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Diffusion

Photograph of a steel gear that has been "case hardened." The outer surface layer was selectively hardened by a high-temperature heat treatment during which carbon from the surrounding atmosphere diffused into the surface. The "case" appears as the dark outer rim of that segment of the gear that has been sectioned. Actual size.

Why study diffusion?

Materials of all types are often heat treated to improve their properties. The phenomena that occur during a heat treatment almost always involve atomic diffusion. Often an enhancement of diffusion rate is desired; on occasion measures are taken to reduce it. Heat-treating temperatures and times, and/or cooling rates are often predictable using the mathematics of diffusion and appropriate diffusion constants. The steel gear shown on this page has been case hardened (**FACTORS THAT AFFECT FATIGUE LIFE**); that is, its hardness and resistance to failure

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by fatigue have been enhanced by diffusing excess carbon or nitrogen into the outer surface layer.

2-1- Introduction

Many reactions and processes that are important in the treatment of materials rely on the transfer of mass either within a specific solid (ordinarily on a microscopic level) or from a liquid, a gas, or another solid phase. This is necessarily accomplished by **diffusion,** the phenomenon of material transport by atomic motion. This chapter discusses the atomic mechanisms by which diffusion occurs, the mathematics of diffusion, and the influence of temperature and diffusing species on the rate of diffusion.

The phenomenon of diffusion may be demonstrated with the use of a *diffusion couple,* which is formed by joining bars of two different metals together so that there is intimate contact between the two faces; this is illustrated for copper and nickel in Figure 2.1, which includes schematic representations of atom positions and composition across the interface. This couple is heated for an extended period at an elevated temperature (but below the melting temperature of both metals), and cooled to room temperature.
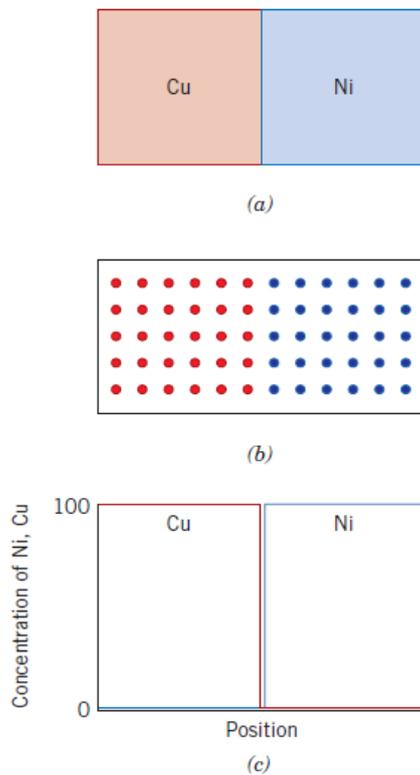
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Figure 2.1 *a*) A copper–nickel diffusion couple before a high-temperature heat treatment. (*b*) Schematic representations of Cu (red circles) and Ni (blue circles) atom locations within the diffusion couple. (*c*) Concentrations of copper and nickel as a function of position across the couple.
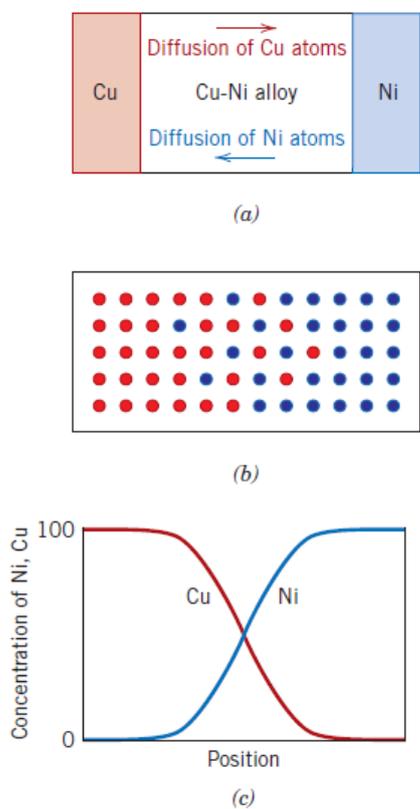
Figur 2.2 (*a*) A copper–nickel diffusion couple after a high-temperature heat treatment, showing the alloyed diffusion zone. (*b*) Schematic representations of Cu (red circles) and Ni (blue circles) atom locations within the couple. (*c*) Concentrations of copper and nickel as a function of position across the couple.

