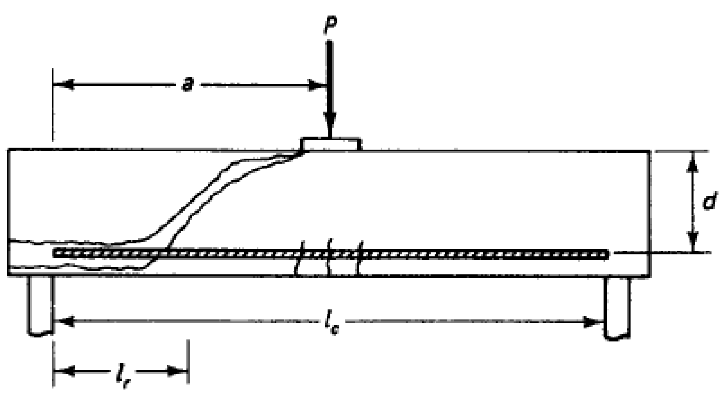
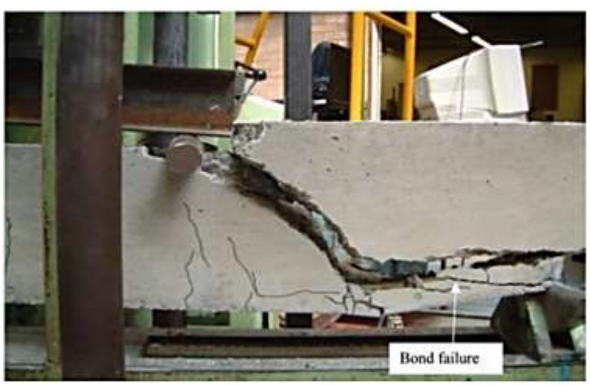
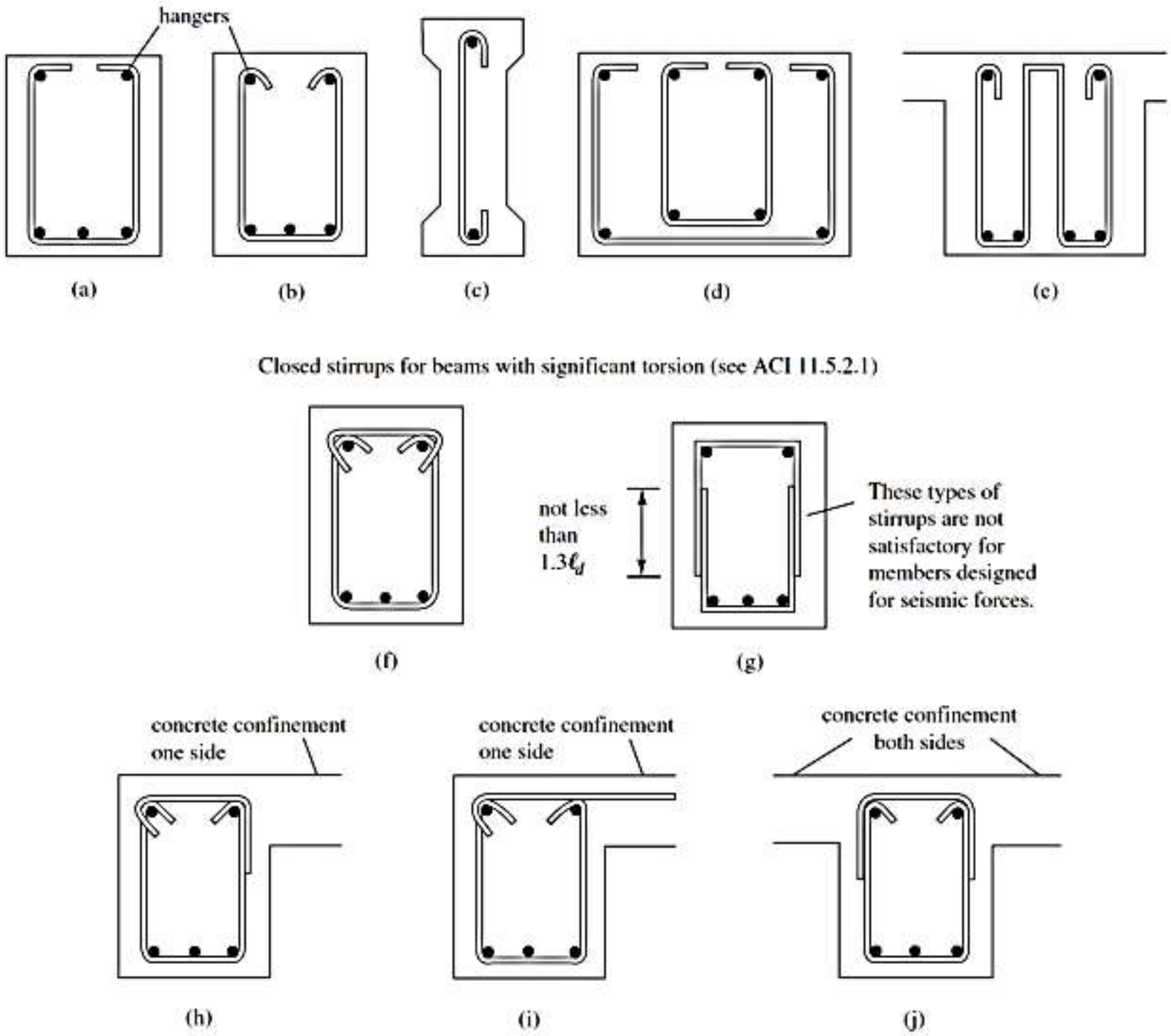
Shear and Diagonal Tension

