)(1-0.75) = (0.5)(0.25) = 0.125$

Since, $0 < 0.125 < 1$

The cavity resonator is stable.

(ii) $(1-1/0.5)(1-1/2) = (1-2)(0.5) = (-1)(0.5) = -0.5$

Since, $g_1 g_2 = -0.5 < 0$

The cavity resonator is unstable.

(iii) $(1-1/3)(1-1/-2) = (1-2)(0.5) = (2/3)(3/2) = 1$

Since, $g_1 g_2 = 1$

The cavity resonator is marginally stable.

Laser Modes

We learned from the previous lectures that in order to obtain the laser beam, the feedback should be used by mirrors, to amplify the beam during its passage in the laser's medium. These mirrors play a large role in influencing the electromagnetic radiation inside the amplifier.

Two types of patterns are produced:

1. **Longitudinal modes** only specific frequencies are possible inside the optical cavity of a laser, according to standing wave condition.
2. **Transverse modes** are created in cross section of the beam, perpendicular to the optical axis of the laser.

Longitudinal modes

The solid line describes a wave moving to the left. On the right side of figure, the superposition of the two waves is shown. Like a standing wave in a string attached to fixed points at both sides, the fixed points of a standing wave are called Nodes. The distance between adjacent Nodes is half the wavelength of each of the interfering waves.

Standing Waves in a Laser

Optical cavity: is created from two mirrors at the both ends of the laser.

Laser mirrors serve two goals:

1. Increase the length of the active medium, by making the beam pass through it many times.

2. Determine the boundary conditions for the electromagnetic fields inside the **Optical Axis of the laser**: the laser beam is ejected out of the laser cavity in the direction of the optical axis.

Condition for creating a standing wave: Two waves of the same frequency and amplitude are moving in opposite directions.

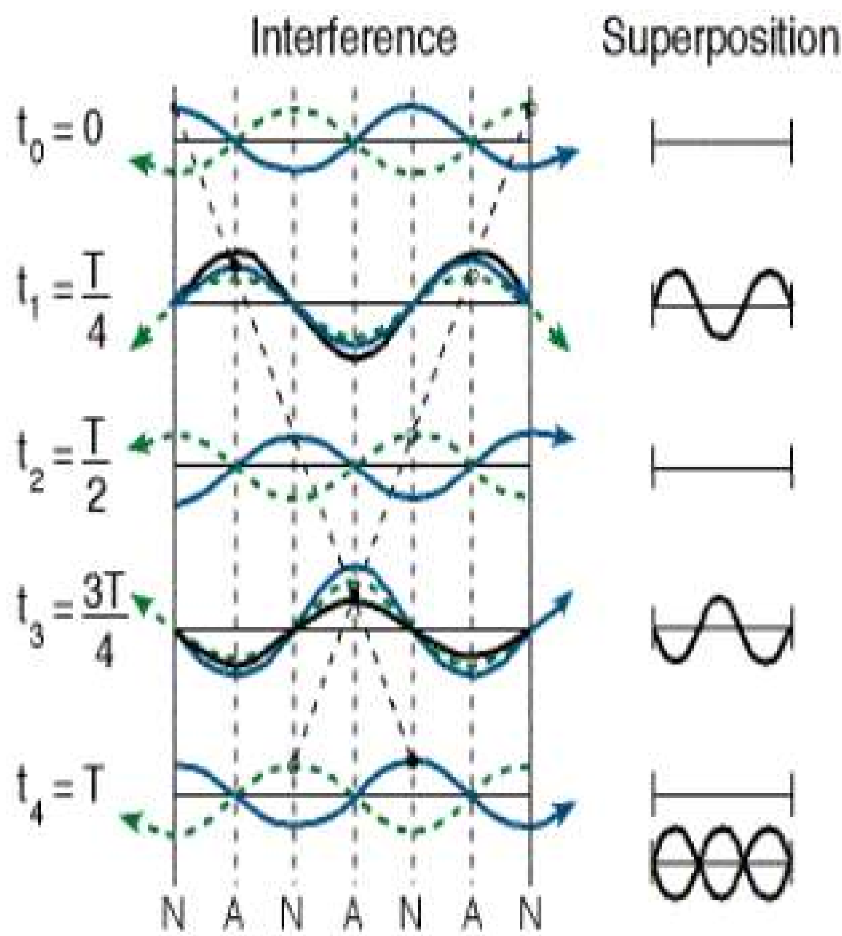


Fig 23: Standing Waves in a Laser

Create a Standing Wave

- The optical path from one mirror to the other and back must be an integer multiplication of the wavelength $(2L = m\lambda)$.

- The wave must start with the same phase at the mirror
- The Length between the mirrors is constant (L), the suitable wavelengths, which create standing waves, must fulfill the condition: $\lambda_m = 2L/m$.

  L : Length of the optical cavity.

  m: Number of the mode (number of antinodes), which is equal to the number of half wavelengths inside the optical cavity.

  λ_m: Wavelength of mode m in matter, inside the laser cavity, is equal to: $\lambda_m = \lambda_\circ/n$.

  $\lambda_\circ$: Wavelength of light in vacuum.

  n: Refraction index of the active medium.

  Since: $c = \lambda_\circ\nu = n\lambda_m\nu_m$, since c is the velocity of light in space.
- The frequency of the longitudinal mode is: $\nu_m = \frac{c}{n\lambda_m}$.

Inserting λ_m to the last equation: $\nu_m = m\left(\frac{c}{2\,n\,L}\right)$.

- The first mode of oscillation: $\nu_1 = \left(\frac{c}{2\,n\,L}\right)$.

This mode is called fundamental longitudinal mode, and it has the basic frequency of the optical cavity.

Separation between axial (longitudinal) modes

If the First mode is m, then:

$$L = m\,\frac{\lambda_m}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-39)$$

If the Second mode is $m+1$, then:

$$L = (m+1)\,\frac{\lambda_{m+1}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-40)$$

It is more convenient to refer to the axial modes by their frequency:

$$\nu_m = \frac{C}{\lambda_m} = m\ \frac{C}{2\,L}$$

$$\nu_{m+1} = \frac{C}{\lambda_{m+1}} = (m+1)\ \frac{C}{2\,L}$$

$$\nu_{m+1} - \nu_m = \frac{C}{2\,L}(m+1-m) = \frac{C}{2\,L} \dots\dots\dots\dots\dots\dots\dots\dots (2-41)$$

The separation between neighboring frequencies is equal to $\frac{C}{2\,L}$ i.e. dependent only on the separation between mirrors and independent of m.

For L=25cm, the separation between neighboring frequencies is $6x10^8$ sec^{-1}.

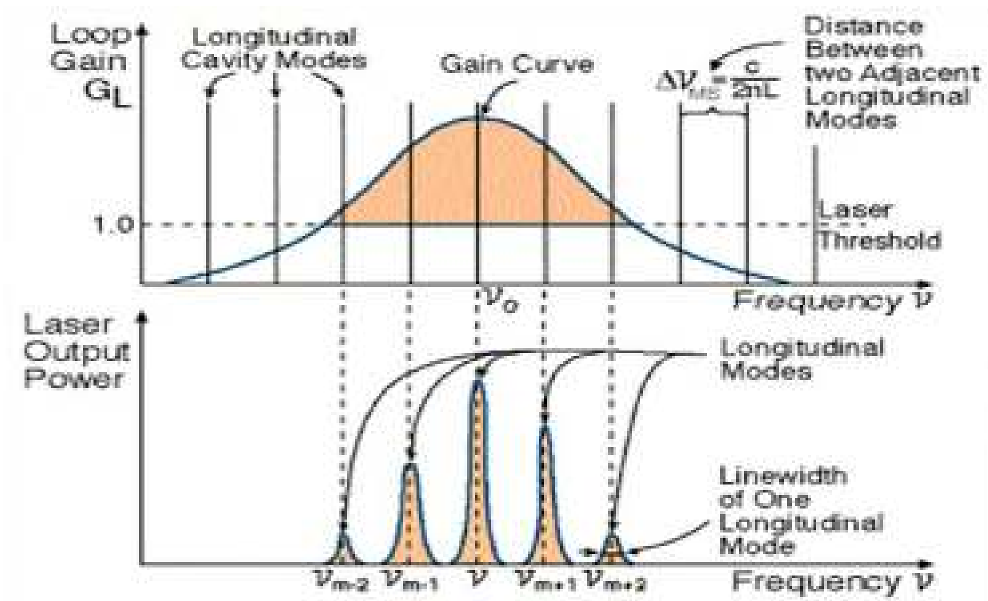
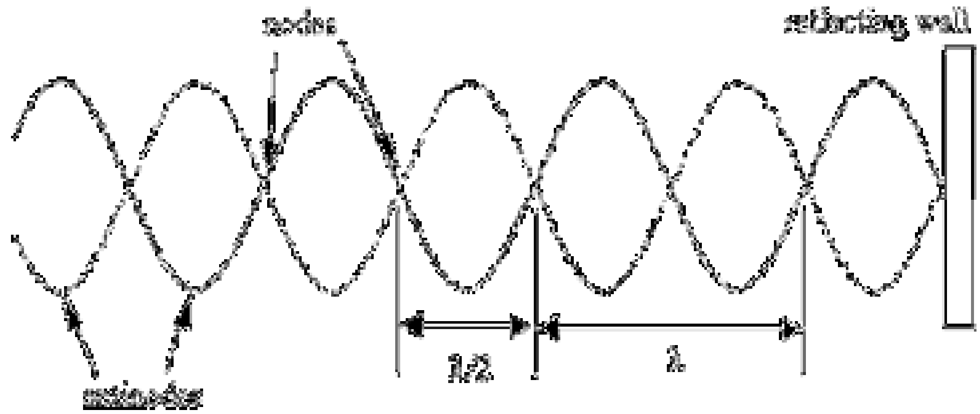


Fig 24: Gain curve of the active mediums and possible longitudinal modes of the laser.

It is clear that a single mode laser can be made by reducing the length of the cavity, such that only one longitudinal mode will remain under the fluorescence curve with G_L>1. The number of possible laser modes that can produce a laser is that achieve the condition of gain is greater than or equal to the loss, as shown in the colored area in the figure above. The approximate number of possible laser modes is given by the width of the Laser bandwidth divided by the distance between adjacent modes.

Standing waves in a string



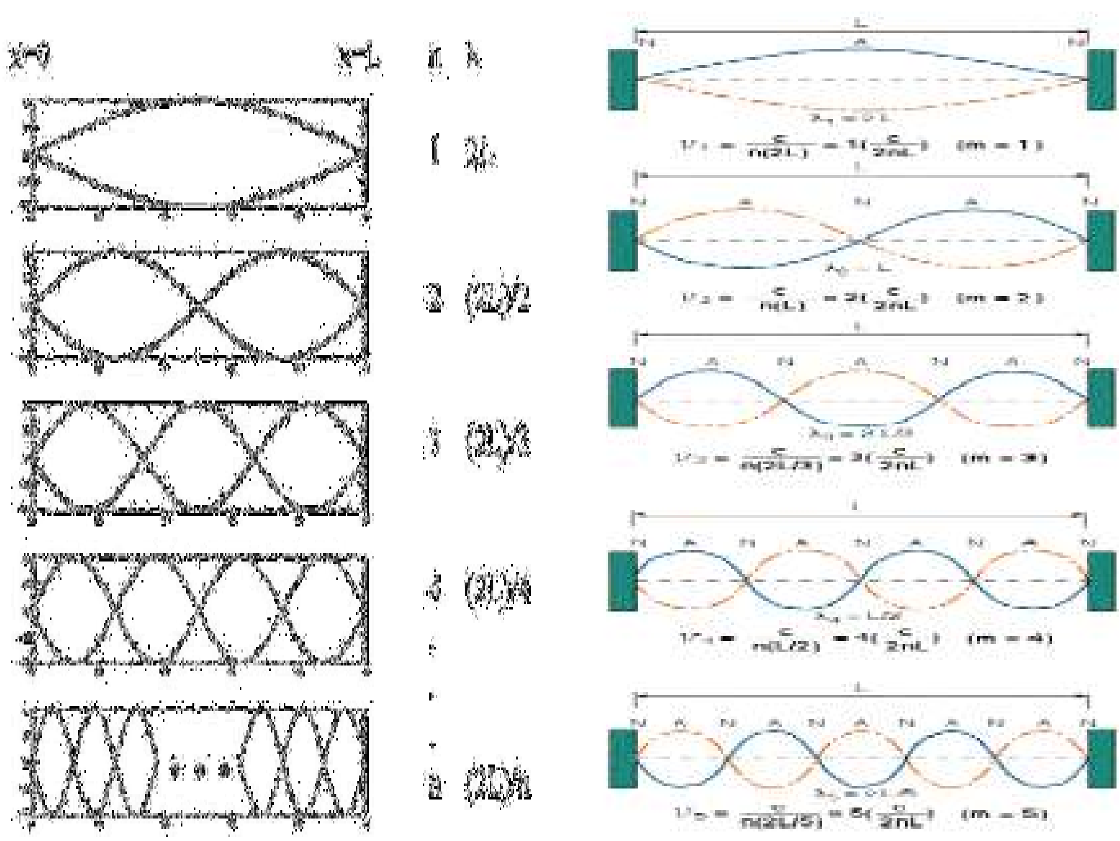


Fig 25: Longitudinal waves with different n value.

Conclusions:

- The frequency of each laser mode is equal to integer (mode number m) times the frequency of the fundamental longitudinal mode.
- From this conclusion it is immediately seen that the difference between frequencies of adjacent modes (mode spacing) is equal to the basic frequency of the cavity $\left(\Delta\nu = \frac{c}{2\,n\,L}\right)$.
- The mode with the longest supported wavelength λ_1 (twice the length of the string) has the lowest possible frequency $\nu = \frac{c}{2L}$. It is called the fundamental mode.
- Subsequent normal modes have shorter wavelengths (integer fraction of $2L$) and higher frequencies (integer of $\left(\frac{c}{2L}\right)$). They are called harmonics of fundamental mode.
- Equation for standing wave with both ends fixed at $x = 0$ and at $x = L$ is: $y(x,t) = 2A\sin(kx)\sin(\omega t)$.