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Chemical analysis will reveal a condition similar to that represented in Figure 2.2 namely, pure copper and nickel at the two extremities of the couple, separated by an alloyed region. Concentrations of both metals vary with position as shown in Figure 2.2*c*. This result indicates that copper atoms have migrated or diffused into the nickel, and that nickel has diffused into copper. This process, whereby atoms of one metal diffuse into another, is termed **interdiffusion,** or **impurity diffusion.**

Interdiffusion may be discerned from a macroscopic perspective by changes in concentration which occur over time, as in the example for the Cu–Ni diffusion couple. There is a net drift or transport of atoms from high- to low-concentration regions. Diffusion also occurs for pure metals, but all atoms exchanging positions are of the same type; this is termed **self-diffusion.** Of course, self-diffusion is not normally subject to observation by noting compositional changes.
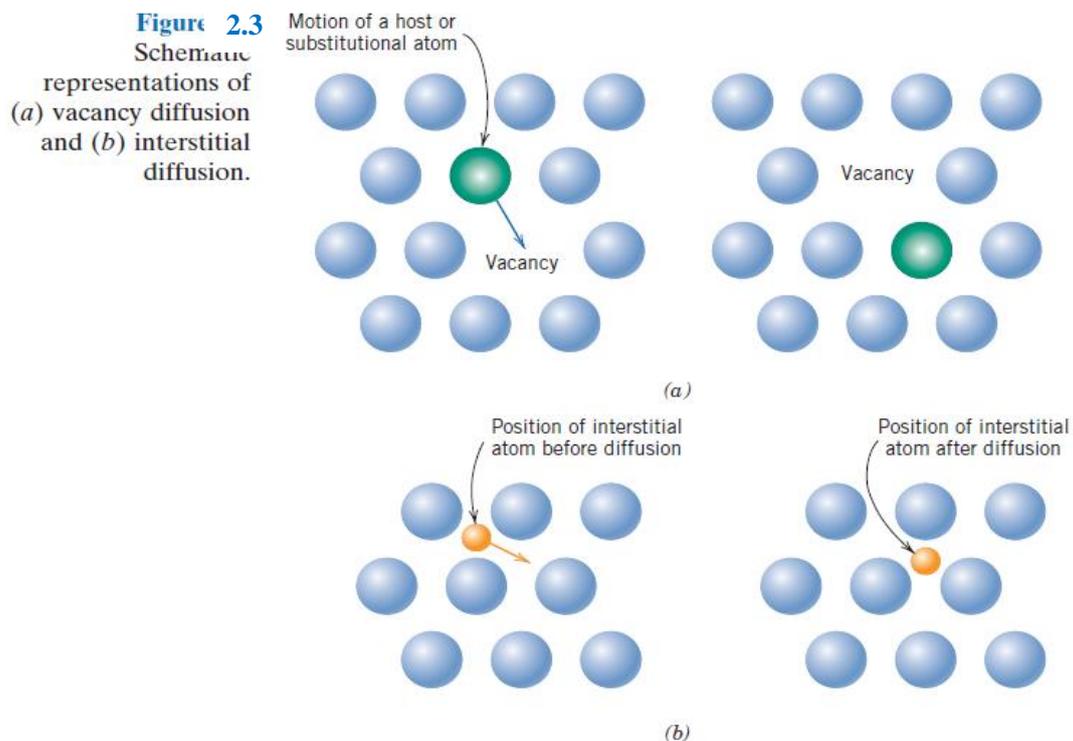
2-2- *Diffusion Mechanisms*

From an atomic perspective, diffusion is just the stepwise migration of atoms from lattice site to lattice site. In fact, the atoms in solid materials are in constant motion, rapidly changing positions. For an atom to make such a move, two conditions must be met: (1) there must be an empty adjacent site, and (2) the atom must have sufficient energy to break bonds with its neighbor atoms and then cause some lattice distortion during the displacement. This energy is vibrational in nature (**Section 1-4**). At a specific temperature some small fraction of the total number of atoms is capable of diffusive motion, by virtue of the magnitudes of their vibrational energies. This fraction increases with rising temperature.

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Several different models for this atomic motion have been proposed; of these possibilities, two dominate for metallic diffusion.

- *Vacancy diffusion*

One mechanism involves the interchange of an atom from a normal lattice position to an adjacent vacant lattice site or vacancy, as represented schematically in Figure 2.3*a*. This mechanism is aptly termed **vacancy diffusion.** Of course, this process necessitates the presence of vacancies, and the extent to which vacancy diffusion can occur is a function of the number of these defects that are present; significant concentrations of vacancies may exist in metals at elevated temperatures (1-1). Since diffusing atoms and vacancies exchange positions, the diffusion of atoms in one direction corresponds to the motion of vacancies in the opposite direction. Both self-diffusion and interdiffusion occur by this mechanism; for the latter, the impurity atoms must substitute for host atoms.