When a simple beam is loaded, as shown in Fig. bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters. The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.





Web Reinforcement



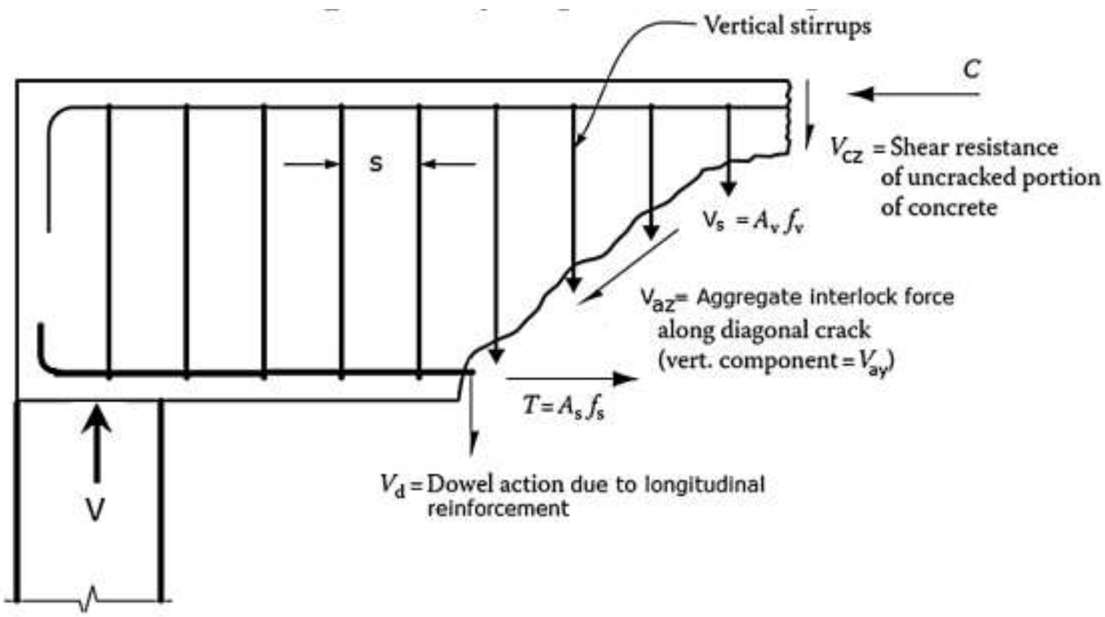
Types of stirrups.