Attention!

Until now it was assumed that the index of refraction (n) is constant along the optical cavity. This assumption means that the length of the active medium is equal to the length of the optical cavity. There are lasers in which the mirrors are not at the ends of the active medium, so L1is not equal to the length of the cavity (L). In such case each section of the cavity is calculated separately, with its own index of refraction:

$$\left(\Delta\nu_{MS} = \frac{c}{2\,n_1\,L_1 + 2\,n_2\,L_2}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2-42)$$

$\Delta\nu_{MS}$: Mode Spacing.

H.W

1. The length of the optical cavity of a Nd-YAG laser is 30 (cm). The length of the laser rod, which makes the active medium, is 10 (cm). The index of refraction of the laser rod is 1.823. The rest of the cavity is air that have an index of refraction of 1.0. Calculate the difference in frequencies between adjacent modes.
2. The length of an optical cavity is 25 (cm). The index of refraction is 1. Calculate the frequencies and the wavelengths of the following modes: m =1, m = 10, m = 100, m = 10^6.
3. The length of the optical cavity in He-Ne laser is 55cm. The Laser bandwidth is 1.5 GHz. Find the approximate number of longitudinal laser modes.

Transverse modes

By studying the distribution of laser intensity on the section area vertically on the optical axis laser, it found that it takes different forms depending on the accuracy of the location of the mirrors and any slight change that leads to the change of these forms, known as Transversal Mode. The number of transverse oscillations modes depends on the shape and size of the mirror. When there are a number of oscillations in the laser product is said to be a multi-mode laser.

By dropping a laser beam on a white screen after being enlarged by a concave lens, the transverse patterns of the laser beam can be examined. Figure 26 illustrates a range of these shapes where green shows the greatest laser intensity and white areas where the laser is missing.

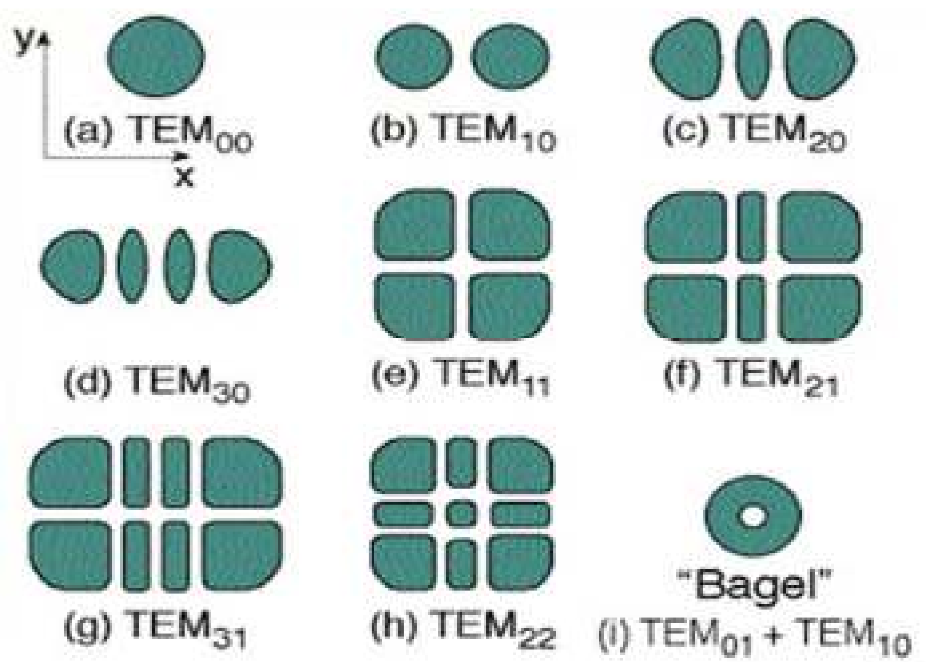


Fig 26: Transverse mode.

NOTES:

- Resonator modes are the distribution of the electromagnetic field within the resonator, which is located at the level of perpendicular to the optical axis of the resonator, which is the transverse modes, and the other is along the length of the optical axis, is the longitudinal modes.

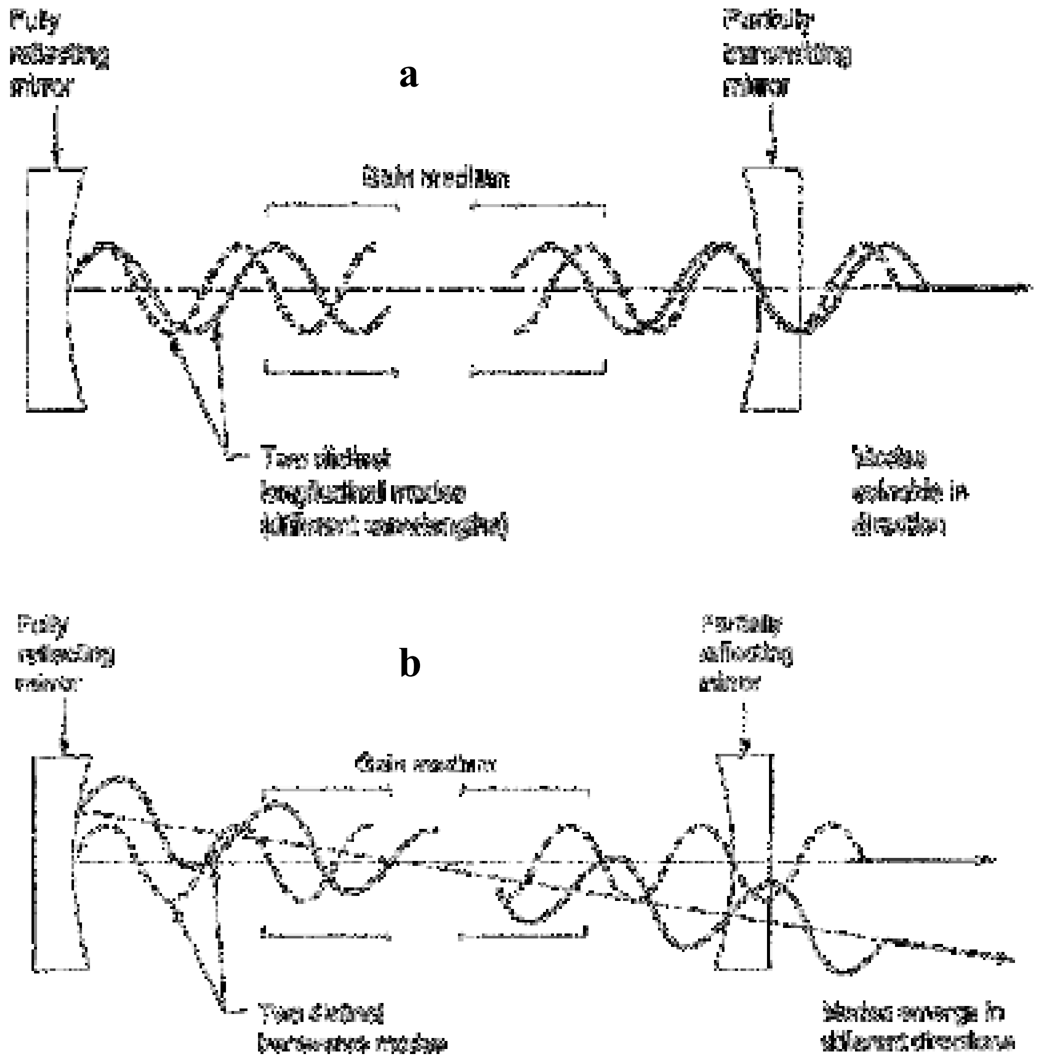


Fig 27: a) Longitudinal b) transverse frequencies modes.

- The number of longitudinal oscillation modes depends on the width of the spectral line and on the length of the resonator. When the resonator length is increasing the frequency interval, (the difference in frequency between any two consecutive modes of the longitudinal or transverse), will be decrease. This result oscillating a larger number of modes within the laser emission line.
- The number of modes under gain curve increases as the length of the laser amplifier (length of the resonator) increases because the frequency interval between the modes decreases with the length of the resonator.
- Typically, transverse modes are slightly off axis of the longitudinal modes, thus, there is significantly greater differences between longitudinal and transverse modes. However, at least one transverse mode will always exist in a cavity of one or more longitudinal modes.
- It is clear that a single mode laser can be made by reducing the length of the cavity, such that only one longitudinal mode will remain under the fluorescence curve with $G_L \geq 1$.
- The number of modes that can produce a laser are those where they achieve the condition that the gain must be greater than or equal to the loss as is evident in the colored area of the figure 24.

Example

The length of the optical cavity in He-Ne laser is 30 cm. The emitted wavelength is 0.6328 μm. Calculate:

1. The difference in frequency between consecutive longitudinal modes.

2. The number of the emitted longitudinal mode at this wavelength.

3. The laser frequency.

Solution:

1. The equation for difference in frequency is the same as for the basic mode:

$$\Delta\nu = \frac{C}{2L} = \frac{\left(3\times10^{8}\frac{m}{s}\right)}{(2\times0.3\ m)} = 0.5\times10^{9}\ Hz = 0.5\ GHz$$

2. from the equation for the wavelength of the n_{th} mode:

$$\lambda_n = 2L/n$$

$$n = 2L/\lambda_n = 2\times0.3/0.6328\times10^{-6} = 0.948\times10^{6}$$

Which means that the laser operate at a frequency, which is almost a million times the basic frequency of the cavity.

3. The laser frequency can be calculated in two ways:

a) By multiplying the mode number from section 2 by the basic mode frequency:

$$\nu = n\times(\Delta\nu) = (0.948\times10^{6})\,(0.5\times10^{9}\ Hz) = 4.74\times10^{14}\ Hz$$

b) By direct calculation:

$$\nu = \frac{c}{\lambda} = \frac{\left(3\times10^{8}\frac{m}{s}\right)}{0.6328\times10^{-6}m} = 4.74\times10^{14}\ Hz$$

Example

The length of the optical cavity in He-Ne laser is 55cm. The Laser bandwidth is 1.5 GHz. Find the approximate number of longitudinal laser modes.

Solution

The distance between consecutive longitudinal modes is:

$$\Delta\nu = \frac{c}{2L} = \frac{\left(3\times10^{8}\frac{m}{s}\right)}{(2\times0.55\ m)} = 2.73\times10^{8}\ s = 0.273\ GHz$$

The approximate number of longitudinal laser modes:

$$N = \frac{Laser\ bandwidth}{\Delta\nu} = (1.5\ GHz)\ /\ (0.273\ GHz) = 5.5 \approx 5\ modes.$$

Oscillating Modes of the Resonator

The resonant cavity of the laser emission is an open resonator consisting of two mirrors. The calculations of the oscillation modes for such cavities include the loss of diffraction from these mirrors as well as the shape and dimensions of the two mirrors in addition to the distance between them in such calculations.

We will study the oscillating modes of the resonator by using two kinds of laser resonators: Plane – Parallel (Fabry–Perot) mirrors resonator and confocal resonator and we will mention some calculations, results and conclusions without going into theoretical details.

a. Plane – Parallel mirrors resonator

Let us assume that the resonator are square in shape with length of 2a, also let L is the distance between them, as shown in figure 28. The resonant frequencies for such cavity given in eq. 1-6, which is:

$$\nu = \frac{c}{\lambda} = \frac{c}{2a}\left(n_x^2 + n_y^2 + n_z^2\right)^{1/2}$$

Which mean:

$$\nu = \frac{c}{2}\left[\left(\frac{n}{L}\right)^2 + \left(\frac{m}{2a}\right)^2 + \left(\frac{l}{2a}\right)^2\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2-43)$$

where $n, m \ and \ l$ are integer numbers.

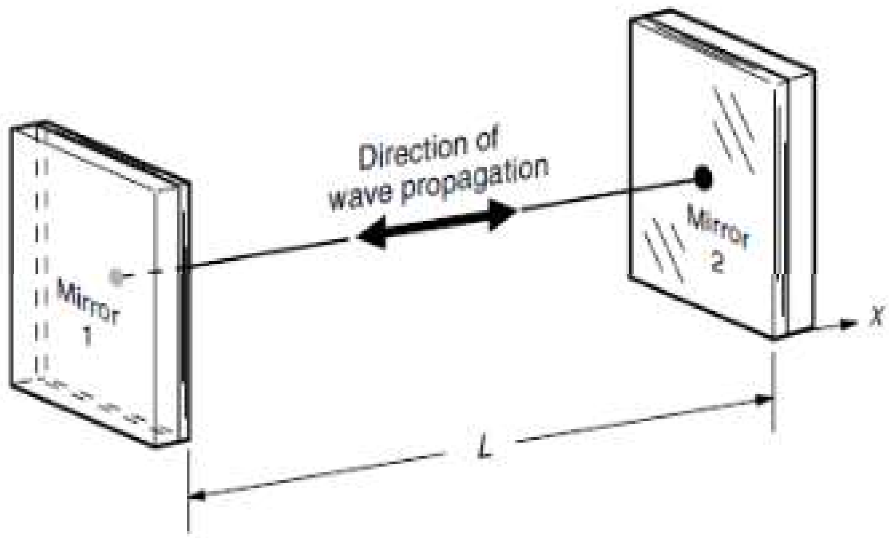


Fig 28: Planar or Fabry–Perot resonator.

If we delete the sides of this cavity i.e. neglect the numbers $m \ and \ l$ as compared to the number n. Therefore, we can write above equation after using chain rules and take only the first two terms as following:

$$\nu = \frac{c}{2}\left(\frac{n}{L} + \frac{1}{2}\frac{(l^2+m^2)}{n}\ \frac{l}{4a^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2-44)$$

Thus, the resonant frequency of any oscillation formula for this open cavity is determined by the amounts $n, m \ and \ l$. We also note from the previous equations that the frequency difference between the two consecutive oscillation modes have the same values of the two amounts l and m but they differ by one value of the amount n, can be expressed according to the following relation:

$$\Delta\nu_n = \frac{c}{2\,L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-45)$$

The oscillation modes that have the same values for amounts l and m and different in the amount of n, differ only in how the field is distributed along the axis of the resonator (z-axis) i.e. longitudinal, such modes are often called longitudinal modes of oscillation or axial modes. Therefore the frequency interval $\Delta\nu_n$ is represent the frequency difference between consecutive longitudinal modes, see figure 29.