Figure 2.3 Schematic representations of (*a*) vacancy diffusion and (*b*) interstitial diffusion.

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- *Interstitial Diffusion*

The second type of diffusion involves atoms that migrate from an interstitial position to a neighboring one that is empty. This mechanism is found for interdiffusion of impurities such as hydrogen, carbon, nitrogen, and oxygen, which have atoms that are small enough to fit into the interstitial positions. Host or substitutional impurity atoms rarely form interstitials and do not normally diffuse via this mechanism. This phenomenon is appropriately termed **interstitial diffusion** (Figure 2.3*b*).

In most metal alloys, interstitial diffusion occurs much more rapidly than diffusion by the vacancy mode, since the interstitial atoms are smaller and thus more mobile. Furthermore, there are more empty interstitial positions than vacancies; hence, the probability of interstitial atomic movement is greater than for vacancy diffusion.

2-3- *Steady – State Diffusion*

Diffusion is a time-dependent process-that is, in a macroscopic sense, the quantity of an element that is transported within another is a function of time. Often it is necessary to know how fast diffusion occurs, or the rate of mass transfer. This rate is frequently expressed as a **diffusion flux** (*J*), defined as the mass (or, equivalently, the number of atoms) *M* diffusing through and perpendicular to a unit cross-sectional area of solid per unit of time. In mathematical form, this may be represented as:

$$J = \frac{M}{At} \qquad ………. (1)$$

where *A* denotes the area across which diffusion is occurring and *t* is the elapsed diffusion time. In differential form, this expression becomes:

$$J = \frac{1}{A}\frac{dM}{dt} \qquad ………. (2)$$

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The units for J are kilograms or atoms per meter squared per second (kg/m^2-s or atoms/m^2-s).

If the diffusion flux does not change with time, a steady-state condition exists. One common example of **steady-state diffusion** is the diffusion of atoms of a gas through a plate of metal for which the concentrations (or pressures) of the diffusing species on both surfaces of the plate are held constant. This is represented schematically in Figure 2.4*a*.
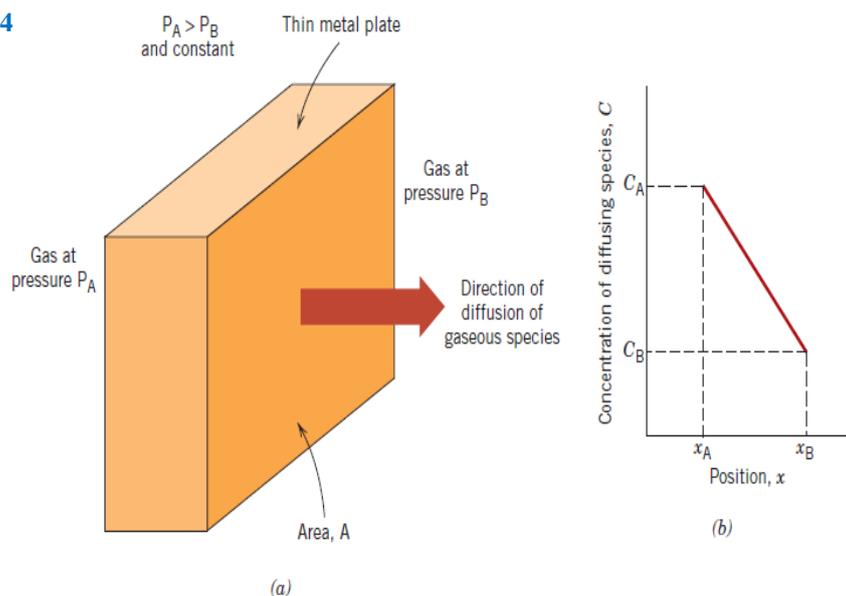
When concentration C is plotted versus position (or distance) within the solid x, the resulting curve is termed the **concentration profile;** the slope at a particular point on this curve is the **concentration gradient:**

$$Concentration\ gradient = \frac{dC}{dx} \qquad \dots\dots\dots.(3)$$

In the present treatment, the concentration profile is assumed to be linear, as depicted in Figure 2.4*b*, and

$$Concentration\ gradient = \frac{\Delta C}{\Delta x} = \frac{C_A - C_B}{x_A - x_B}$$

Figure 2.4
(*a*) Steady-state diffusion across a thin plate. (*b*) A linear concentration profile for the diffusion situation in (*a*).