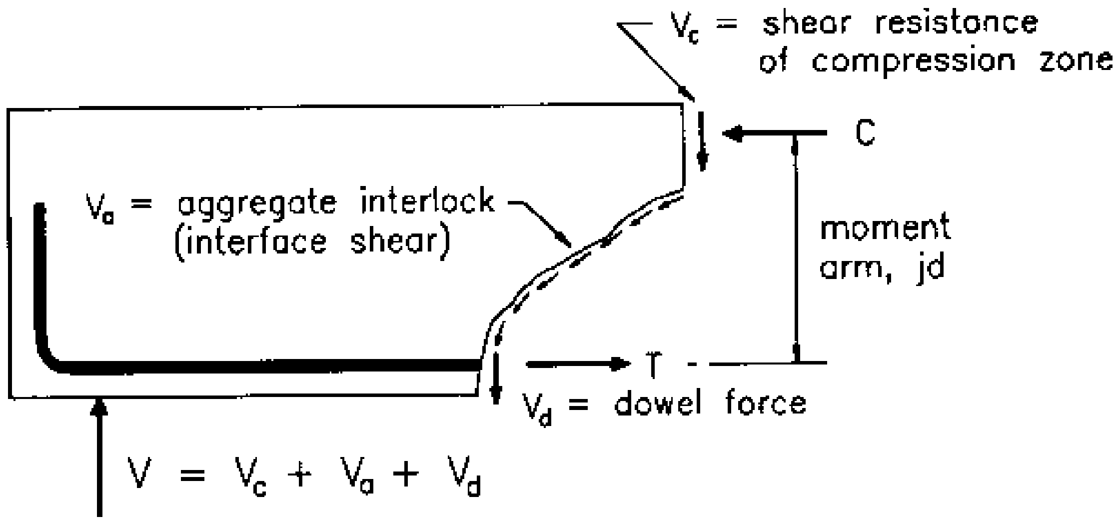






Shear Strength of Beam





C- Design of Web Reinforcement :-

* $S = (A_v f_y d)/V_s$
* $V_s = V_n - V_c = \frac{V_u}{\phi} - V_c$

$$V_s = \frac{V_u}{\phi} - V_c$$

* $S = \frac{A_v f_y (\sin\alpha + \cos\alpha)}{V_s}$

* When $V_s \leq \frac{1}{3}\sqrt{f'_c}\, b_w d \; (= 2V_c)$

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \frac{3 A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* when $V_s > \frac{1}{3}\sqrt{f'_c}\, b_w d \; (= 2V_c)$

$$S_{max} \leq \begin{cases} d/4 \\ 300\,mm \\ \frac{3 A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* If $V_s > \frac{2}{3}\sqrt{f'_c}\, b_w d \; (= 4V_c)$ The Section must be changed.

Design Procedure for Web Reinforcement

1- Analyzing the beam and draw S.F. Diagram

2- Find shear force design (V_{ud}) and find (ϕV_c) from the equations below according to the kind of loadings:-

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d$$

$$V_c = \left(1+\frac{N_u}{14A_g}\right)\left[\frac{\sqrt{f'_c}}{6} b_w d\right]$$

$$V_c = \left(1+\frac{0.3N_u}{A_g}\right)\left[\frac{\sqrt{f'_c}}{6} b_w d\right]$$

3- If • $V_{ud} \leq \phi V_c/2 \Rightarrow$ No need for shear reinforcement

• $\phi V_c/2 \leq V_{ud} \leq \phi V_c \Rightarrow$ Minmum shear reinf.

The maximum distance between stirrups is calculated by

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases} \quad [\text{the min. value}]$$

The stirrups will be continued to $V_u = \frac{\phi V_c}{2}$ and after that there is no need for shear reinf.

4- If ($V_{ud} > \phi V_c$), then find shear force design for steel (ϕV_s). If this force is greater than ($4\phi V_c$) then the beam section must be changed, if not the distance between stirrups (Max distance) will be find from the equations below according to (V_s) value

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \end{cases} \text{the min Value} \quad \text{if } V_s \leq \frac{1}{3}\sqrt{f_c'}\, b_w d \; (=2V_c)$$

$$S_{max} \leq \begin{cases} d/4 \\ 300\,mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \end{cases} \text{the min Value} \quad \text{if } V_s > \frac{1}{3}\sqrt{f_c'}\, b_w d \; (4V_c)$$

5- Calculating the distance between the stirrups at critical section (S_o), if thés distance greater or equal to (S_{max}), then the distance from support face to the point, which at this point ($V_u = \phi V_c/2$) will found and using ($S = S_{max}$). If

$(S_o < S_{max})$, then we find the distance which, after this distance we will reinforce by minmum reinforcement, and after that we find the distance which there is no need to shear reinforcement.

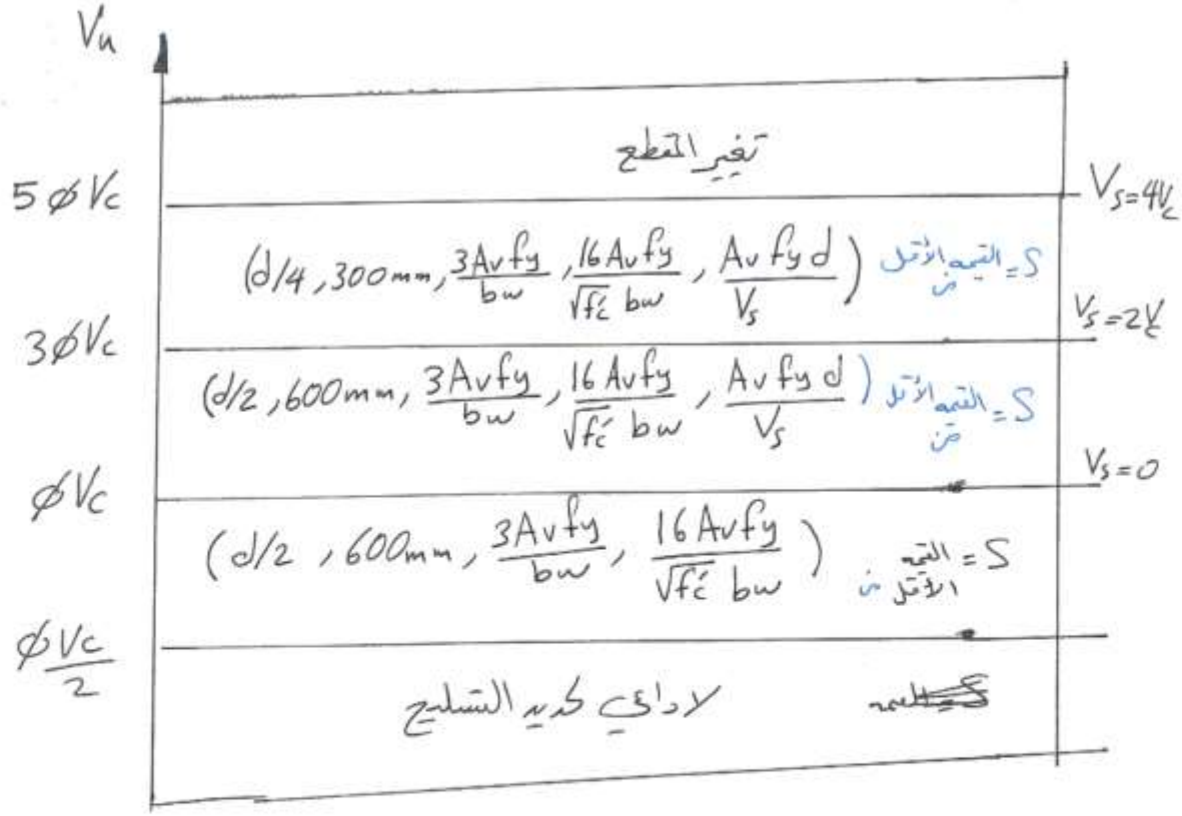
6- Find the distance between stirrups for the region between critical section and the point of min. reinf. by using the following eq.