Now, if the oscillation modes are different in the amount m or l or both and have the same value of amount n, such modes are called transverse modes of oscillation. The frequency difference between the two consecutive transverse modes differ by one value of the amount m can be expressed according to the following relation:

$$\Delta\nu_m = \frac{c\,L}{8\,n\,a^2}\left(m + \frac{1}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-45)$$

For resonator with practical values of L, $\Delta\nu_n$ will be within a few hundred of MHz, while $\Delta\nu_m$ or $\Delta\nu_l$ will be within a few MHz.

The use of the term longitudinal and transverse modes may be combination and may erroneously indicating to two separate types of oscillations modes (longitudinal and transverse). But the truth is, as we have mentioned, there are three specific amounts to define the oscillation mode which are the numbers $n, m\ and\ l$ and also both the electric and magnetic fields of the radiation are almost perpendicular to the resonator axis (z-axis). The change of these two fields in the direction of the transverse on the axis of the resonator is determined by the two amounts $l\ and\ m$, while expressing the change of field towards the z-axis (longitudinal) by the amounts n. When we talk about a transverse oscillation mode, we give it a specific amount of the number m and the number l, while leaving the number n to take any value. Also the longitudinal mode is defined by a certain amount of the number n while the values of l and m are ignored. The figure 29 shows the frequencies of the oscillation modes of the resonator with two plane mirrors and parallel.

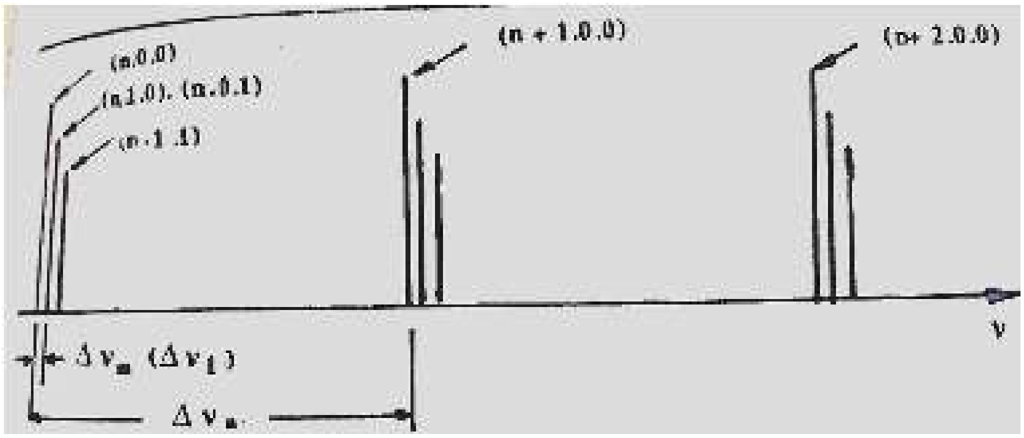


Fig 29: Resonant frequencies of resonator with two plane-parallel mirrors.

The number of longitudinal oscillation modes depend on the spectral line width and the length of the laser resonator. Increasing in the length of resonator leads to decrease in the frequency interval between any two modes and thus results in a larger number of modes within the laser emission line, see figure 30.

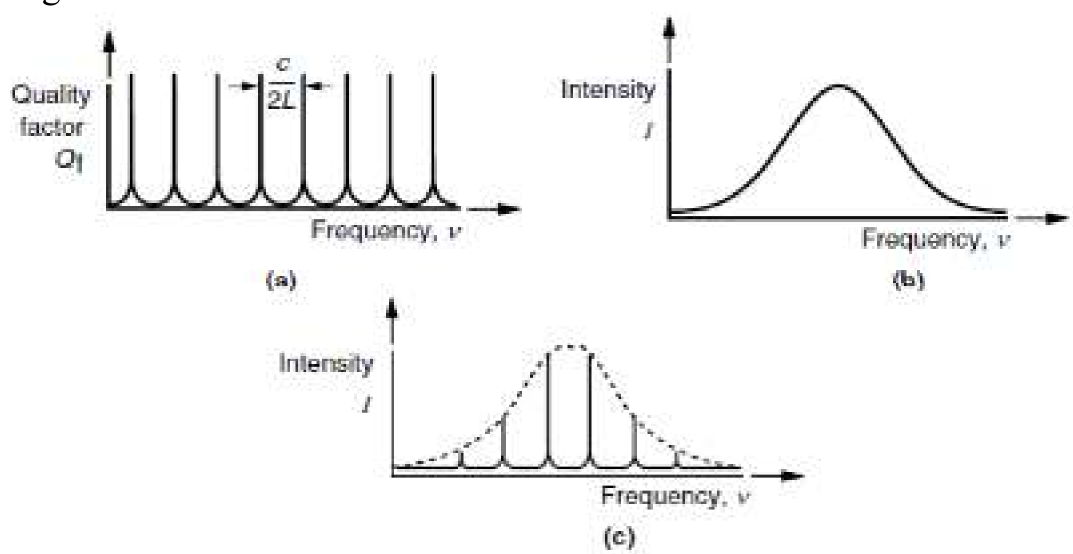


Figure 30: Longitudinal modes in a laser output (a) Modes that can exist within the cavity (b) Overall laser transition linewidth (c) Output of the laser indicating the modes within it.

While the number of transverse modes depends on the shape and size of the mirror and on other laser building values.

When there are a number of oscillation modes in the laser product is said to be a multi-mode laser (multi-mode operation), see for example figure 31.

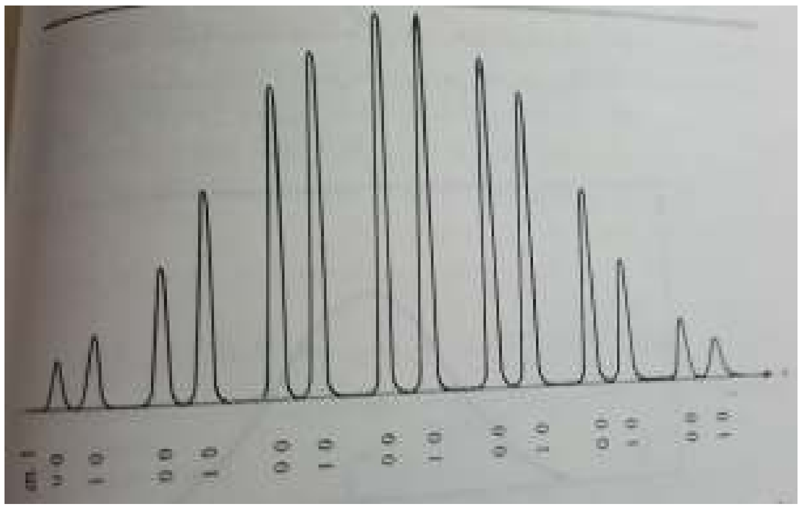


Fig.31: Oscillating of two transverse modes for each longitudinal mode.

Oscillating of a single transverse mode can be practically obtained by inserting loss factors that work to reduce other transverse modes and forced the multiple laser modes to work in a certain mode.

In some types of laser, a circular-shaped barrier used which can be controlled to operate on the optical axis of the resonator. Its purpose is to block all transverse oscillation modes except the mode TEM_{00}. This is possible because these modes by their nature do not exactly match the axis of the resonator so suffer from the loss by the barrier and it will reduce before the loss comes on the mode TEM_{00} that exactly match the axis of the resonator.

Figure 32 shows the distribution of field with perpendicular direction on the resonator axis (x-axis), and foe transverse oscillation mode with each m and

l are equal to zero, such this oscillation mode called **Transverse Electric Magnetic** field (TEM_{00}) and described by symmetric. Foe transverse oscillation mode with $l = 1$ and $m = 0$ or vice versa, ($l = 0$ and $m = 1$), that is antisymmetric oscillation mode and known as TEM_{01} or TEM_{10} mode, where the change in the x-direction corresponds to the value $l = 1$ and the change in the y-direction corresponds to the value $m = 0$.

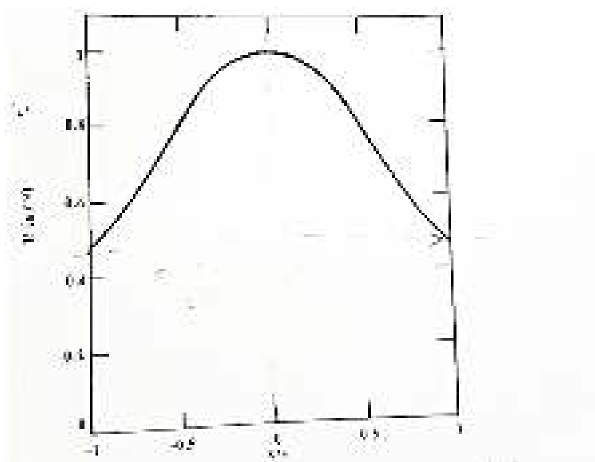


Fig.32: amplitude distribution for symmetric oscillation mode for resonator with two plane-parallel mirrors.

For such modes, the diffraction loss that produced from the resonator mirrors and for certain wavelength be larger for antisymmetric oscillation mode (TEM_{10}, for example) than for symmetric oscillation mode (TEM_{00}).

Conclusion

If we consider two modes that have the same values of m and l, but with a unit difference in the n value, the corresponding difference in frequency of oscillation will be

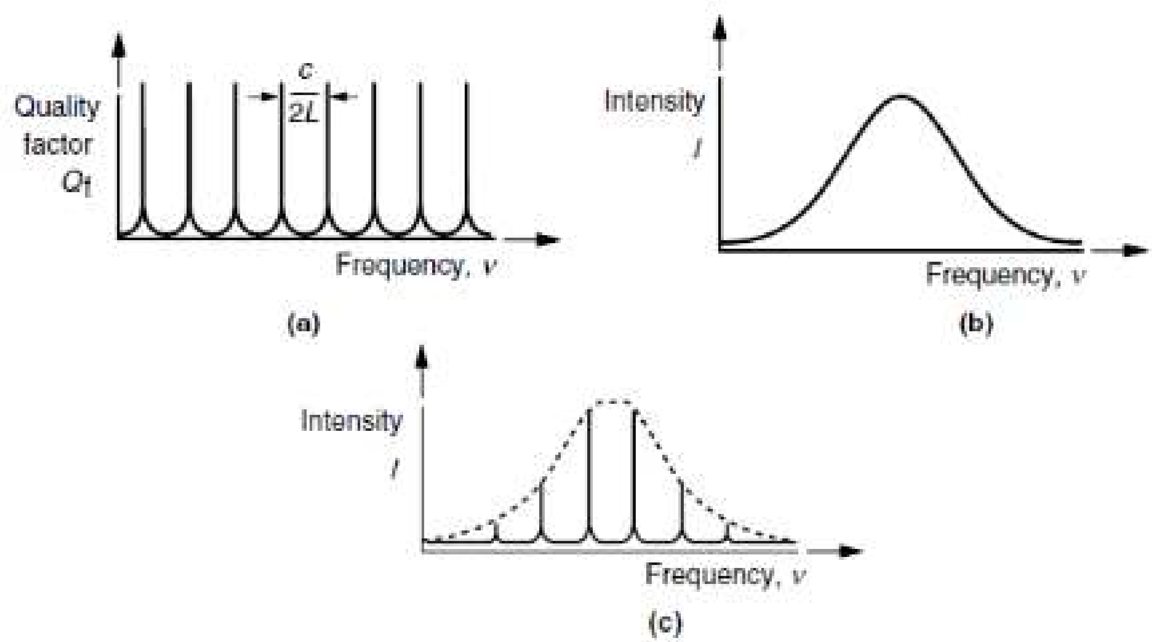
$$\Delta\nu_n = \frac{c}{2\,L}$$

These two contiguous modes will differ only in their field distribution along the longitudinal (or x) axis. Thus, the different modes resulting from the different values of n are referred to as the *longitudinal* or *axial modes* even though they are separated at discrete frequencies in the spectral domain.

For illustration, we note that a typical value for the mirror spacing in a laser would be about L=1500 nm, then:

$$\Delta\nu_n \approx \frac{3\times10^{11}}{2\times1500} = 100\ MHz$$

Thus, the separation between the longitudinal modes for this cavity is 100 MHz, (Figure 33a). The laser transition linewidth (Figure 33b), is normally much wider, say, 1 GHz for a Doppler broadened. As a result, the output of the laser will consist of a number of oscillating discrete frequencies, the axial modes, within the broadened linewidth (Figure 33c).



cavity. (b) Overall laser transition linewidth. (c) Output of the laser indicating the modes within it.

When the modes have the same n value but differ in their values for m and l, the resulting field distributions differ in the transverse direction, and are thus referred to as the *transverse modes* of the cavity. In essence, the transverse modes give an indication of the distribution of intensity within the beam cross section. If we consider adjacent transverse modes where $m = l$ but the value of n does not change, and with $\Delta m = 1$, then we have from equation (2-45)

$$\Delta \nu_m = \frac{c\,L}{8\,n\,a^2}\left(m + \frac{1}{2}\right)$$

Or

$$\Delta \nu_m \approx \Delta \nu_n \frac{\lambda\,L}{8\ a^2}\left(m + \frac{1}{2}\right)$$

And considering the same example used for the axial modes, but with the mirror dimensions a=b=10 mm, and with the beam in the infrared region of wavelength $\lambda \approx \frac{2}{3} \times 10^{-3}$ mm, then since $m \approx 1$, we find that

$$\Delta \nu_m \approx 100 \times \frac{1500 \times \frac{2}{3} \times 10^{-3}}{8 \times 100}\left(1 + \frac{1}{2}\right) \approx 0.2\ MHz$$

Thus, $\Delta \nu_n \gg \Delta \nu_m$ indicating that the separation in frequency between longitudinal modes is much greater than that between transverse modes, see figure 34.

The transverse modes show as a pattern of spots and are often referred to as transverse electromagnetic or TEM modes. They are characterized by two integers (m, l) that indicate the number of modes in two orthogonal directions and are thus designated by TEM_{ml}.