$P_A > P_B$ and constant | Thin metal plate

Gas at pressure P_A

Gas at pressure P_B

Direction of diffusion of gaseous species

Area, A

(*a*)

Concentration of diffusing species, C

C_A

C_B

x_A x_B
Position, x

(*b*)

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For diffusion problems, it is sometimes convenient to express concentration in terms of mass of diffusing species per unit volume of solid (kg/m^3 or g/cm^3). The mathematics of steady-state diffusion in a single (x) direction is relatively simple, in that the flux is proportional to the concentration gradient through the expression:

$$J = -D \frac{dC}{dx} \qquad \dots \dots \dots (4) \quad \textbf{Fick's first law.}$$

The constant of proportionality D is called the **diffusion coefficient,** which is expressed in square meters per second. The negative sign in this expression indicates that the direction of diffusion is down the concentration gradient, from a high to a low concentration.

Example problem 2.1

Diffusion Flux Computation

A plate of iron is exposed to a carburizing (carbon-rich) atmosphere on one side and a decarburizing (carbon-deficient) atmosphere on the other side at 700°C (1300°F). If a condition of steady state is achieved, calculate the diffusion flux of carbon through the plate if the concentrations of carbon at positions of 5 and 10 mm (5×10^{-3} and 10^{-2} m) beneath the carburizing surface are 1.2 and 0.8 kg/m^3, respectively. Assume a diffusion coefficient of 3×10^{-11} m^2/s at this temperature.

Solution

Fick's first law, Equation 5.3, is utilized to determine the diffusion flux. Substitution of the values above into this expression yields

$$J = -D \frac{C_A - C_B}{x_A - x_B} = -(3 \times 10^{-11} \, m^2/s) \frac{(1.2 - 0.8) \, kg/m^3}{(5 \times 10^{-3} - 10^{-2}) \, m}$$

$$= 2.4 \times 10^{-9} \, kg/m^2\text{-s}$$

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2-4- *Nonsteady-State Diffusion*

Most practical diffusion situations are nonsteady-state ones. That is, the diffusion flux and the concentration gradient at some particular point in a solid vary with time, with a net accumulation or depletion of the diffusing species resulting. This is illustrated in Figure 2.5, which shows concentration profiles at three different diffusion times.
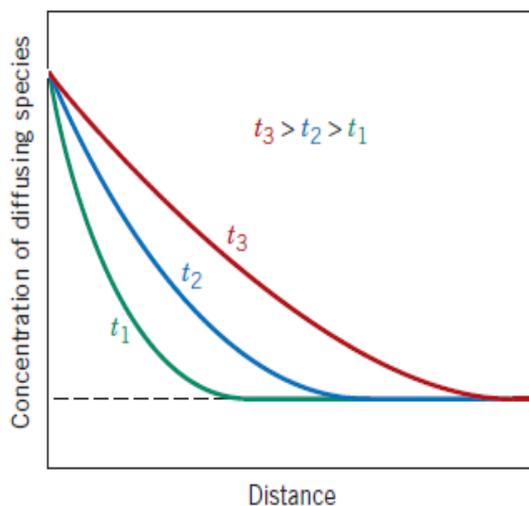

Figure 2.5 Concentration profiles for nonsteady-state diffusion taken at three different times, t_1, t_2, and t_3.

Under conditions of nonsteady state, use of Equation **Fick's first law** is no longer convenient; instead, the partial differential equation:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) \qquad \dots\dots\dots(5)$$

Known as **Fick's second law,** is used. If the diffusion coefficient is independent of composition (which should be verified for each particular diffusion situation), this equation simplifies to:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \qquad \dots\dots\dots(6)$$

Solutions to this expression (concentration in terms of both position and time) are possible when physically meaningful boundary conditions are specified.

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One practically important solution is for a semi-infinite solid in which the surface concentration is held constant. Frequently, the source of the diffusing species is a gas phase, the partial pressure of which is maintained at a constant value. Furthermore, the following assumptions are made:

1. Before diffusion, any of the diffusing solute atoms in the solid are uniformly distributed with concentration of C_o.

2. The value of x at the surface is zero and increases with distance into the solid.

3. The time is taken to be zero the instant before the diffusion process begins.

These boundary conditions are simply stated as:

$$\text{For } t = 0, \ C = C_o \quad at \ 0 \leq x \leq \infty$$

$$\text{For } t > 0, \ C = C_s (the \ constant \ surface \ concentration) \quad at \ x = 0$$

$$C = C_o \quad at \ x = \infty$$

Application of these boundary conditions to Equation of **Fick's second law,** yields the solution:

$$\frac{C_x - C_o}{C_s - C_o} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right) \qquad \ldots\ldots\ldots. (7)$$