$$S = \frac{A_v f_y d}{V_s}$$

and the distance between stirrups will be changed according to the methods used before.

- If the distance between stirrups is small, then we use bigger (Ø bar) or use stirrups with (⅂⊔⊓Γ) shape.

7. Clarify the position, kind & radius of stirrups (Ø bar of stirrups) on the beam diagram



مخطط تسليح القص حسب المقاومة التصميمية للقص (Vu)

Shear Strength of Concrete :-

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = 0.3\sqrt{f_c'}$$

because of reduction in area which was caused by flexural cracking, the shear strength of beam is less than that found in the equation above, & it is find by the following equation.

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{6}\sqrt{f_c'}$$

That means bending moment may caused decreasing in shear strength to about half it's magnitude.

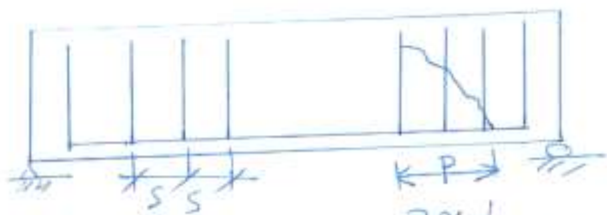
The stresses of shear in the case of cracking depend on the ratio between bending moment to shear & it is also depend on the longitudenal steel reinforcement ratio, because this steel reinforcement lead to decrease the cracks caused by bending & then increase of concrete resistance to radial cracks, i.e. increasing shear strength of concrete.

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{7}\left(\sqrt{f_c'} + 120\rho\frac{Vd}{M}\right) \leq 0.3\sqrt{f_c'}$$

Shear Strength of Web Reinforcement:-

$V_s = n A_v f_y$

(n = No. of stirrups; A_v = area of shear reinforcement)



$P \simeq d$

$n = \frac{P}{S}$ or $n = \frac{d}{S}$

$$V_s = \frac{A_v f_y d}{S} \quad \text{(vertical stirrups)}$$

$$V_s = \frac{A_v f_y d}{S} (\sin\alpha + \cos\alpha)$$

(inclined stirrups)

$$V_n = V_c + V_s = V_c + \frac{A_v f_y d}{S}$$

$$V_u \leq \phi V_n$$

In the case of there is no concentrated force between the face of the support & in the distance equal to (d), so the critical section for maximum shear force is taken in distance about (d) from the face of the support. The distance (S) from the face of the support to (d) is equal to the space calculated at the distance (d) from the face of the support.

If the conditions above are not occure then the critical section is taken at the face of the support.

According to ACI-code

$$V_c = \left(\sqrt{f_c'} + 120 \rho_w \frac{V_u d}{M_u}\right)\frac{b_w d}{7} \leq 0.3\sqrt{f_c'}\, b_w d$$

& the $\frac{V_u d}{M_u}$ must be ≤ 1.0

The equation above is used for researchs & Programming but for design the code give this eq. :-

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w d$$

If there is an axial compressive force, the shear resistance will increase and can be found by this eq.

$$V_c = \left(1 + \frac{N_u}{14 A_g}\right)\left(\frac{\sqrt{f_c'}}{6}\right) b_w d$$

where :- N_u is a compressive force (N)
A_g is a total section area.

If there is an axial tension force, then, the shear resistance will decrease & can be found by the following eq. :-

$$V_c = \left(1 + \frac{0.3 N_u}{A_g}\right)\left(\frac{\sqrt{f_c'}}{6}\right) b_w d$$

where :- N_u is tension force in (N) with Negative sign (−).