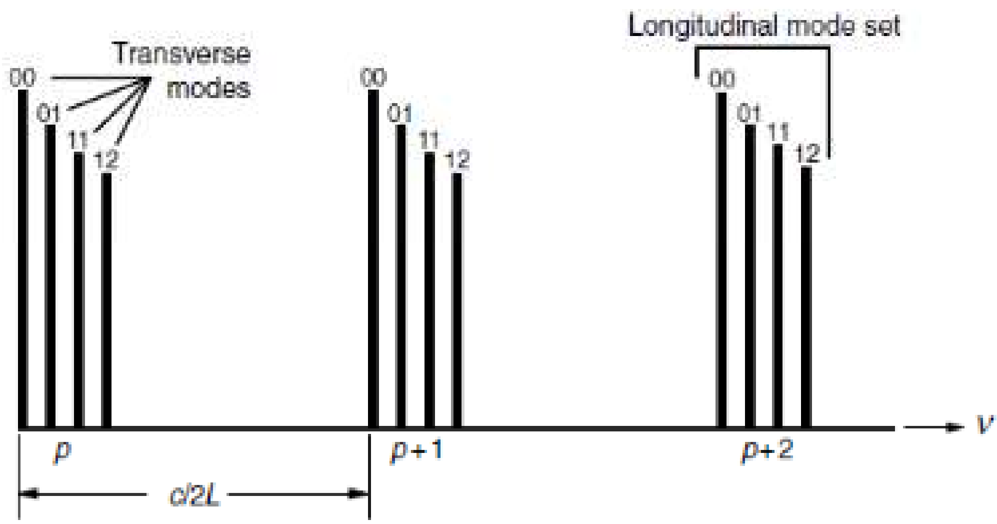


Figure34: Resonant frequencies of a Fabry–Perot resonator.

The m and l values are typically small, about 10 or less, and the larger the values, the more complex and spread out are the beam patterns.

b. Confocal Resonator

In the same way, calculations of Boyd and Gordon came to the oscillation modes for confocal resonator with length of L, and in order to simplify the calculations, the mirror is chosen with a square shape with side length of 2a and make the distance (L) to much larger than (a).

Therefore, the distance between two facing points can be approximated by distance (L), then the intensity of radiation can be calculated at the sides of mirrors. Figure 35a represent the intensity distribution at the mirrors for TEM_{00} oscillation mode. It is a radii Gaussian shape (radial), and if we look to the mirror, we will see luminous circular spot with size usually given by amount(w_z), which is the distance from the center of mirror, since the intensity of radiation reduced to $\frac{1}{e^2}$ from its value at the center of mirror.

From calculation of intensity for Gaussian shape, w_z gives as follow:

$$w_z = \left(\frac{\lambda L}{\pi}\right)^{\frac{1}{2}} \dots\dots\dots (2-46)$$

where w_z called the spot size at the mirror, for example it is equal to 0.3 mm for wavelength of 600 nm and resonator length of 0.5 m.

Calculations of Boyd and Gordon for this resonator gives also the distribution for antisymmetric oscillation modes, as example for the mode TEM_{01} (L=1, M=0), figure 35b shows the variation of field intensity towards the radius along y-axis science the distribution towards x-axis corresponds to the figure 35a. The shape of the spot at the mirror appears as shown in figure 26.

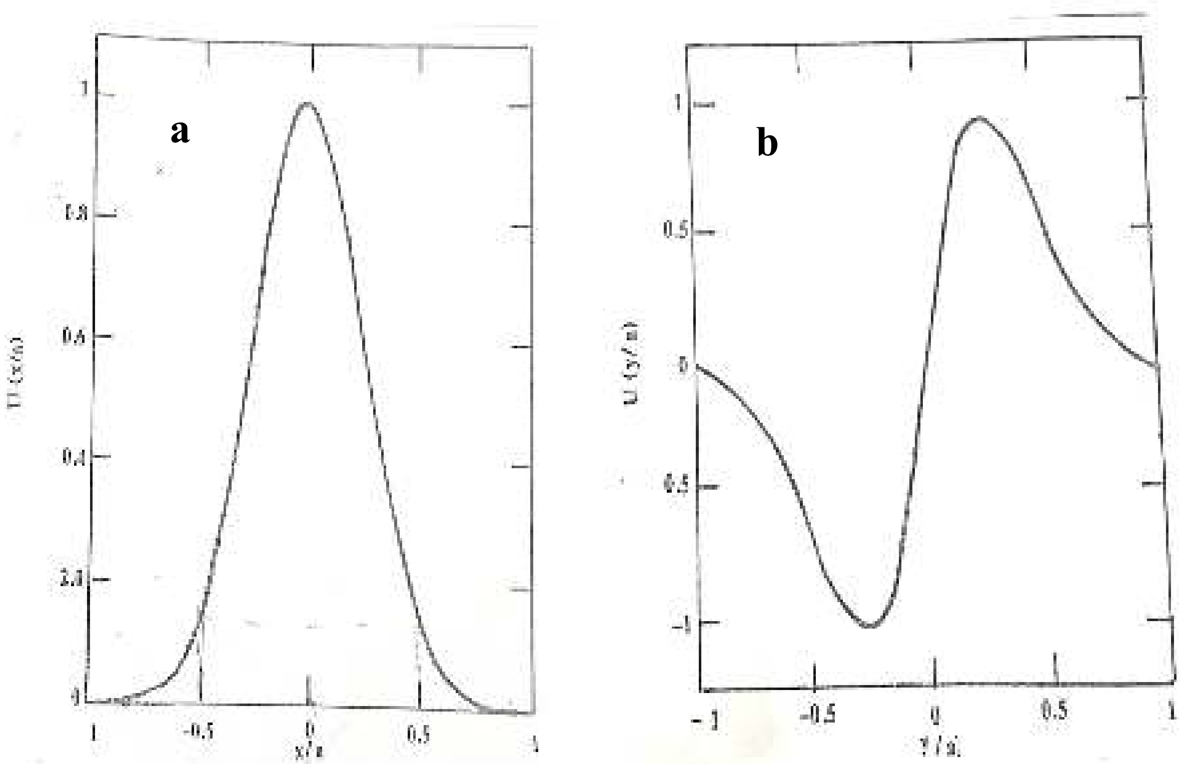


Figure 35: Intensity of oscillation mode for confocal resonator: a) Symmetric. b) Antisymmetric

After identifying the distribution shape of the field intensity at the mirrors of the resonator, it is necessary to identify for such distribution at any position, inside or out the resonator.

When applying the diffraction theorem in the resonator calculations and for confocal resonator, a relation can be found that expresses the spot size that drawn by the beam w_z at position Z. If we choose the center of the resonator to represent the point of origin $(Z = 0)$, see figure 36, the spot size will be get as follow:

$$w(Z) = w_o \left[1 + \left(\frac{2\,Z}{L}\right)^2\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-47)$$

Where w_o is the spot size at the center of resonator i.e. at $Z = 0$ and given as:

$$w_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-48)$$

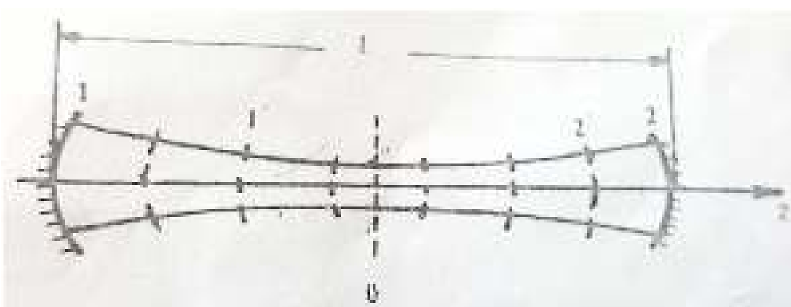


Figure 36: Spot size of TEM00 oscillation mode for confocal resonator.

Figure 36 shows the dimensions of beam (spot size) as a function to the position along the resonator axial which is drawing according to the equation (2-48). In this figure, we note the beam has a waist represent the smallest size for the spot and called *beam waist* which is at the position $Z = 0$, i.e (w_o) is spot size at waist beam. In addition we can note the spot size at the mirrors (at the position $\left(Z = \pm\frac{1}{2}\right)$ will be:

$$w = \left(\frac{L\,\lambda}{\pi}\right)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2-49)$$

Above equation is agreement with the equation (2-46). As we see the spot size at the mirrors larger with $\sqrt{2}$ times than its value at the center of resonator. This result is expected if we remember that the mirrors of resonator make to focal the beam in the resonator center.

The laser beam is also describe with its wave front and the radius of curvature of the wave front is expressed at a position inside resonator or along axial of it (along Z-axis) by the amount $R(Z)$ at position Z:

$$R(Z) = Z\left[1 + \left(\frac{L}{2\,Z}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2-50)$$

Figure 36 also shows the wave front at different positions, indicated by intermittent lines. At $Z = 0$ the $R(0) = \infty$, i.e. the shape of wave front is plane, which is as expected because of considerations of symmetry mode. While at the position $Z = \pm\frac{1}{2}$ (at the mirrors) equal to L, which is as expected because of the surface of mirror represent curveted front for the wave.

Line Selection

Lasers often undergo transition simultaneously on a number of transition lines or wavelengths. Thus, there may be a number of these transition lines present in the output. Since each line contributes to the output power, the power may be relatively high under such circumstances. However, in situations where a higher degree of mono-chromaticity is desired, the laser should be made to oscillate on only one of the transitions. This can be done by inserting a wavelength dispersive element such as a prism or diffraction grating into the cavity (Fig. 37).

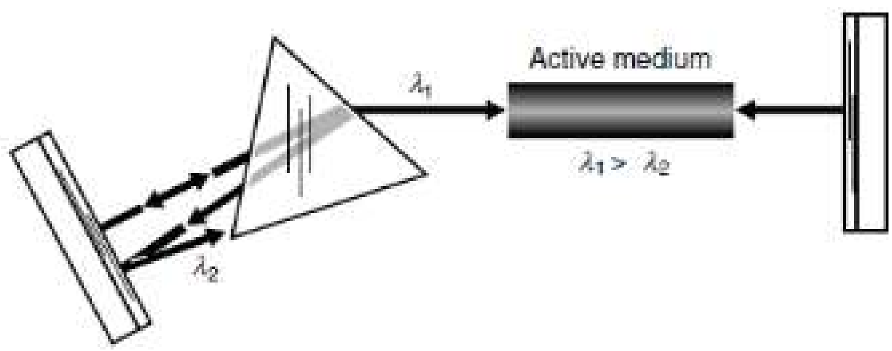


Figure 37: Wavelength selection.

As the beam propagates through the prism, the individual wavelengths undergo different levels of refraction. The wavelength whose ray is directed normal to the mirror is reflected back into the cavity, while the other wavelengths experience losses, resulting in only one wavelength oscillating. The oscillating wavelength can be changed, that is, the laser tuned, by rotating the prism or diffraction grating. Even when the laser is oscillating on a single line, that line itself may be broadened and could be oscillating on a number of modes.

Mode Selection

As explained, the number of modes that oscillate within a laser cavity can be very large. This has an influence on the properties of the output beam. For example, the directionality of the output beam is reduced as the number of transverse modes increases, and likewise, the spectral purity is reduced as the number of longitudinal modes increases since the beam then contains a large number of discrete frequency components.

Although these characteristics may be acceptable for some applications, there are other situations where they may not be appropriate. Case in point is when a laser is used for alignment. This requires low beam divergence, and the lowest divergence is obtained with the TEM_{00} mode. Thus for such applications, it might be necessary to suppress all the higher transverse modes for the laser to operate only in the TEM_{00} mode. Mode selection techniques are used to change the oscillating modes of a laser. In the next two subsections, we consider various methods for selecting transverse and longitudinal modes of a laser.

Transverse Mode Selection

The field distribution of the laser beam is such that the Gaussian or TEM_{00} mode is smaller in size than all the other transverse modes. In fact, the higher the order of the mode, the wider the beam becomes. Furthermore, the energy of the higher order modes tends to be concentrated away from the resonator axis. Thus, the higher order modes can be eliminated by placing a diaphragm in the cavity and having it normal to the cavity axis. If the aperture of the diaphragm is made small enough, the higher order modes are severely attenuated, leaving only the TEM_{00} mode. One disadvantage with this technique is that reduction of the aperture also increases losses associated even with the fundamental mode, thereby reducing the overall output power available.

Longitudinal Mode Selection

Longitudinal mode selection is used when it is desirable to have a highly monochromatic output beam. There are various techniques available for reducing the number of longitudinal modes that oscillate within a laser cavity. These include cavity length variation, the Fabry–Perot etalon, and the Fox–Smith interferometer.

- ***Cavity Length Variation***

As discussed earlier, the longitudinal modes are spaced apart by:

$$\Delta\nu_n = \frac{c}{2\,L}$$

From above equation, it is evident that decreasing the cavity length L increases the spacing between the discrete frequency modes. Thus, if the cavity length is decreased to the point that the frequency spacing is greater than the transition linewidth (Fig. 38), then only one mode will oscillate. This technique is effective for cavities where the laser linewidths are relatively small, such as for gas lasers.

One drawback of the technique, though, is that decreasing the cavity length decreases the volume of active material available for lasing, which can significantly reduce the output power. For solids and liquids where the laser linewidths are much greater, this technique is usually not appropriate.

