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Direct Search method البحث عن مسار

Hooke & Jeeves method:

This method consist of two phases:-

- a. **Exploration Search phase**, in this step we establish direction & Improvement.
- b. **Pattern Move**, which extracts the current solution vector another point in the solution space.

The procedure is as follows:

1. Choose an initial base point $x^{\circ}=\left(x_1^{\circ}+x_2^{\circ}+..........+x_n^{\circ}\right)$ & an initial step vector $p=\left(\Delta x_1+\Delta x_2+..........+\Delta x_n\right)$

   مقدار الزيادة x_1 وx_2 يجب أن تكون Δ متساوية

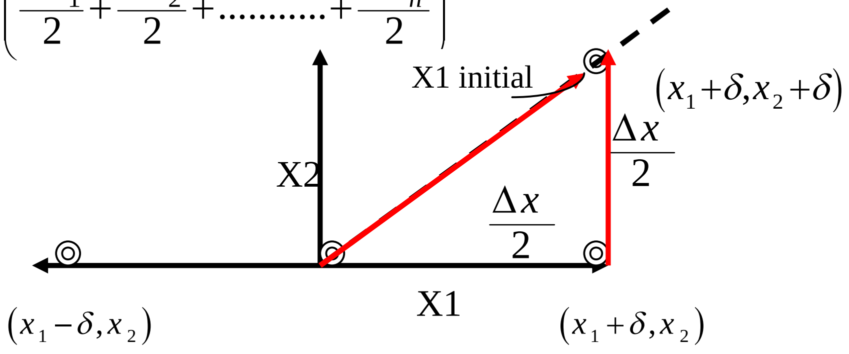
2. Carry out an exploration استكشاف about x° as indicated :
   - a. Evaluate قيم the objective function $f\left(x^{\circ}\right)$
   - b. Each variable is now changed in turn by adding the step length and evaluate the function if the function success , replace x_i° by $x_i^{\circ}+\Delta x_i$,

     if not try $x_i^{\circ}-\Delta x_i$,

     if neither step gives a success , leaves x_i° unchanged & considered all n variables.

     We have a new base point x^1.
   - c. If $x^1 = x^{\circ}$ the exploration is repeated with reduced step length -

     $$p=\left(\frac{\Delta x_1}{2}+\frac{\Delta x_2}{2}+..........+\frac{\Delta x_n}{2}\right)$$



   - d. If $x^1 \neq x^{\circ}$ we make a pattern move.

3. Pattern move procedure is as follows:
   - a. move from the base point x^1 in the direction x^1-x° & evaluate the function at the pattern move point.

     $$x^2=x^{\circ}+2\left(x^1-x^{\circ}\right)\Rightarrow x^2=2x^1-x^{\circ}$$ call this base point 2
   - b. Continue exploration

4. Terminate the process when the step size < tolerance.

$$\Delta x_i \prec (tolerance)$$

Example1: Min Z = $3x_1^2 + x_2^2 - 12x_1 - 8x_2$

Start with an initial value $x^\circ = (1,1)$ and initial step size p = (0.5 , 0.5).

Solution:

Step	*Computation*	*X_1*	*X_2*	*f (X_1, X_2)*	*Remark*
0	***Initial***	***1***	***1***	***-16***	***Initial value***
1	***Exploration***	***1.5***	***1***	***-18.25***	***Success***
		1.5	***1.5***	***-21***	***Success***

Base 1 (1.5 , 1.5) , Set 1 (1 , 1)

$x_1^2 = 2(1.5) - 1 = 2$

$x_2^2 = 2(1.5) - 1 = 2$

2	***Pattern Move***	***2***	***2***	***-24***	***Success***

Base 2 (2 , 2) , Set 1 (1 , 1)

$x_1^3 = 2 + 0.5 = 2.5, P(2.5, 2)$

$x_1^3 = 2 - 0.5 = 1.5, P(1.5, 2)$

$x_2^3 = 2 + 0.5 = 2.5, P(2, 2.5)$

3	***Exploration***	***2.5***	***2***	***-23.25***	***Failure***
		1.5	***2***	***-23.25***	***Failure***
		2	***2.5***	***-25.75***	***Success***

Base 3 (2 , 2.5) , Set 2 (1.5 , 1.5)

$X_1^4 = 2(2) - 1.5 = 2.5$

$X_2^4 = 2(2.5) - 1.5 = 3.5$

4	***Pattern Move***	***2.5***	***3.5***	***-27***	***Success***

Base 4 (2.5 , 3.5) , Set 2 (1.5 , 1.5)

5	***Exploration***	***3***	***3.5***	***-24***	***Failure***
		2	***3.5***	***-27.75***	***Success***
		2	***4***	***-28***	***Success***

Base 5 (2 , 4) , Set 3 (2 , 2.5)

6	***Pattern Move***	***2***	***5.5***	***-25.75***	***Failure***
	Exploration	***2.5***	***5.5***	***-25***	***Failure***
		1.5	***5.5***	***-25***	***Failure***
		2	***6***	***-24***	***Failure***

The Method of Rotating Coordinates (Resonbrock Method)

Resonbrock unconstrained procedure

To minimize the objective function f(x1,x2,……..,xn) the algorithm of the procedure on the following steps:

1. A starting point $x_1^\circ, x_2^\circ, ..., x_n^\circ$ and initial step size $\lambda_i, i = 1, 2, ..., n$ are selected and the objective function evaluated.
2. The first variable x_1 is stepped a distance λ_1 parallel to the axis & the function evaluated.