Shear Design of Beams:-

a- Minimum Shear Reinforcement:-

Theoretically there is no need to shear reinforcement when the shear force is less than concrete strength design:-

$$V_u \leq \phi V_c$$

& the following equation is used to find shear strength of concrete

$$V_c = \frac{1}{6} \sqrt{f'_c}\, b_w d$$

But the ACI code requires provision of at least a minimum area of web reinforcement equal to:-

$$A_v = \frac{1}{16} \sqrt{f'_c} \frac{b_w S}{f_y} \geq \frac{b_w S}{3 f_y}$$

when $V_u > \frac{\phi V_c}{2}$

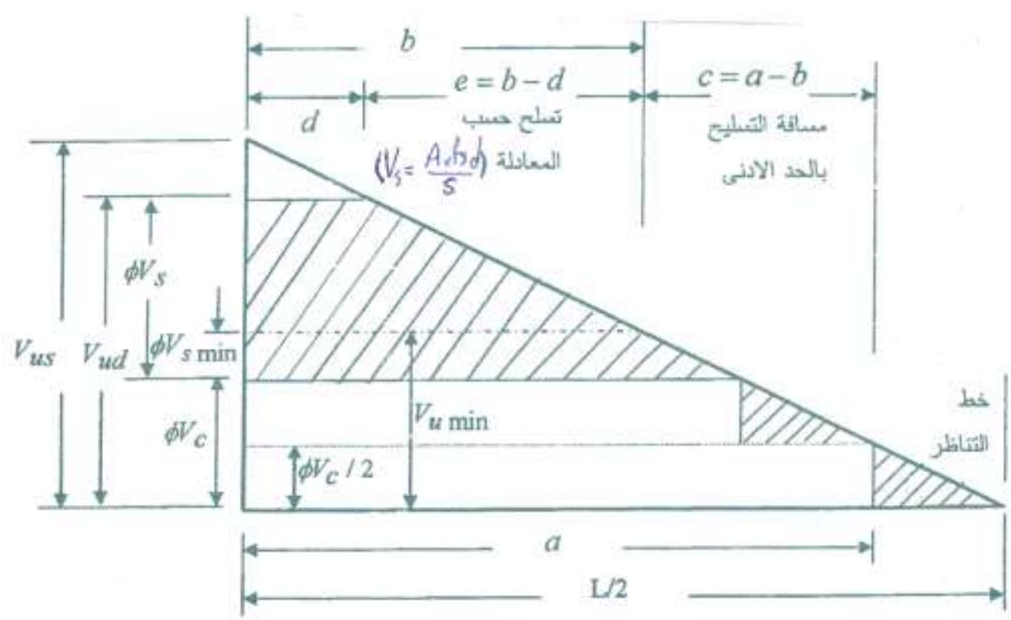
$$S_{max} \leq \begin{cases} \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \\ \frac{3 A_v f_y}{b_w} \end{cases}$$

There is no need for shear reinforcement when

$$V_u \leq \phi \frac{V_c}{2}$$

B- Region of Web Reinforcement

- When (V_{ud}) Shear force design at critical section less than ($\phi V_c/2$), there is no need to stirrups
- When (V_{ud}) larger than ($\phi V_c/2$) & less or equal (ϕV_c) then the beam must reinforced by minmum shear reinforcement for the distance varied from the face of the support to the point, that at this point the shear force equal to ($\phi V_c/2$).
- If the (V_{ud}) (shear force design) at critical section is greater than (ϕV_c), then there will be categories according to the Fig. below.



This Fig. represents the shear force diagram for a half Uniformly distributed load simply supported beam. These catogaries are:-

1- The distance between critical section and face of the support. The shear reinforcement at this distance equal to the same amount of reinforcement for critical section. That means the distance between the Stirrups at critical section (S_o). The first stirrup will be putted at distance equal to $(S_o/2)$ from the support face.

2- The distance from the point reinforced with minimum shear reinforcement (b) to the critical section which is called (e) and it reinforced according to equation $\left(V_s = \frac{A_v f_y d}{S}\right)$. The minmum shear reinforcement means that, the distance between Stirrups, is the maximum distance (S_{max}). The distance (b) is determined by calculating the minimum shear strength of reinforcement (i.e. $S = S_{max}$)