Where C_x represents the concentration at depth x after time t. The expression $erf(x/2\sqrt{Dt})$ is the Gaussian error function, values of which are given in mathematical tables for various $x/2\sqrt{Dt}$ values; a partial listing is given in Table 2.1. The concentration parameters that appear in Equation (7) are noted in Figure 2.6, a concentration profile taken at a specific time. Equation (7) thus demonstrates the relationship between concentration, position, and time, namely, that C_x being a function of the

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dimensionless parameter $x/2\sqrt{Dt}$ may be determined at any time and position if the parameters C_o, C_s and D are known.

Table 2.1 Tabulation of error function values

z	erf(z)	z	erf(z)	z	erf(z)
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.0	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999
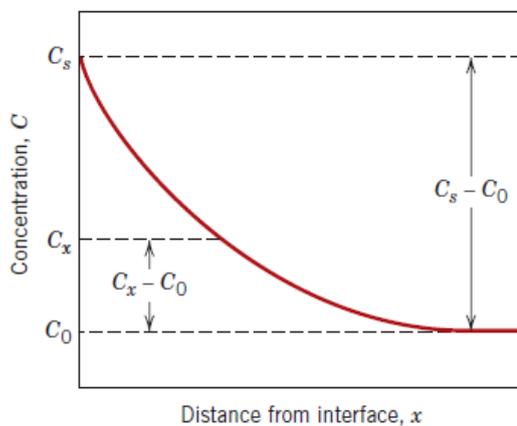
Figure 2.6 Concentration profile for nonsteady-state diffusion; concentration parameters relate to Equation 5.5.

Suppose that it is desired to achieve some specific concentration of solute, C_1, in an alloy; the left-hand side of Equation (7) now becomes:

$$\frac{C_1 - C_o}{C_s - C_o} = constant$$

This being the case, the right-hand side of this same expression is also a constant, and subsequently:

$$\frac{x}{2\sqrt{Dt}} = constant$$

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$$\therefore \frac{x^2}{Dt} = constant$$

Some diffusion computations are thus facilitated on the basis of this relationship, as demonstrated in Example Problem 2.3.

Example problem 2.2.

Nonsteady-State Diffusion Time Computation I

For some applications, it is necessary to harden the surface of a steel (or iron-carbon alloy) above that of its interior. One way this may be accomplished is by increasing the surface concentration of carbon in a process termed **carburizing**; the steel piece is exposed, at an elevated temperature, to an atmosphere rich in a hydrocarbon gas, such as methane (CH_4).

Consider one such alloy that initially has a uniform carbon concentration of 0.25 wt% and is to be treated at 950°C (1750°F). If the concentration of carbon at the surface is suddenly brought to and maintained at 1.20 wt%, how long will it take to achieve a carbon content of 0.80 wt% at a position 0.5 mm below the surface? The diffusion coefficient for carbon in iron at this temperature is 1.6×10^{-11} m²/s; assume that the steel piece is semi-infinite.

Solution

Since this is a nonsteady-state diffusion problem in which the surface composition is held constant, Equation 5.5 is used. Values for all the parameters in this expression except time t are specified in the problem as follows:

$$C_0 = 0.25 \text{ wt\% C}$$
$$C_s = 1.20 \text{ wt\% C}$$
$$C_x = 0.80 \text{ wt\% C}$$
$$x = 0.50 \text{ mm} = 5 \times 10^{-4} \text{ m}$$
$$D = 1.6 \times 10^{-11} \text{ m}^2/\text{s}$$

Thus,

$$\frac{C_x - C_0}{C_s - C_0} = \frac{0.80 - 0.25}{1.20 - 0.25} = 1 - \text{erf}\left[\frac{(5 \times 10^{-4} \text{ m})}{2\sqrt{(1.6 \times 10^{-11} \text{ m}^2/\text{s})(t)}}\right]$$

$$0.4210 = \text{erf}\left(\frac{62.5 \text{ s}^{1/2}}{\sqrt{t}}\right)$$

We must now determine from Table 5.1 the value of z for which the error function is 0.4210. An interpolation is necessary, as

z	erf(z)
0.35	0.3794
z	0.4210
0.40	0.4284

$$\frac{z - 0.35}{0.40 - 0.35} = \frac{0.4210 - 0.3794}{0.4284 - 0.3794}$$

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or

$$z = 0.392$$

Therefore,

$$\frac{62.5 \ s^{1/2}}{\sqrt{t}} = 0.392$$

and solving for t,

$$t = \left(\frac{62.5 \ s^{1/2}}{0.392}\right)^2 = 25,400 \ s = 7.1 \ h$$

Example problem 2.3.