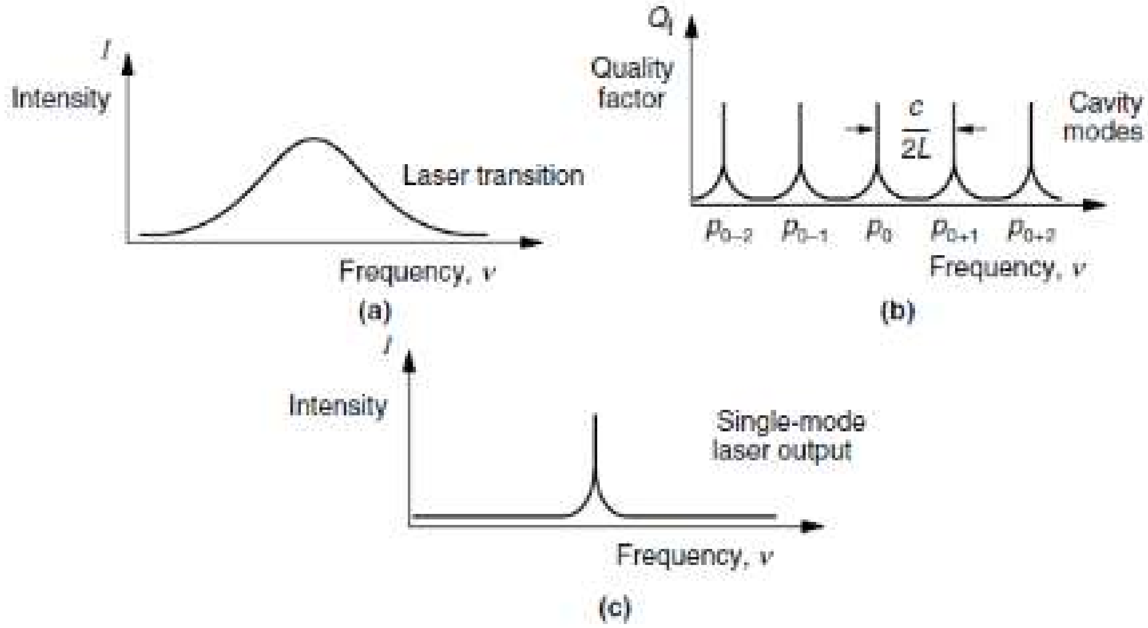


Figure 38: Longitudinal mode selection by frequency spacing. (a) Overall laser transition linewidth. (b) Modes that can exist within the cavity. (c) Output of the laser, indicating the single mode within it.

- ***Fabry–Perot Etalon***

Another technique involves the use of a Fabry–Perot etalon that may consist of a glass block with two of its faces ground parallel to a high degree of accuracy. This is inserted in the laser cavity as shown in Fig. 39. Let the thickness of the block or spacing between the parallel faces be h, and let the normal to the block be inclined at an angle θ to the cavity axis. A beam that is incident on the etalon is partially reflected from the first surface at point A, and the partially transmitted portion is reflected from the second surface at point B and retransmitted through the first surface at point A′. If the initially reflected beam OAP and retransmitted beam OBP, along with those resulting from multiple reflections, destructively interfere for certain modes, then those modes will have low loss. Since the incident beam OA is in the medium with the lower refractive index, it undergoes a 180◦ (π) phase shift on reflection at A, and it can be shown that OBP undergoes a phase shift, φ, of:

$$\phi = \frac{2n\pi\,\nu}{c}\,2h\cos\theta' \quad \ldots\ldots (2-51)$$

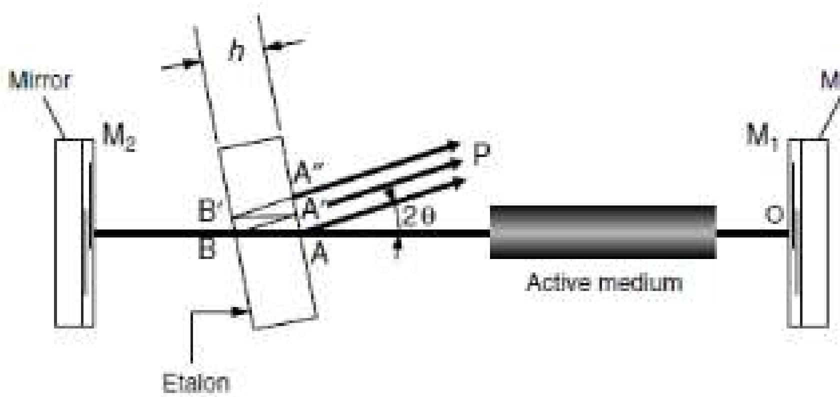


Figure 39: Fabry–Perot etalon setup.

where n is the refractive index of the block, ν is the mode frequency, c is the velocity of light in free space, and θ′ is the angle of refraction of the beam within the etalon. For destructive interference, the phase difference between the two rays must be 180˚, and thus we have:

$$\frac{4n\pi\,\nu}{c}\,2h\cos\theta' - \pi = (2m-1)\pi,\ m = 1, 2, 3 \ldots\ldots \qquad (2-52)$$

From equation (2-52), we find that the modal frequencies that will be associated with minimum cavity loss when the etalon is inserted in the resonator are given by:

$$\nu = \frac{mc}{2nh\cos\theta'} \quad \ldots\ldots (2-53)$$

with contiguous modes that encounter minimum loss being separated by:

$$\Delta\nu = \frac{c}{2nh\cos\theta'} \quad \ldots\ldots (2-53)$$

Thus by decreasing the thickness of the etalon, the separation between modes can be increased to the point that only one mode oscillates in the resonator. Furthermore, the orientation θ of the etalon can be adjusted such that only the mode at the center of the linewidth oscillates. In the same vein, the laser can be tuned (oscillating frequency varied) over a narrow frequency range by adjusting θ.

- ***Fox–Smith Interferometer***

The Fox–Smith interferometer, shown schematically in Fig. 40, is another technique that is used for longitudinal mode selection. It consists of two mirrors, M_1 and M_2, along with a beam splitter, A. The beam from O is partially reflected from the splitter as OAP, while the rest of the beam first undergoes 100% reflection at M1, and then yet another 100% reflection at M_2 as OABACP. Just as in the case of the Fabry–Perot etalon, beam OAP undergoes a 180° phase shift after being reflected.

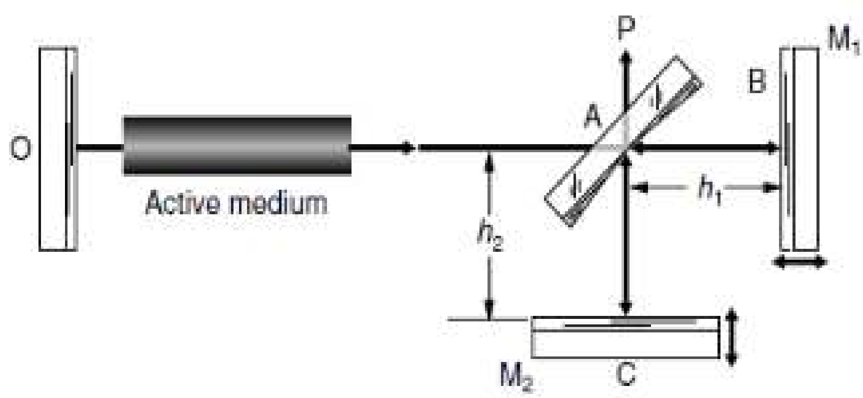


Figure 40: Fox–Smith interferometer.

The shift in phase of beam OABACP is:

$$2\frac{2\pi}{\lambda}(h_1 + h_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-54)$$

where h_1 and h_2 are the distances of M_1 and M_2, respectively, from the beam splitter.

Thus for destructive interference, we have:

$$2\frac{2\pi}{\lambda}(h_1 + h_2) - \pi = (2m-1)\pi, \ m = 1, 2, 3 \dots\dots \qquad (2-54)$$

resulting in a frequency difference between contiguous modes with low losses of

$$\Delta\nu = \frac{c}{2n(h_1+h_2)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2-53)$$

where n is the refractive index of the medium in which the beam propagates. Again, by reducing $h_1 + h_2$, the modal separation can be increased until only one mode oscillates

Beam Modification

Some of the basic characteristics of a pulsed beam, such as the peak power, pulse duration (pulse width), and pulse repetition rate (pulse frequency), can be changed during the beam generation to enable certain desired beam properties to be achieved. For example, a higher peak power may be required to initiate cutting or welding in some materials, with the power level that is needed being significantly reduced once the material melts. We start with a discussion on the measure of energy losses in the laser cavity, that is, quality factor (Q_f), which constitutes the framework for controlling the peak power and pulse duration. This is followed by a discussion on Q-switching, which is a method that is used in producing pulsed laser outputs of relatively short duration, and different methods for achieving Q-switching.

Quality Factor

The high-quality resonance cavity stores energy well while the low-quality resonance cavity does not. In addition, the high quality factor is accompanied by a relatively narrow spectral line, while the low quality factor is accompanied by a relatively broad spectral line. This relationship between Q_f and spectral line widths can be expressed simply as follows:

$$Q_f = \frac{Resonant\ frequency}{Spectral\ line\ width} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-54)$$

Or

$$Q_f = \frac{\nu}{\Delta\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-55)$$

In fact, the laser medium works to feed the oscillation modes by energy. Theoretically, if the dissipation energy is zero, the quality factor has infinite value, but in practice there must be a loss. This means that the spectral line has a simple broad. Currently, the spectral line width value can be reduced to reach 1 Hz.

Q-Switching

The output of some pulsed solid-state lasers such as the ruby laser usually consists of a number of random spikes, each of about a microsecond duration, with the individual spikes spaced apart by about 1μs, and with peak powers of the order of kilowatts. The entire pulse duration may be about 1 ms. Q-switching is a technique that is used to produce laser outputs of higher power (of the order of megawatts) and shorter duration (of the order of nanoseconds). It must be noted, however, that even though the output power is increased, the total energy content of the pulse is not, and may even be less.

To understand the principle behind Q-switching, we consider a laser cavity in which a shutter is placed in front of one of the mirrors. When the shutter is closed, it prevents light energy from reaching the second mirror, and thereby being reflected back into the cavity. In other words, most of the light energy is lost and oscillation cannot take place. The Q-value of the cavity is then very low (Fig. 41a-1), since the losses are high (Fig. 41a-2). As pumping of the laser continues (Fig. 41a-3), the population inversion keeps building up (Fig. 41a-4), far in excess of the threshold value, without any oscillation taking place. When a significantly high value of population inversion has been achieved, the shutter is suddenly opened to reduce the losses.

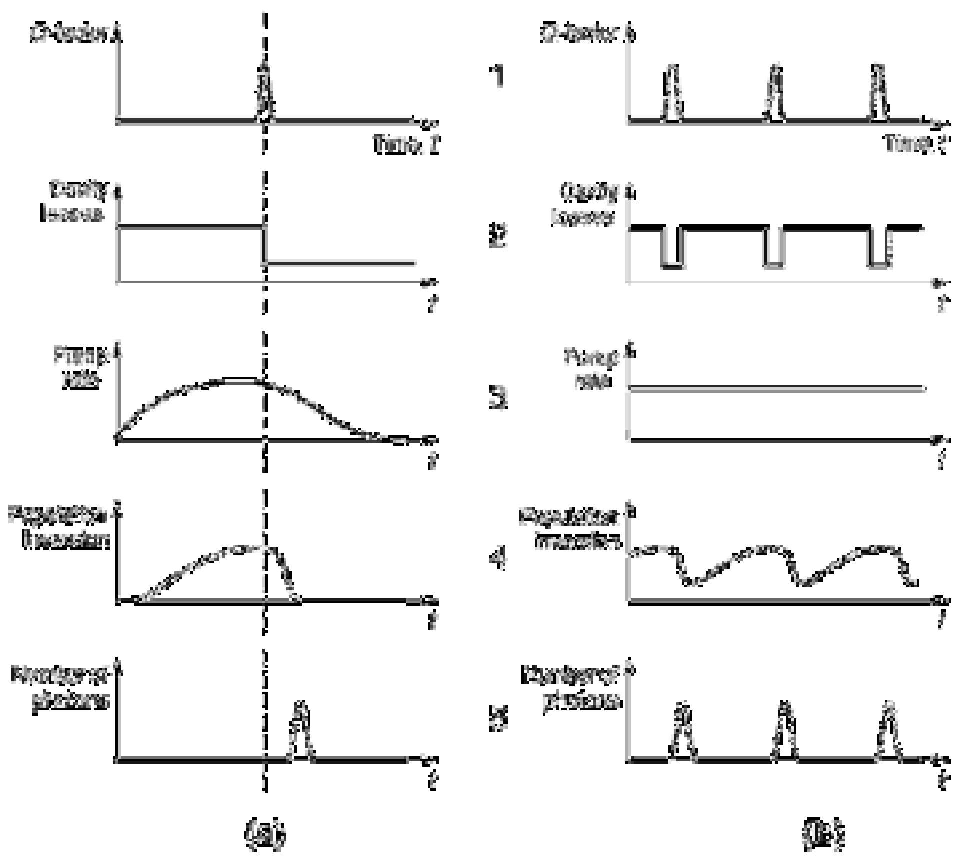


Figure 41: Time evolution of the Q-switching process. (a) Single pulse. (b) Continuous pulsing.

At this point, the gain of the laser (due to the high population inversion) is much greater than the losses. The high energy accumulated as a result of the large difference between the instantaneous and threshold population inversions is then released as an intense beam of short duration (Fig. 41a-5). Opening of the shutter to reduce the losses increases the Q-value of the cavity. Hence, the name Q-switching.

Q-switching can either result in a single pulse, in which case the pump rate is also pulsed (Fig. 41a), or it can be repetitively pulsed, in which case pumping is continuous (Fig. 41b). The necessary conditions for a laser to be Q-switchable are:

1. The lifetime, τu, of the upper level has to be longer than the cavity buildup time, t_c, that is,

$$\tau_u > t_c$$

This enables the upper level to store the extra energy pumped into it.