$$V_{s\,min} = \frac{A_v f_y d}{S_{max}}$$

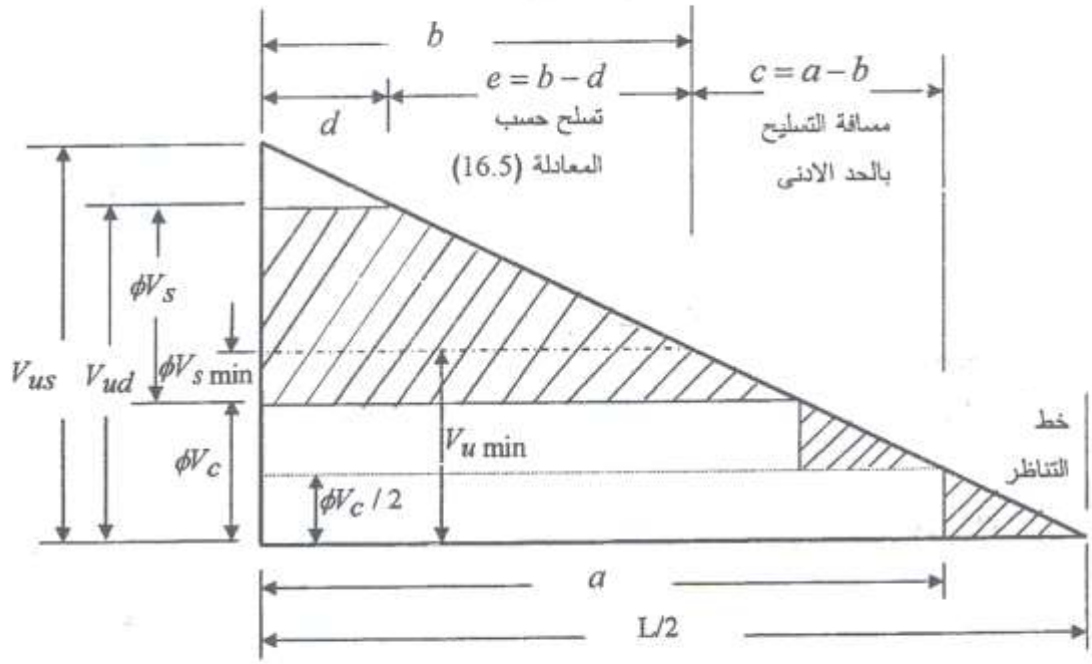
After that, minmum shear strength design ($V_{u\,min}$) will calculated.

$$V_{u\,min} = \phi V_{s\,min} + \phi V_c$$

From equilibrium or the trianguls theory (b) can be found. At this point shear strength design equal to ($V_{u\,min}$).

3- The distance from the point which there is no need to shear reinf. (at distance (a) from support face) to the point, which the shear reinforcement at this point is equal to minimum reinforcement (point (c)). This distance will reinforce by minimum reinforcement ($S = S_{max}$). (a) can be found by force equilibrium or the trinanguls theory. Shear Force design at distance (a) equal to ($\phi V_c/2$)

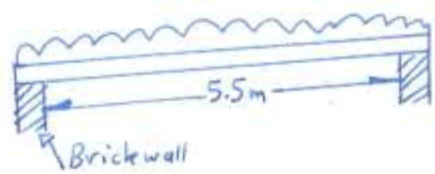
- There is no need for shear reinforcement between point (a) and the point which, at this point the shear force equal to Zero.


b
d
e = b − d
تسلح حسب
المعادلة (16.5)
c = a − b
مسافة التسليح
بالحد الادنى
Vus
Vud
ϕVs
ϕVs min
ϕVc
ϕVc / 2
Vu min
خط
التناظر
a
L/2

E.x.:- Design shear reinforcement for the beam shown below for the following data:-

$b = 300mm$, $d = 500mm$, $LL = 40kN/m$, DL including self weight $= 34kN/m$, $f_y = 300$ MPa, $f'_c = 30$ MPa.

Solution:-



* $W_u = 1.6 \times 40 + 1.2 \times 34 = 104.8\ kN/m$

* finding shear force at the face of the support.

$V_{us} = 104.8 * \frac{5.5}{2} = 288.2\ kN$

* Finding shear force at critical section.

$V_{ud} = V_{us} - W_u d = 288.2 - 0.5 \times 104.8 = 235.8\ kN$

* $\phi V_c = 0.75 \left(\frac{1}{6}\sqrt{f'_c} \times b \times d\right)$

$= 0.75\left(\frac{1}{6}\sqrt{30} \times 300 \times 500\right) * 10^{-3} = 102.698\ kN$