Nonsteady-State Diffusion Time Computation II

The diffusion coefficients for copper in aluminum at 500 and 600°C are 4.8×10^{-14} and 5.3×10^{-13} m²/s, respectively. Determine the approximate time at 500°C that will produce the same diffusion result (in terms of concentration of Cu at some specific point in Al) as a 10-h heat treatment at 600°C.

Solution

This is a diffusion problem in which Equation 5.6b may be employed. The composition in both diffusion situations will be equal at the same position (i.e., x is also a constant), thus

$$Dt = \text{constant} \tag{5.7}$$

at both temperatures. That is,

$$D_{500} t_{500} = D_{600} t_{600}$$

or

$$t_{500} = \frac{D_{600} t_{600}}{D_{500}} = \frac{(5.3 \times 10^{-13} \ \text{m}^2/\text{s})(10 \ \text{h})}{4.8 \times 10^{-14} \ \text{m}^2/\text{s}} = 110.4 \ \text{h}$$

2-5- *Factors That Influence Diffusion*

- *Diffusing Species*

The magnitude of the diffusion coefficient D is indicative of the rate at which atoms diffuse. Coefficients, both self- and interdiffusion, for several metallic systems are listed in Table 2.2. The diffusing species as well as the host material influence the diffusion coefficient. For example, there is a significant difference in magnitude between self-diffusion and

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carbon interdiffusion in iron at 500 C, the *D* value being greater for the carbon interdiffusion (3.0×10^{-21} vs. 2.4×10^{-12} ^{m2}/s).This comparison also provides a contrast between rates of diffusion via vacancy and interstitial modes as discussed above. Self-diffusion occurs by a vacancy mechanism, whereas carbon diffusion in iron is interstitial.

- *Temperature*

Temperature has a most profound influence on the coefficients and diffusion rates. For example, for the self-diffusion of Fe in α-Fe, the diffusion coefficient increases approximately six orders of magnitude (from 3.0×10^{-21} to 1.8×10^{-15} m^2/s) in rising temperature from 500 to 900°C (Table 2.2). The temperature dependence of the diffusion coefficients is related to temperature according to:

$$D = D_o exp\left(-\frac{Q_d}{RT}\right) \qquad ………. (8)$$

Where:

D_o= a temperature-independent preexponential (m^2/s).

Q_d= the **activation energy** for diffusion (J/mol or eV/atom).

R= the gas constant, 8.31 J/mol-K or 8.62×10^{-5} eV/atom-K

T= absolute temperature (K).

The activation energy may be thought of as that energy required to produce the diffusive motion of one mole of atoms. A large activation energy results in a relatively small diffusion coefficient. Table 2.2 also contains a listing of and values for several diffusion systems.

Taking natural logarithms of Equation (8) yields:

$$lnD = lnD_o - \frac{Q_d}{R}\left(\frac{1}{T}\right) \qquad ………. (9)$$

Or in terms of logarithms to the base 10

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$$logD = logD_o - \frac{Q_d}{2.3R}\left(\frac{1}{T}\right) \qquad \ldots\ldots\ldots (10)$$

Since D_o, Q_d and R are all constants, Equation (10) takes on the form of an equation of a straight line:

$$y = b + mx$$

Where y and x are analogous, respectively, to the variables $\log D$ and $1/T$. Thus, if $\log D$ is plotted versus the reciprocal of the absolute temperature, a straight line should result, having slope and intercept of $-Q_d/2.3R$ and $\log D_o$ respectively. This is, in fact, the manner in which the values of Q_d and D_o are determined experimentally. From such a plot for several alloy systems (Figure 2.7), it may be noted that linear relationships exist for all cases shown.

Table 2.2 A Tabulation of Diffusion Data.