2. The pumping duration, t_p, has to be longer than the cavity buildup time, t_c. Preferably, it has to be at least as long as the lifetime of the upper level, t_u, that is,

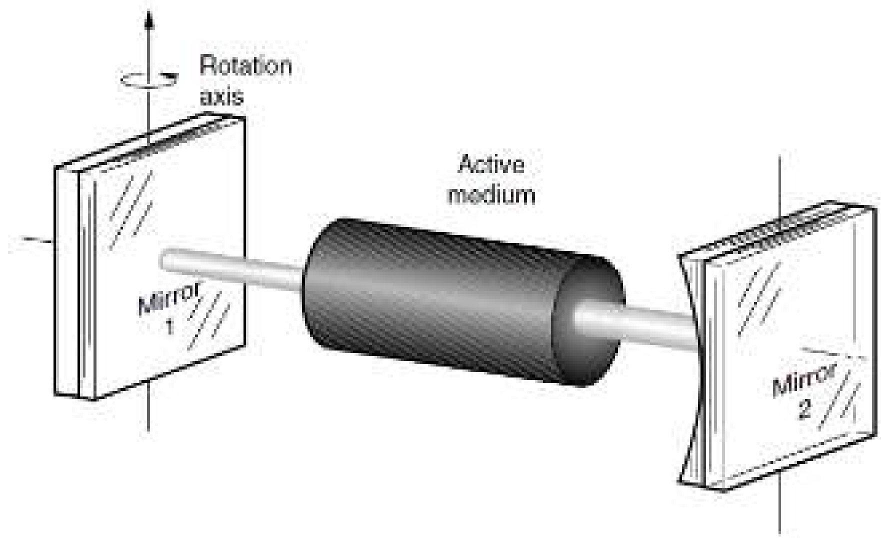
$$t_p > t_c \text{ and } t_p \geq t_u$$

3. The initial cavity loss must be high enough during pumping to prevent oscillation during that period.

4. The change in Q_f value must be sudden.

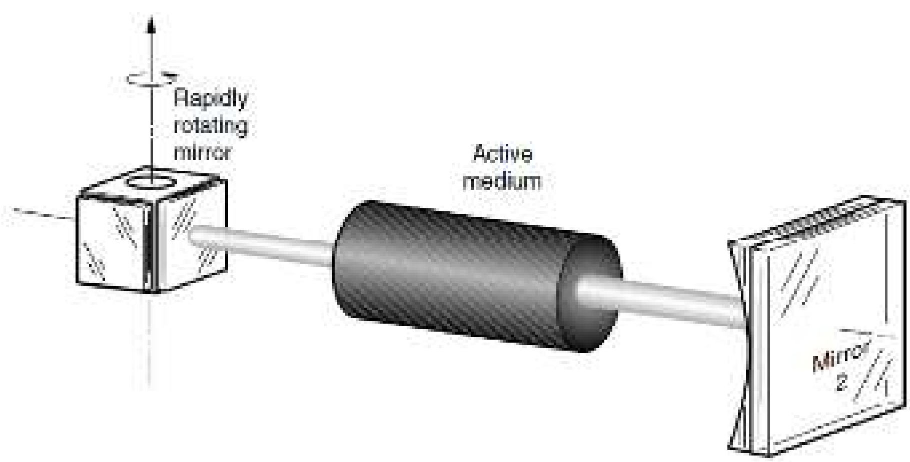
There are various techniques available for Q-switching:

1- Mechanical Shutters.



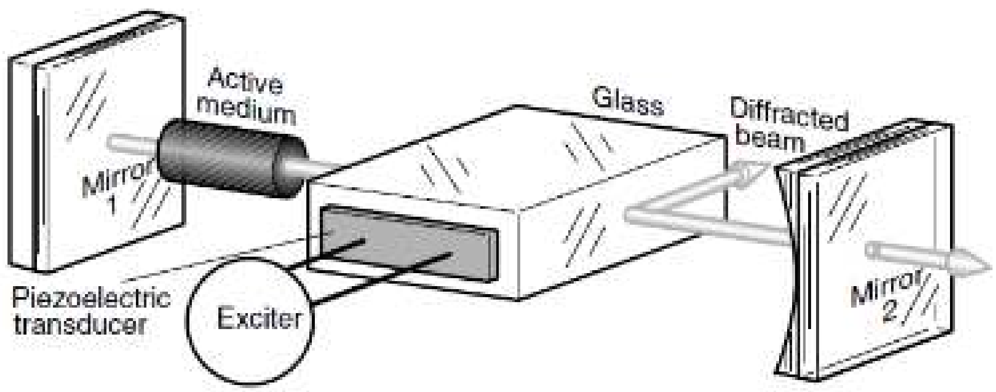
Mirror rotation for mechanical shutter action.

2- Electro-Optic Shutters



Quadrilateral mirror assembly for mechanical Q-switching.

3- Acousto-Optic Shutters
4- Passive Shutters



Acousto-optic modulation.

Mode-Locking

Mode-locking is used to generate pulses of higher peak power (of the order of gigawatts or more) than can be achieved by Q-switching and of very short duration (of the order of picoseconds or even femtoseconds). A laser cavity generally sustains a large number of oscillating modes. The oscillation of each of these modes is normally independent of other modes. Mode-locking is achieved by combining a number of distinct longitudinal modes of a laser in phase, with each mode having a slightly different frequency. Mode-locking can be classified into active and passive Mode-locking.

In active mode locking, an external signal is used to modulate or vary cavity losses or quality factor, Qf (t), at a frequency equal to the intermodal separation, while Passive mode-locking involves the use of nonlinear optical material.

Types of Lasers

There are various types of lasers. Each one has different characteristics that depend, largely, on the active medium used for laser action. In this chapter, we first discuss the major types of lasers, based primarily on the active medium. In addition, we also discuss recent developments in laser technology, especially with regard to industrial lasers, specifically those used for materials processing. The principal laser categories include the following.

1- Solid-state lasers.
2- Gas lasers.
3- Liquid dye lasers.
4- Semiconductor (diode) lasers.
5- Free electron lasers.

1- Solid-State Lasers

Solid-state lasers normally use an insulating crystal or glass as the host lattice. In it is embedded the active medium, which is either a dopant or an impurity in the host material. The crystal host material does not participate directly in the lasing action. The dopant is the component that participates directly in laser action. It is normally a transition metal or rare earth element, and it substitutes for some of the atoms in the host material, rather than being an interstitial impurity. For the solid-state lasers, the active medium is shaped in the form of either a rod or a slab. Since the rod form is the traditional shape for the active medium.

The rod is normally cylindrical in shape, with ends that are ground and polished to be plane and parallel. Its dimensions depend on the active medium being used. The ends of the rod may either be silvered (one completely and the other partially), in which case the rod constitutes the optical cavity, or the rod may be placed between two external mirrors.

Pumping is commonly done using a flash lamp, and the configuration involves either a cylindrical cavity with elliptical cross-section or a helical flashtube. The rod temperature may be controlled by circulating air or liquid around it. Otherwise, the heat generated can change the cavity dimensions and consequently, the cavity modes. Most solid-state lasers generate pulsed beams, even though some generate continuous wave outputs. Their coherence lengths are thus relatively short, making them unsuitable for a number of interference-based applications. However, the significant amount of energy available in each pulse makes them attractive to applications that require such energy bursts. These include resistor trimming, initiation of thermonuclear fusion, and spectroscopic research.

Common types of solid-state lasers include the ruby, Nd:YAG, and Nd:Glass lasers. These are further discussed in the following sections.

1.1- The Ruby Laser

The significance of the ruby laser is that it was the first laser to be successfully made and operated. The rod is a single crystal of ruby, which consists of crystalline aluminum oxide (Al_2O_3) that is doped with chromium. The chromium constitutes about 0.05% of the rod by weight and replaces some of the aluminum ions. The resulting material is pinkish in color. The aluminum oxide (sapphire) is the host lattice while the chromium ions, Cr^{3+}, constitute the active medium. The size of the ruby rod normally ranges from 0.5 to 1.0 cm in diameter and from 5 to 20 cm in length.

The energy levels involved in the lasing action are those of the chromium ions, and these are illustrated in Fig. 37 This is a three-level system. The ground level (level 1) is indicated as E_1.

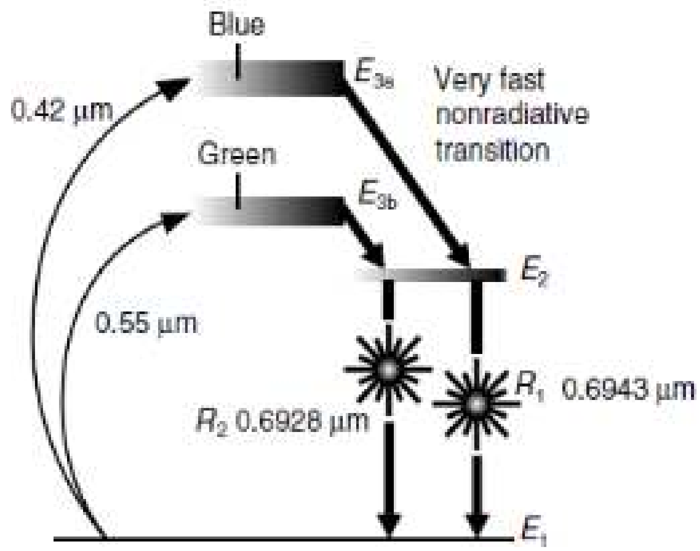


Figure 37: Energy levels of the ruby laser.

The pump level (level 3), indicated by E_3, has two bands, E_{3a} that has a mean wavelength of 0.42µm that corresponds to blue and E_{3b} that has a mean wavelength of 0.55µm corresponding to green. The upper lasing level (level 2), E_2, constitutes two sub-bands, E_{2a} with a wavelength of 0.6943µm (commonly referred to as R_1) and E_{2b} with a wavelength of 0.6927µm (commonly referred to as R_2). The R_1 transition usually dominates. Atoms that are pumped to level 3 where the lifetime is about 10−4ms. decay rapidly through non-radiative transition to level 2, which is a metastable state, with a lifetime of about 3 ms. The lasing transition normally occurs between E_{2a} and E_1 (the R_1 transition), even though the R_2 transition can also occur under special circumstances. The transition is homogeneously broadened due to interaction between the chromium ions and lattice phonons, with a FWHM of about 330 GHz.

Pumping may be done using a xenon flash lamp operated at a pressure of about 500 Torr, which provides a pulsed white light of the order of gigawatts with a period of about a millisecond and a pulse rate of one per second. The pumping action excites chromium ions from the ground state (level 1) to level 3 due to the absorption of radiation in the blue and green wavelength range, when the threshold light intensity is achieved. The excited ions undergo rapid transition to the metastable state corresponding to the upper lasing level. Due to the relatively long lifetime of this level (level 2), its population keeps increasing as pumping continues, until a population inversion is eventually achieved between levels 2 and 1. Lasing action is then triggered by the initial small spontaneous emission that naturally results from atoms in the higher energy states. The photons from such spontaneous emission are radiated in all directions. Those corresponding to the 2–1 transition and that are directed parallel to the cavity axis then stimulate the chromium ions in the upper laser level to radiate.

Lasing action ceases when the lamp stops operating. The net result is an output pulse from the laser. Even during a single pulse of about a millisecond, a number of sharp peaks that result from spiking can be observed.

The output of a ruby laser may be about 10–50 MW with a pulse duration of about 10–20 ns. Since ruby lasers are three-level lasers, high-threshold energy is needed to excite at least half of the Cr^{3+} ions to achieve population inversion. High levels of energy are thus required to operate them, resulting in extensive heating of the laser rod. This, coupled with the low thermal conductivity of ruby, makes them difficult to operate in continuous mode. As a result, they are not widely used anymore.

1.2- Neodymium Lasers

Neodymium (Nd) lasers have either a crystal or glass material as the host lattice, and this is doped with neodymium ions, Nd^{3+}, that constitutes the active medium. The energy levels are thus those of the neodymium ions and are illustrated in Fig. 38 These are four-level lasers. The ground level (level 0) is the E_0 energy level. Again, the pump level (level 3) has two bands, with E_{3a} having a mean wavelength of 0.73 μm, while E_{3b} has a mean wavelength of 0.8 μm.

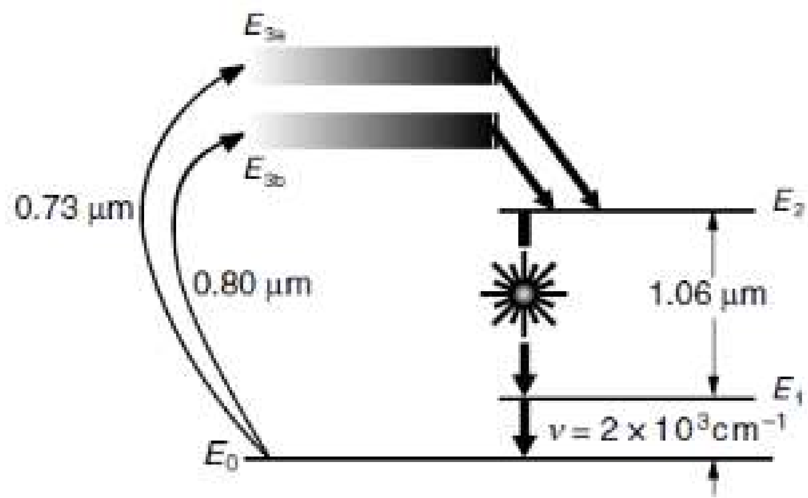


Figure 38: Energy levels of neodymium lasers.

The upper lasing level (level 2) is E_2 with a lifetime of about 0.23 ms., and from there, transition occurs to the lower lasing level (level 1), E_1. This transition also primarily homogeneously broadened with a FWHM of about 195 GHz, and a transition wavelength of 1.06 μm in the infrared range. Atoms are pumped from level 0 to level 3, from which they undergo rapid non-radiative decay to level 2, which is a metastable state. The energy difference between level 1 and the ground state, level 0, is such that $h\nu \gg k_{BT}$ at room temperature, so that level 1 is practically empty under thermal equilibrium conditions. Furthermore, transition from level 1 to level 0 is very fast, and occurs by non-radiative processes. Any atoms that make a transition to level 2 then essentially result in a population inversion. Neodymium lasers thus have a much lower inversion threshold requirements than ruby lasers. The traditional pumping schemes are similar to those outlined for the ruby laser. There is a variety of host materials that are used for neodymium lasers, but the most common ones are YAG and glass.

1.2.a The Nd:YAG Laser

The host lattice for this laser is a crystal of yttrium aluminum garnet (YAG) with the chemical composition $Y_3\ Al_5\ O_{12}$, where the Nd^{3+} ions (about 0.1–2%) substitute for some of the Y^{3+} ions. Operation of the Nd:YAG laser can be either in the continuous wave (CW) or in pulsed mode, depending on whether pumping is continuous or intermittent. A xenon flash lamp may be used at medium pressure (500–1500 Torr) or a krypton lamp at high pressure (4–6 atm), and the rod size is similar to that of the ruby laser. The power output in CW mode varies from 150 W to 6 kW, and that in Q-switched pulsed mode is of the order of 50MW with a pulse duration of about 20 ps, at a repetition rate of 1–100 Hz. The high-power (up to 2 kW) CW laser may be achieved by having three Nd:YAG rods in line as a single oscillator, with each rod being arc lamp pumped. The very high powers (up to 6 kW) are obtained by pumping with a diode laser. The efficiency of both pulsed and CW Nd:YAG lasers typically ranges between 1 and 3%. Applications of Nd:YAG lasers include laser surgery and materials processing, for example, welding, cutting, drilling, and surface modification.