Check if there is need for shear reinforcement

$V_{ud} = 235.8 > \phi V_c = 102.698$ ∴ there is a need for shear reinforcement

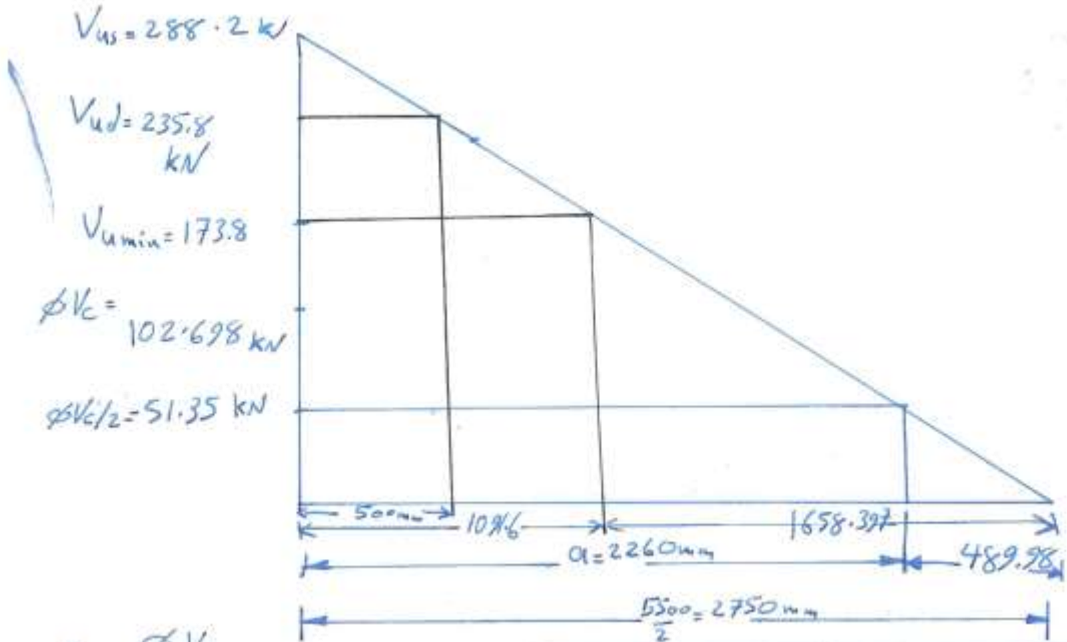
* $\phi V_s = V_{ud} - \phi V_c = 235.8 - 102.698 = 133.1\ kN$

$V_s = \frac{133.1}{\phi} = \frac{133.1}{0.75} = 177.47\ kN$

* $4 * \phi V_c = 4 * 102.698 = 410.792\ kN$

∵ $\phi V_s < 4\phi V_c$
$133.1\ kN < 410.792\ kN$

∴ The section is adiquate for shear [illegible]



* $\frac{\phi V_c}{2} = \frac{102.698}{2} = 51.35\ kN$

$\because \phi V_s < 4\phi V_c$

$\therefore S_{max} \leq \begin{cases} d/2 = 250\ mm \checkmark \\ 600\ mm \\ \frac{3A_v f_y}{b_w} = \frac{3 \times 2 \times 79 \times 300}{300} = 474\ mm \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} = \frac{16 \times 2 \times 79 \times 300}{\sqrt{30} \times 300} = 461\ mm \end{cases}$

$\therefore$ Use $S_{max} = 250\ mm$

* Find spacing of reinforcement at critical section.

$S_o = \frac{A_v f_y d}{V_s} \Longrightarrow$ use ϕ 10mm for stirrups.

$\therefore A_v = 2 * \frac{\pi}{4} \times 10^2 = 157.0\ mm^2$

$S_o = \frac{157 * 300 * 500}{177.47 * 10^{-3}} = 132.7\ mm < S_{max} = 250\ mm$

$\therefore$ use $S_o = 130\ mm$ c/c

* Finding the distance in which there is no need for reinf.:

$\frac{x}{51.35} = \frac{2750}{288.2} \Rightarrow x = 489.98\ mm$

$a = 2750 - 489.98 = 2260\ mm$

- We can find the distance (a) by other method.

$V_{us} - W_u * a = \phi V_c / 2$

$288.2 - 104.8 * a = 51.35 \implies a = \frac{51.35 - 288.2}{-104.8}$

$\therefore a = 2.260$ m

* Determine the distance, which it is after reinforced by minimum reinforcement (shear reinforcement).

$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 * 157 * 300 * 500 * 10^{-3}}{250} = 70.650$ kN

$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8$ kN

$V_{us} - W_u b = 173.8$

$288.2 - 104.8 * b = 173.8 \implies b = \frac{173.8 - 288.2}{-104.8}$

$b = 1.0916$ m

or $\frac{x}{173.8} = \frac{2750}{288.2} \implies x = 1658.397 \implies x_1 = 2750 - 1658.397$

$\implies x_1 = 1091.6$ mm

* Distribution of shear reinforcement along the beam

a- Put the first stirrups at a distance equal to $\frac{S_o}{2} = \frac{130}{2} = 65$ mm

$\therefore$ Put the first Stirrups at a distance equal to 60 mm from the face of the support.

b- Number of the stirrups (130mm)

$n = \frac{1092 - 60}{130} = 7.938$ Use $8 \phi 10$ stirrup @ 130 mm c- So, the distance from the face of the support which reinforced for shear until now equal to:-