Diffusing Species	Host Metal	$D_0(m^2/s)$	Activation Energy Q_d		Calculated Values	
			kJ/mol	eV/atom	T(°C)	$D(m^2/s)$
Fe	α-Fe (BCC)	2.8×10^{-4}	251	2.60	500	3.0×10^{-21}
					900	1.8×10^{-15}
Fe	γ-Fe (FCC)	5.0×10^{-5}	284	2.94	900	1.1×10^{-17}
					1100	7.8×10^{-16}
C	α-Fe	6.2×10^{-7}	80	0.83	500	2.4×10^{-12}
					900	1.7×10^{-10}
C	γ-Fe	2.3×10^{-5}	148	1.53	900	5.9×10^{-12}
					1100	5.3×10^{-11}
Cu	Cu	7.8×10^{-5}	211	2.19	500	4.2×10^{-19}
Zn	Cu	2.4×10^{-5}	189	1.96	500	4.0×10^{-18}
Al	Al	2.3×10^{-4}	144	1.49	500	4.2×10^{-14}
Cu	Al	6.5×10^{-5}	136	1.41	500	4.1×10^{-14}
Mg	Al	1.2×10^{-4}	131	1.35	500	1.9×10^{-13}
Cu	Ni	2.7×10^{-5}	256	2.65	500	1.3×10^{-22}

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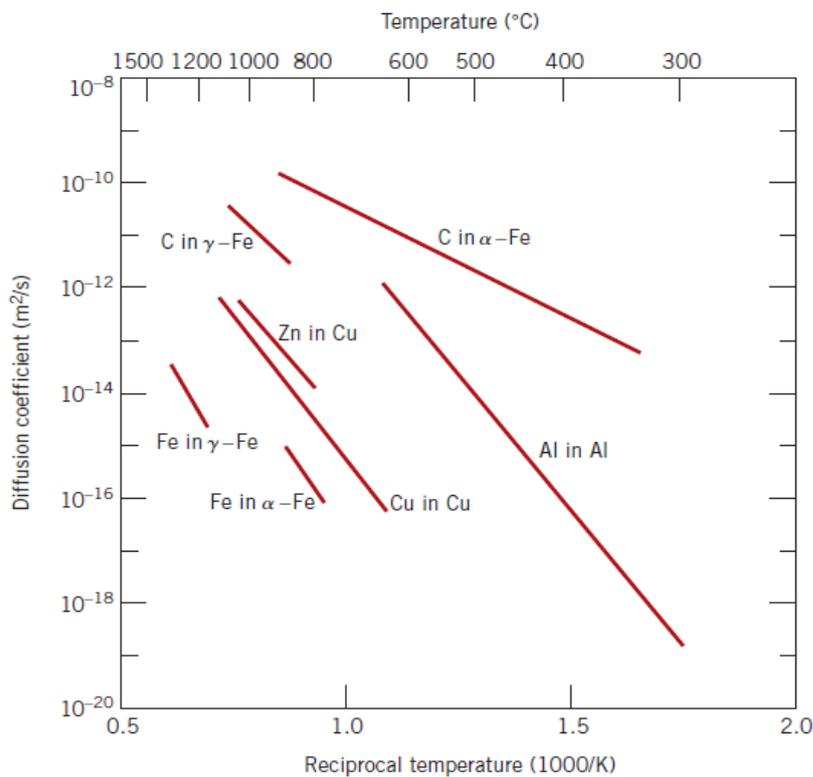*Dr. Mohammed Ghazi*
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Figure 2.7 Plot of the logarithm of the diffusion coefficient versus the reciprocal of absolute temperature for several metals. [Data taken from E. A. Brandes and G. B. Brook (Editors), *Smithells Metals Reference Book,* 7th edition, Butterworth-Heinemann, Oxford, 1992.]

Example problem 2.4

Diffusion Coefficient Determination

Using the data in Table 5.2, compute the diffusion coefficient for magnesium in aluminum at 550°C.

Solution

This diffusion coefficient may be determined by applying Equation 5.8; the values of D_0 and Q_d from Table 5.2 are 1.2×10^{-4} m^2/s and 131 kJ/mol, respectively. Thus,

$$D = (1.2 \times 10^{-4} \text{ m}^2/\text{s}) \exp \left[-\frac{(131{,}000 \text{ J/mol})}{(8.31 \text{ J/mol-K})(550 + 273 \text{ K})} \right]$$

$$= 5.8 \times 10^{-13} \text{ m}^2/\text{s}$$

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Example problem 2.5

Diffusion Coefficient Activation Energy and Preexponential Calculations

In Figure 5.8 is shown a plot of the logarithm (to the base 10) of the diffusion coefficient versus reciprocal of absolute temperature, for the diffusion of copper in gold. Determine values for the activation energy and the preexponential.