1.2.b The Nd:Glass Laser

The host lattice for the Nd:Glass laser is glass such as silicate, phosphate, or fluoride glass, with 1–5% Nd^{3+}. It has the advantage that the rod size can be much larger (up to 1m in length and about 50mm in diameter) than that for Nd:YAG, since glass can be easily made with high-quality (optical homogeneity and free of residual stresses) and large sizes because of its lower melting temperature. With power output per unit volume being comparable to that of Nd:YAG, relatively larger power outputs can be obtained with this type of laser. Since glass has an amorphous structure, it has short-range order. Thus, the environment and therefore the field of each ion is different from that of any other ion. This variation of ion environments in the glass matrix results in additional inhomogeneous broadening, and thus a much broader bandwidth. The Nd:Glass laser therefore normally operates in multimode. Very short pulse periods, of the order of 5 ps, with high-output powers, can thus be obtained when the output is mode-locked.

One major disadvantage of Nd:Glass lasers is the low thermal conductivity of glass, which is about an order of magnitude smaller than that of Nd:YAG. This limits their usefulness for CW operation, and are thus only used for low-rate pulsed outputs, with a typical output pulse rate of 1 Hz. The relatively high-peak pulse outputs (up to 20 TW) of Nd:Glass lasers enable them to be used in laser-induced nuclear fusion reactions. However, the low pulse rates limit their applicability in manufacturing to such operations as drilling and spot welding, and in the cases where they are acceptable, are able to produce superior quality holes at depth-to-diameter ratios up to 50:1, much higher than what can be achieved with the Nd:YAG laser.

2- Gas Laser

Gas lasers are among the most common form in the laser industry. The power levels range from several kilowatts (carbon dioxide (CO_2) lasers) to milli Watt (helium–neon (He–Ne) lasers). They can be operated in either the continuous mode or pulsed mode, with output frequencies ranging from ultraviolet to infrared.

As the name implies, gas lasers use a gaseous medium as the active medium. Common examples are the He-Ne and CO_2 lasers. The broadening mechanisms in gas lasers are not as strong as those in solids. Thus, the resulting linewidths, determined primarily by Doppler broadening, are relatively small. This is because collision broadening is relatively small due to the low pressures normally used in gas lasers. As a result, the energy levels are relatively narrow, and thus a sharp emission line is essential for excitation. Optical pumping, with its broad emission spectrum, is therefore not suitable for pumping gas lasers, since it would result in inefficient pumping. Electrical pumping is thus the most common means of exciting the active medium in gas lasers. Pumping is also done by chemical means, with an electron beam, or by gas-dynamic expansion.

When classified by the active medium, one can identify four principal types of gas lasers:

1. Neutral atom lasers.

2. Ion lasers.

3. Metal vapor lasers.

4. Molecular lasers.

2.1 Neutral Atom Lasers

The most common example of neutral atom lasers (using inert gases as the active medium) is the He–Ne laser, which was also the first laser to generate CW output. The active medium in this case consists of 1 part neon to 10 parts helium. This is a four-level laser, and Figure 40 illustrates the energy level schemes of He and Ne. The helium atoms are more easily or efficiently excited to the higher levels by electron collision than are neon atoms. Thus, the electrons that are accelerated by the passage of a discharge through the mixture excite the helium atoms to the metastable higher energy levels denoted by HE_{3a} and HE_{3b}.

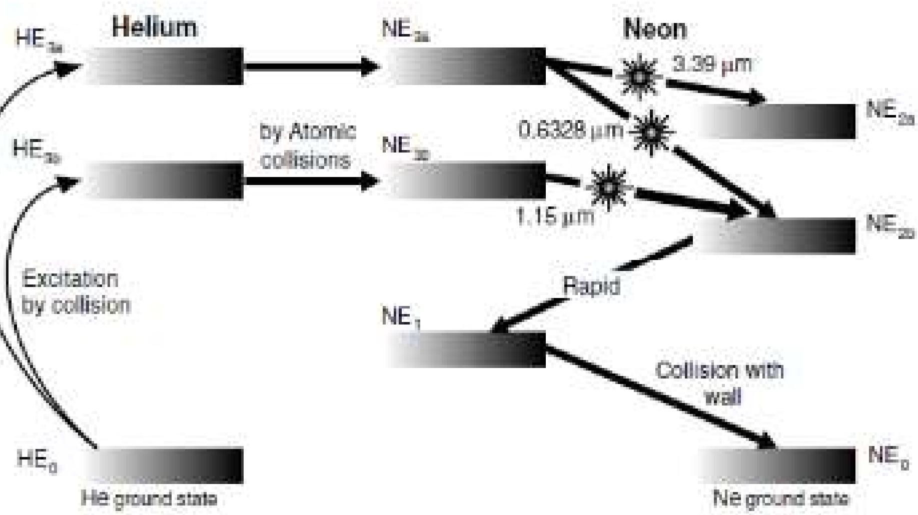


Figure 40: Energy levels of the He–Ne system.

These energy levels of helium happen to coincide with some of the excited states of neon, the NE_{3a} and NE_{3b} levels. The excited helium atoms are thus able to transfer energy to neon atoms, which are in the ground state when collision takes place between the two. This results in de-excitation of the helium atoms involved to the ground state while the neon atoms are excited to the higher energy level. This process of energy transfer through collision is referred to as resonant collision or resonant energy transfer, and is so called because the corresponding energy levels coincide. The process increases the population of the NE_{3a} and NE_{3b} levels of neon relative to the lower NE_{2a} and NE_{2b} levels, resulting in a population inversion between the two sets of levels.

Lasing action occurs between energy levels of the neon atoms. The role of helium is thus to assist in the pumping process. The metastable state of the excited helium atoms (staying at the high level for a relatively long time) makes the energy transfer process more efficient. The decay time of the upper lasing levels of neon (about 100 ns) is about an order of magnitude greater than that of the lower lasing levels (about 10 ns), thereby satisfying a necessary condition for CW operation.

Even though there are a large number of possible transitions between the sublevels of the Ne laser transition states, the principal or strongest ones are shown in Fig. 40, with transition at wavelengths of 0.633, 1.15, and 3.39 μm. The last two wavelengths fall in the infrared region, while the first one results in the red light, and is the most common mode in which the laser is used.

After transition, the atoms in the lower lasing level relax spontaneously to the NE_1 level which is metastable. Thus, if the temperature of the neon gas is high enough, electrons from the NE_1 state may jump back to the NE_{2a} and NE_{2b} levels (Boltzmann's law). This phenomenon is known as radiation trapping, and will cause the population inversion between the upper lasing level and the NE_{2a} and NE_{2b} levels to decrease, resulting in quenching.

The three main factors that affect radiation trapping and the population inversion of He–Ne lasers are current density, total and partial gas pressures of He and Ne, and diameter of the discharge tube. With a higher current density in the discharge tube, the temperature of the gas becomes higher. Therefore, there is an optimum current density that provides maximum pumping power without quenching laser action.

The gas pressures of He and Ne influence the population of the NE_1 state. If the population of the NE_1 state is increased due to a pressure change, the chances of radiation trapping are increased. The optimum pressures have been found to be about 1mmHg (1 Torr) for helium and about 0.1mmHg for neon. Finally, in order to reduce the population of the NE_1 level and thus reduce the chances for radiation trapping, the NE_1 neon atoms must collide with the discharge tube walls to de-energize. If the inner diameter of the discharge tube wall is too large, there will be less chance of collisions with the wall. Thus, the tube wall must be made small enough to ensure a high-enough probability of wall collisions. In practice, tube diameters range from about 1 to 6 mm. Beyond that range, the output power is reduced because the population inversion is lessened due to a fewer number of wall collisions by atoms in the NE_1 state.

Figure 41 is a schematic of the He–Ne laser. The discharge tube is typically 20–80 cm in length. At the ends are the mirrors, which may be either internal (sealed inside the discharge tube) or external to the sealed tube. The internal system has the disadvantage that it needs to be replaced periodically due to erosion by the discharge. On the contrary, with the external system, the windows at the ends of the tube reflect part of the beam away, resulting in losses. This problem is mitigated by positioning the windows at the Brewster angle θB (see Section 9.4), which is given by

$$tan\theta_B = n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2-51)$$

wh

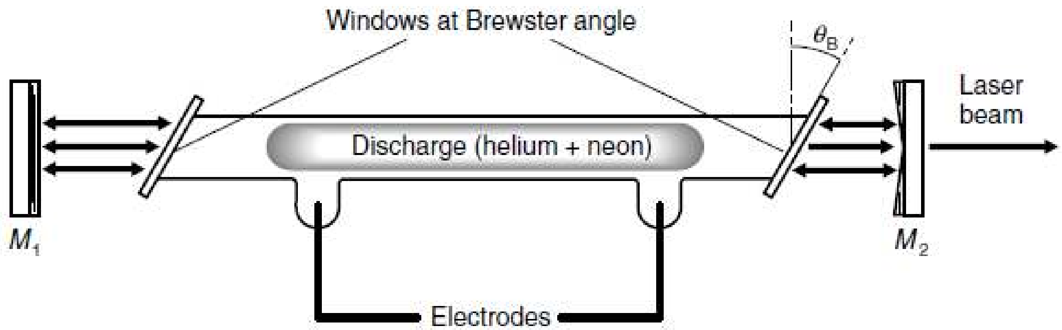


Figure 41: Schematic of a He–Ne laser (external mirrors).

The output beam of the He–Ne laser has a bandwidth of about 1.6 GHz due primarily to Doppler broadening. Thus, there may be several axial modes operating simultaneously. A single axial mode operation can be obtained by reducing the cavity length to about 10–15 cm. This enables high stability to be achieved by controlling the cavity length. The outputs of He–Ne lasers typically range from 0.5 to 50 mW, with an input power requirement of 5–10 W. The overall efficiency is thus of the order of 0.02 %.

Common applications for He–Ne lasers include position sensing, character or barcode reading, alignment, displacement measurement by interferometry, holography.

2.2 Ion Lasers

Ion gas lasers are generally four-level lasers, and the active medium is an ionized inert gas, with a typical operating pressure of about 1 Torr. There are a variety of ion lasers, including argon, krypton, xenon, and mercury ion lasers. These have essentially the same design. We shall use the argon laser for illustration.

The energy level scheme of the argon ion (Ar+) laser is shown in Fig. 42 Excitation to the higher laser level, E_3, can occur in three ways:

1. Through collision of (Ar+) ions in their ground state with electrons.

2. Through collision of (Ar+) ions in the metastable state with electrons.

3. Through radiative cascading from higher levels (to the E_3 level).

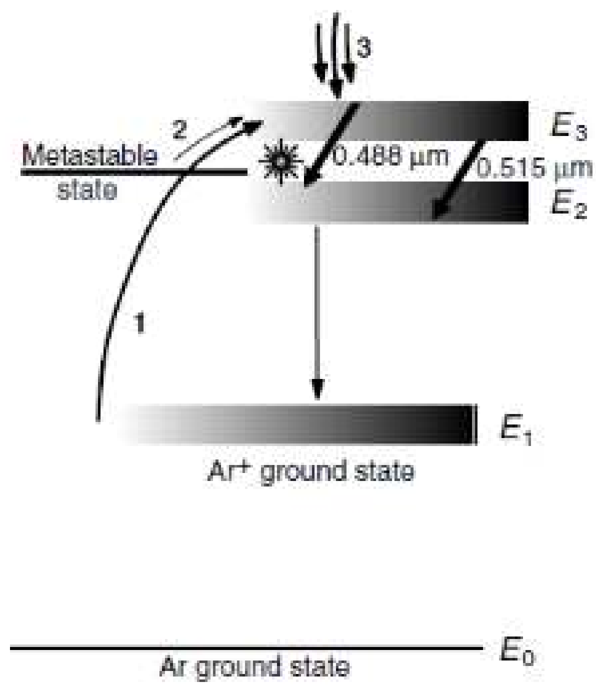


Figure 42: Energy levels of the Ar^+ system.

Laser transition occurs between the E_3 (higher) and E_2 (lower) levels, with the lifetime of the E_3 level being about 10^{-8}, which is an order of magnitude greater than that of the E_2 level (about 10^{-9}), thereby satisfying the condition for CW operation.

The pumping process involves two stages instead of one. In the first stage, the neutral atom is ionized by colliding with an electron in the discharge tube. The ion may then be excited to a higher energy level by another collision with an electron, in the second stage. Since pumping requires two stages, the pumping process, and therefore the ion laser in general, is inefficient. The efficiency is approximately 0.1%. Current densities of around 1000 A/cm^2 are therefore required to maintain the threshold population inversion in ion lasers.

The high-current densities necessary for laser operation raise the temperature of the ions in the discharge tube to very high levels (of the order of 3000 K). This has the effect of broadening the Doppler linewidth to about 3.5 GHz. Furthermore, with the ions at such high temperatures, collisions with the tube walls would normally cause significant damage. This damage might be reduced by increasing the diameter of the tube wall, but that requires higher pumping currents for the same power output. Thus, discharge tubes must be made of ceramic materials to withstand collisions with the hot ions.

Wall damage may be further reduced by applying a static magnetic field along the tube axis in the discharge zone. This is done by wrapping a current-carrying coil around the tube. It forces the electrons to undergo a spiral motion along the tube axis, thereby confining them to the tube center, and reducing contact with the walls. This solution also has the benefit of increasing the electron density along the tube center, which increases the pumping rate and thus the output power.