$60 + 8 * 130 = 1100$ mm

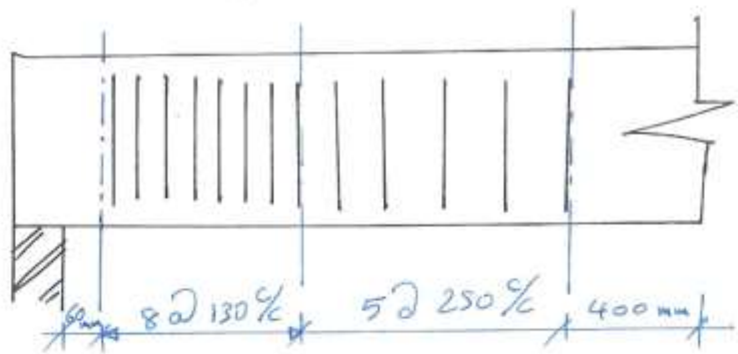
∴ No of stirrups of 250mm c/c $= \frac{2260 - 1100}{250}$

$= 4.64$

∴ use 5 ø 10mm stirrups @ 250 c/c

So, the space which reinforced to shear equal to $1100 + 5(250) = 2350$ mm

So, the region which not reinforced for shear is equal to $2750 - 2350 = 400$ mm



or from equilibrium

$V_{us} = W_u a = \phi V_c/2$

$288.2 - 104.8a = 51.35\ kN \Rightarrow a = 2260\ mm$

finding the distance which after this distance the shear reinforcement in minmun magnitude

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 500 \times 10^{-3}}{250}$$

$= 71.1\ kN$

$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8\ kN$

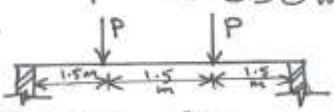
$V_{us} - W_u b = 288.2 - 104.8b = 173.8 \Rightarrow b = 1092\ mm$

the distance between critical section & the point of minmum shear reinf. is small

$e = b - 500 = 1092 - 500 = 592\ mm$

So use the same shear reinf. for critical section

Ex.:- A reinforced concrete girder with a rectangular section, loaded by two concentrated loads, each of them consist of 80kN service Live load & 60 kN service dead load. The width of this girder equal to 300mm & its effective depth equal to 550mm. Design this girder for shear.



Solution:- 1- $h = 550 + 100 = 650$ mm (if we assume the reinforcement in 2 layrs)

$\therefore W_g = 0.65 \times 0.3 \times 24 = 4.68$ kN/m

$W_u = 4.68 \times 1.2 = 5.62$ kN/m

$P_u = 1.2 * 60 + 1.6 * 80 = 200$ kN

$V_{us} = 200 + 4.5 \times \frac{5.62}{2} = 212.65$ kN

2- Calculate V_{ud}

$V_{ud} = 212.65 - 5.62 \times 0.55 = 209.6$ kN

$\phi V_c = 0.75 \left(\frac{1}{6}\right) \sqrt{30} \times 300 \times 550 * 10^{-3} = 112.96$ kN

$\frac{\phi V_c}{2} = 56.48$ kN

3- $V_{ud} = 209.6$ kN $> \phi V_c = 112.96$ kN $\therefore$ There is need for shear reinf.

Calculate $\phi V_s = 209.6 - 112.96 = 96.6$ kN

$\therefore V_s = 128.8$ kN

$\because \phi V_s < 4 \phi V_c$ $\therefore$ the section is adiqute for shear

$\because \phi V_s < 2 \phi V_c$

Use ϕ bar 10mm for strirups

$$\therefore S_{max} \begin{cases} 600\text{ mm} \\ d/2 = 275\text{ mm} \\ \frac{3 A_v f_y}{b_w} = 3 * 2 \times \frac{\pi}{4} \times 10^2 \times 300 = 474\text{ mm} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} = \frac{16 * 2 \times 79 \times 300}{\sqrt{30} \times 300} = 462\text{ mm} \end{cases}$$

Distance from (0 – 1.5) m

- from shear force diagram

$V_u = 204.22\,kN > \phi V_c = 112.96\ kN$

∴ all the distance will reinforced for shear

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 550}{275} \times 10^{-3} = 71.1\ kN$$

$V_{u\,min} = 71.1 + 112.96 = 184.06\ kN$

$V_{u\,min} < 204.22\ kN$

So, we don't use $S = S_{max}$

Distance (3 – 15) m

$V_u = 4.22 < \phi V_c/2 = 56.48\ kN$

∴ There is no need for shear Reinf.

Note :- (Because the variation in shear in the region (0 – 1.5) is small, the distance between the stirrups is still with the same reinforcement for shear for S_o)

Put the firs stirrup in the distance equal to $S_o/2 = \frac{200}{2} = 100$ mm from the face of the support

So, the No other stirrups

$$n = \frac{1500 - 100}{200} = 7$$

i.e. (7 @ 200 mm c/c)

Use $S_{max} = 275\,mm$

5- Calculate (S_o) $\quad S_o = \frac{A_v f_{yd}}{V_s} = \frac{2 * \frac{\pi}{4} * 300 * 550}{128.8 * 1000} = 202\,mm$

USE $\Rightarrow S_o = 200\,mm < S_{max}$