Solution

From Equation 5.9b the slope of the line segment in Figure 5.8 is equal to $-Q_d/2.3R$, and the intercept at $1/T = 0$ gives the value of $\log D_0$. Thus, the activation energy may be determined as

$$Q_d = -2.3R\,(\text{slope}) = -2.3R\left[\frac{\Delta(\log D)}{\Delta\left(\dfrac{1}{T}\right)}\right]$$

$$= -2.3R\left[\frac{\log D_1 - \log D_2}{\dfrac{1}{T_1} - \dfrac{1}{T_2}}\right]$$

where D_1 and D_2 are the diffusion coefficient values at $1/T_1$ and $1/T_2$, respectively. Let us arbitrarily take $1/T_1 = 0.8 \times 10^{-3}\,(\text{K})^{-1}$ and $1/T_2 = 1.1 \times 10^{-3}\,(\text{K})^{-1}$. We may now read the corresponding $\log D_1$ and $\log D_2$ values from the line segment in Figure 5.8.

[Before this is done, however, a parenthetic note of caution is offered. The vertical axis in Figure 5.8 is scaled logarithmically (to the base 10); however, the actual diffusion coefficient values are noted on this axis. For example, for $D = 10^{-14}\,\text{m}^2/\text{s}$, the logarithm of D is -14.0 *not* 10^{-14}. Furthermore, this logarithmic scaling affects the readings between decade values; for example, at a location midway between 10^{-14} and 10^{-15}, the value is not 5×10^{-15} but, rather, $10^{-14.5} = 3.2 \times 10^{-15}$].

Thus, from Figure 5.8, at $1/T_1 = 0.8 \times 10^{-3}\,(\text{K})^{-1}$, $\log D_1 = -12.40$, while for $1/T_2 = 1.1 \times 10^{-3}\,(\text{K})^{-1}$, $\log D_2 = -15.45$, and the activation energy, as

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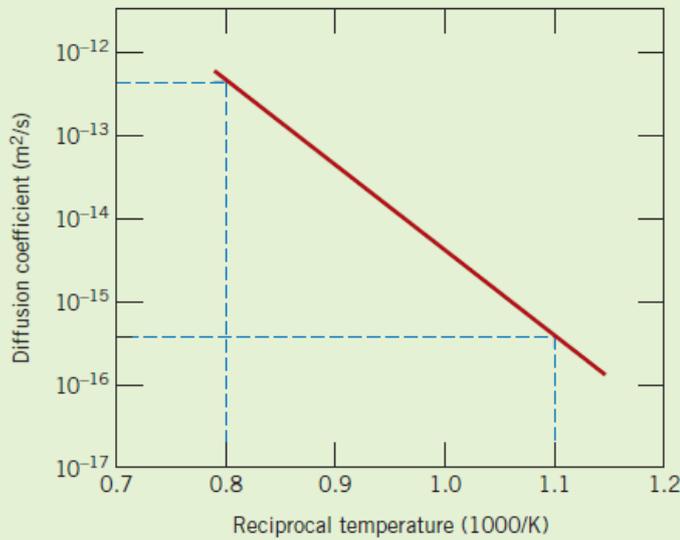**Dr. Mohammed Ghazi**
Chapter (2)

Figure 5.8 Plot of the logarithm of the diffusion coefficient versus the reciprocal of absolute temperature for the diffusion of copper in gold.

determined from the slope of the line segment in Figure 5.8 is

$$Q_d = -2.3R \left[\frac{\log D_1 - \log D_2}{\dfrac{1}{T_1} - \dfrac{1}{T_2}} \right]$$

$$= -2.3 \, (8.31 \text{ J/mol-K}) \left[\frac{-12.40 - (-15.45)}{0.8 \times 10^{-3} \, (\text{K})^{-1} - 1.1 \times 10^{-3} \, (\text{K})^{-1}} \right]$$

$$= 194{,}000 \text{ J/mol} = 194 \text{ kJ/mol}$$

Now, rather than trying to make a graphical extrapolation to determine D_0, a more accurate value is obtained analytically using Equation 5.9b, and a specific value of D (or $\log D$) and its corresponding T (or $1/T$) from Figure 5.8. Since we know that $\log D = -15.45$ at $1/T = 1.1 \times 10^{-3} \, (\text{K})^{-1}$, then

$$\log D_0 = \log D + \frac{Q_d}{2.3R} \left(\frac{1}{T} \right)$$

$$= -15.45 + \frac{(194{,}000 \text{ J/mol})(1.1 \times 10^{-3} \, [\text{K}]^{-1})}{(2.3)(8.31 \text{ J/mol-K})}$$

$$= -4.28$$

Thus, $D_0 = 10^{-4.28} \text{ m}^2/\text{s} = 5.2 \times 10^{-5} \text{ m}^2/\text{s}$.