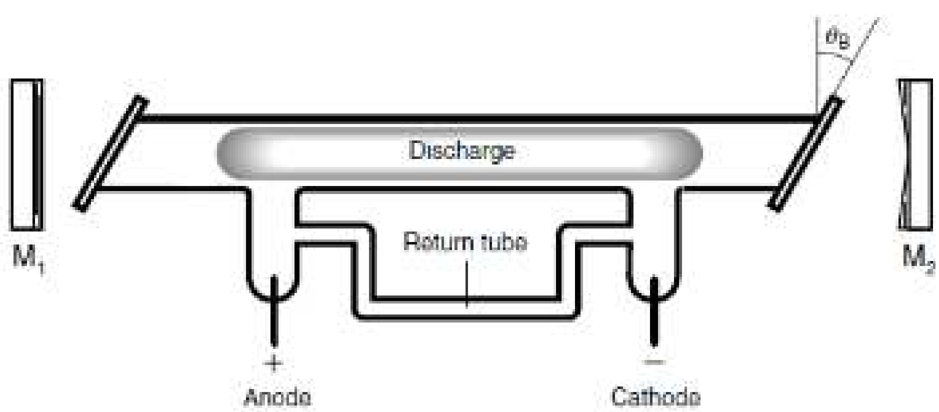


Figure 43: **Schematic of an Ar$^+$ laser.**

In addition to raising the temperature of the discharge tube, the high-current density causes the ions to migrate toward the cathode. Since the ions have a lower mobility than electrons, there is a tendency for the ions to accumulate at the cathode. This might result in the discharge being extinguished due to a loss of active medium. To prevent this from happening, a return path is provided between the anode and the cathode (Fig. 43). The return tube is made longer than the discharge tube to prevent the discharge from passing through the return tube (path of least resistance).

Output of the Ar^+ ion laser occurs at a number of wavelengths, with the strongest being the 0.4881 μm (blue) and 0.515 μm (green) wavelengths. Power outputs typically range from about 1 to over 20W CW, but can also be lower than 1W. Ar^+ ion lasers are used in laser printers, in surgery, for pumping dye lasers, and in spectroscopy. Other ion lasers include the krypton (Kr^+) ion laser with an output wavelength of 0.6471 μm and a CW output of 5-6 mW, and the xenon (Xe^+) ion laser with an output wavelength of 0.5395–0.995 μm, and a pulsed peak power output of about 200 W.

2.3 Metal Vapor Lasers

The design of a metal vapor laser is similar to that of the He–Ne laser, since both use He to assist the pumping process. However, the metal vapor laser employs a metal vapor instead of a second inert gas. The metal vapor is generated by a metal component that is contained in a small reservoir close to the anode, where it is heated to produce the vapor in the discharge tube. The metal vapor lasers are used in spectroscopy, and photochemical experiments. Typical CW output powers for these lasers fall between 50 and 100 mW, filling the range between He–Ne and Ar+ ion lasers.

There are a variety of metal vapor lasers, but the most common ones are the helium–cadmium (He–Cd) and helium–selenium (He–Se) lasers. Figure 44 illustrates the energy level scheme of the He–Cd system.

Collision between the excited He atoms (He^*) and Cd atoms in their ground state result in transfer of energy to the Cd atoms causing their ionization and excitation to the higher energy level:

$$He^* + Cd \rightarrow He^{+} (Cd^{+})^* + e^{-}$$

Laser transition occurs on the $CE_{3b} \rightarrow CE_{2a}$ line with a wavelength of 0.325 µm in the ultraviolet range, and the $CE_{3a} \rightarrow CE_{2b}$ line with a wavelength of 0.4416 µm in the blue range. Efficiencies may be up to 0.02 %, while the oscillation bandwidths are about 5 GHz.

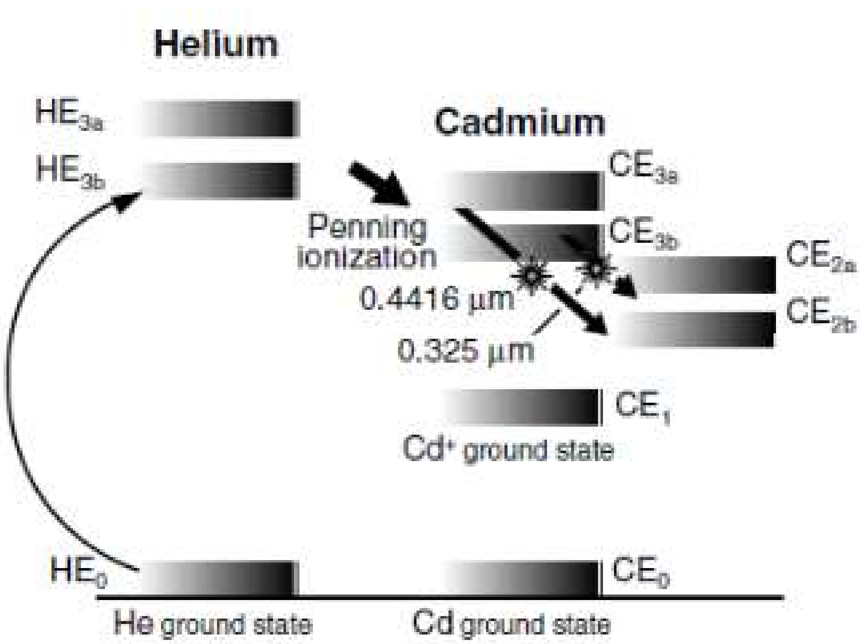


Figure 44: Energy levels of the He–Cd system.

2.4 Molecular Gas Lasers

Unlike the neutral atom and ion gas lasers, molecular gas lasers derive their transitional energy from a molecule. The overall energy of a molecule is normally composed of four principal components:

1- Electronic energy, which results from electron motion about each nucleus.
2- Vibrational energy, which results from vibrations of the constituent atoms about an equilibrium position.
3- Rotational energy, which results from rotation of the molecule as a whole about an axis.
4- Translational energy, which results from linear or curvilinear motion of the molecule as a whole.

Each electronic energy level (Fig. 45a), consists of a number of vibrational levels (Fig. 45b) and each vibrational level in turn consists of a number of rotational levels (Fig. 45c). In Fig. 45, E_1 refers to the ground level, and E_2 refers to the first excited state for the case where the atoms of the molecule are held fixed at a nuclear separation distance r.

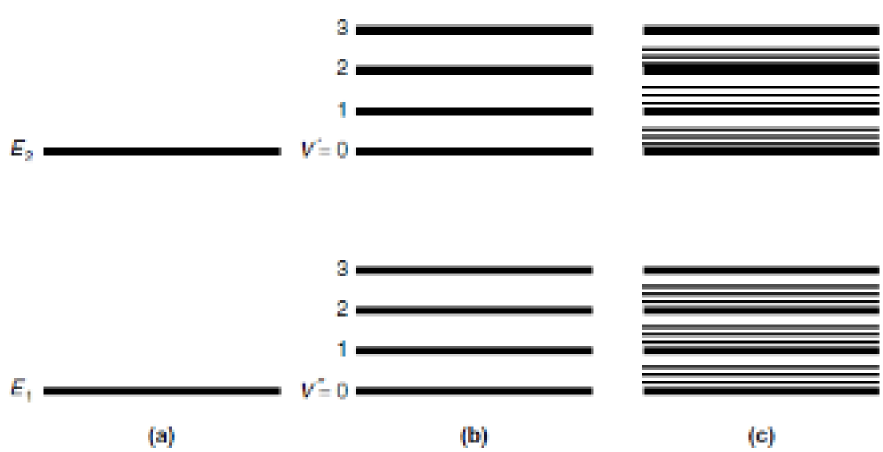


Figure 45: Energy levels of a molecule. (a) Electronic. (b) Vibrational. (c) Rotational.

The electronic, vibrational, and rotational energies are quantized, and transitions between the various levels can produce laser oscillations. Translational energy, on the contrary, is non- quantized, and is thus not useful for laser oscillations. There are three common transitions associated with the quantized energies. These are normally used to categorize the lasers as:

1. Vibrational–rotational.

The CO_2 laser is the most common example. It has the advantage of relatively high efficiencies, about 10–30 %. Common applications of the CO_2 laser include materials processing, communications, spectroscopy, and surgery.

2. Vibrionic (vibrational–electronic).

A Common example is the nitrogen (N_2) laser with an output wavelength of 0.3371 μm in the ultraviolet range. Another type of vibrionic laser is the hydrogen (H_2) laser with output wavelengths of 0.160 and 0.116 μm in the vacuum ultraviolet range.

3. Pure rotational.

Since laser action is relatively difficult to achieve in rotational lasers, these are not further discussed.

4. The excimer laser.

A common examples are the noble gases (Ar_2^*, Kr_2^*, Xe_2^*) or their oxides, (e.g., ArO^*). For the Xe_2^* excimer, transition occurs at a wavelength of 0.172 μm with a linewidth of $\Delta\lambda = 0.015$ μm, tunable, while the output wavelength of the Ar_2^* excimer laser is about 0.125 μm. The efficiency of the laser is about 1 %. Excimer lasers are useful in applications such as isotope separation where the ultraviolet output is very useful. XeCl excimer lasers have been used in laser-assisted chemical vapor deposition for semiconductor manufacture. Generally, excimer lasers are useful for removal processes such as photochemical reactions, or more specifically, photoablation, and are typically used for polymers, ceramics, and glass. One application involves making holes in the nozzles of ink jet components for printers. Other applications include wire stripping and burr removal.

3. **DYE Lasers**

The dye laser is a form of liquid laser where the active medium is an organic dye, which is solid that is dissolved in a solvent such as water, ethyl alcohol, or methyl alcohol. Various dyes are used, including scintillator dyes with wavelengths less than 0.4 µm; coumarin dyes with wavelengths in the range 0.4–0.5 µm; xanthene dyes (e.g., rhodamine 6 G) from 0.5–0.7 µm; and polymethine dyes in the range 0.7–1.0 µm.

The CW output power of dye lasers is of the order of 10–100mW, while the pulsed outputs may average about 100Wwith peak powers of about 1kW. The broad oscillation bandwidth enables the output of dye lasers to be mode locked to produce very short pulses of the order of picoseconds. The tunable characteristics of dye lasers make them very useful for spectroscopy, pollution detection, photochemical processing, and isotope separation.

Tunable solid-state lasers are easier to operate, and also generate output powers in comparison with dye lasers. Furthermore, they do not involve the use of toxic materials that may pose significant disposal problems. On the contrary, dye lasers can be tuned over a wider range of the electromagnetic spectrum, and are also capable of generating much shorter pulses.

4. Semiconductor (DIODE) Lasers

Semiconductor lasers, also known as diode lasers, are based on the generation of photons when electrons in the conduction band of an appropriate semiconductor material recombine with holes in its valence band. The efficiencies for semiconductor lasers is up to 50 %. The output wavelengths range from 0.7 to 30µm in the infrared region. Semiconductor lasers are particularly suited to optical fiber communications. There are several forms of semiconductor lasers. These include the following:

1. Homojunction lasers.

2. Heterojunction lasers.

3. Quantum well lasers.

4. Separate confinement heterostructure lasers.

5. Distributed feedback lasers.

6. Vertical-cavity surface-emitting lasers (VCSELs).

5. Free Electron Laser

Compared to the other lasers that we have discussed thus far, with free electron lasers, energy levels of atoms or molecules are not involved in the laser radiation. Even though they are not commercially available, we briefly introduce them here because of their potential as high-power, high-efficient lasers of excellent beam quality, and with the capability of being tuned over a wide range of wavelengths, from ultraviolet through the infrared.

The principle of operation is based on electrons that are accelerated to speeds approaching that of light, and become coupled by Coulomb interaction to a beam of photons traveling in the same direction. Part of the electron-beam energy can be transferred to the photon beam under suitable circumstances. To achieve this, a magnetic arrangement is used to induce a transverse oscillatory (wiggly) motion to the electron beam (Fig. 46). Thus, the electron beam develops a velocity component perpendicular to the direction of travel, which can then couple with the electric field of the light beam that is naturally transverse to the direction of travel

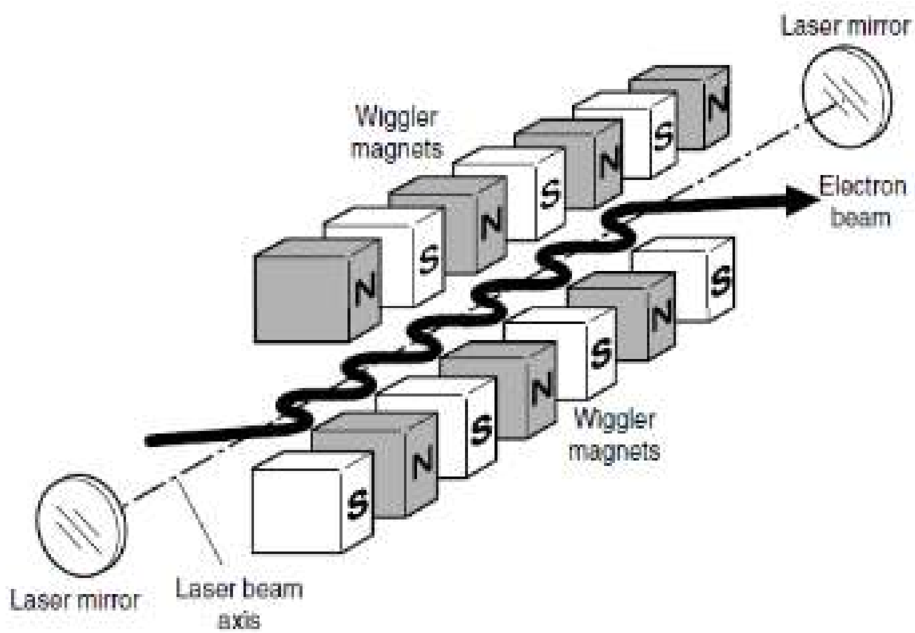


Figure 46: Schematic of a free electron laser.

Free electron lasers have the advantage that they can be tuned to specific wavelengths, and over a very wide range. They are also capable of producing intense laser pulses with high accuracy, and since there is no active medium in the laser cavity, high power levels can be achieved without overheating the cavity. However, they are rarely used because they currently require large electron-beam accelerators to operate. In addition, they have very low efficiencies (of the order of 0.001–0.01 %).