If the value of the objective function decreased , the move is termed a success & λ_1 increased by a factor $\alpha\left(\alpha \geq 1\right)$.

If the value of the objective function increased , the move is termed a failure & λ_1 decreased by a factor $\beta\left(0 \prec \beta \prec 1\right)$ & The direction of movement reversed.

The value of $\alpha \,\&\, \beta$ recommended by Rosenborck are $\alpha = 3 \,\&\, \beta = 0.5$.

3. The next variable $x_i, i = 1, 2, ..., n$ is in turn stepped a distance λ_i parallel to the axis. The same acceleration or deceleration in step 2 are applied for all variables in consecutive repetitive sequence until one success & one failure have been encountered at least in all n direction. i.e. continue the search sequentially along the direction $S_1^j, S_2^j, ..., S_i^j, ...S_n^j,$ until at least one step has been successful & one step has failed in each of the n-direction.

4. Compute the new set of direction $S_1^{j+1}, ..., S_i^{j+1}, ...S_n^{j+1}$ for use in next $\left(j+1\right)^{th}$ Stage of minimization by using the gram-Schmidt orthogonalization procedure.
   a. Compute a set of independent directions p1,p2,…….pi,………pn. as

$$p_{n,n} = \left[p_1 \quad p_2 \quad .. \quad . \quad p_n\right]$$

$$= \left[S_1^j, S_2^j, ...S_n^j,\right] \begin{bmatrix} A_1 & 0 & . & 0 \\ A_2 & A_2 & . & 0 \\ . & . & . & . \\ A_n & A_n & . & A_n \end{bmatrix}$$

In which

A_i = The algebraic sum of all the successful step length in the corresponding direction $S_i^j, i = 1, 2, ...n$

P_1 = The vector joining the starting point & the final point obtained after the sequence of the searches in the $\left(j\right)^{th}$ stage.

$P_2, P_3, ……, P_n$ = The algebraic sum of the successful step length in the all directions except the first one, & so on these linearly independent vector $P_1, P_2, ……, P_n$ can be used to generate a new set of orthogonal directions.

b. Set $D_1^j = P_1^j$

Matrix

Constan

With $$S_1^j = \frac{D_1^j}{\sqrt{D_1^{(j)T} . D_1^j}}$$

C. Compute $$D_i^j = P_i^j - \sum_{m=1}^{i-1}\left[\overbrace{\underbrace{P_{m+1}^{(j)T} . S_m^j}_{\text{Constan}} . S_m^j}^{\text{Matrix}}\right]$$

With $$S_1^j = \frac{D_i^j}{\sqrt{D_i^{(j)T} . D_i^j}}$$

5. Take the best point obtained in the present stage , & repeat the same procedure of searching from step 2 on words.

i.e. $$new\ \overrightarrow{\left[x_{L,i}^j\right]} = old\ \overrightarrow{\left[x_{L,i}^j\right]} + \lambda_i \overrightarrow{\left[S_{L,i}^j\right]}$$

where:

L = variable index, L = 1,2,……..,n
i = direction index, I = 1,2,………,n
j = stage index.

6. The procedure terminates when the convergence criterion is satisfied.

Example: Min f(x)= $x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$

Initial base

$$x = \begin{bmatrix}0\\0\end{bmatrix} \& f(x) = 0$$

$$S_1^0 = \begin{bmatrix}1\\0\end{bmatrix}, S_1^0 = \begin{bmatrix}0\\1\end{bmatrix}, \lambda = \begin{bmatrix}0.8\\0.8\end{bmatrix}, \beta = 0.5 \& \alpha = 3$$

Solution:

Stage 1:

$$\left[x_1\right]_{new} = \left[x\right]_{old} + \lambda_1\left[S_1\right]$$

$$\left[x_1\right]_{new} = \begin{bmatrix}0\\0\end{bmatrix} + 0.8\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0.8\\0\end{bmatrix} \Rightarrow f\,(0.8,0) = 2.08\left(failure\right)$$

$$[x_2]_{new}=\begin{bmatrix}0\\0\end{bmatrix}+0.8\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\0.8\end{bmatrix}\Rightarrow f\ (0,0.8)=-0.16(Success)$$

point	X_1	X_2	λ_1	λ_2	f(x)	Remark
0	0	0	-	-	0	initial
1	0.8	0	0.8	-	2.08	failure
2	0	0.8	-	0.8	-0.16	success
3	-0.4	0.8	-0.4	-	-0.88	success
4	-0.4	3.2	-	2.4	4.4	failure
5	-1.6	0.8	-1.2	-	0.8	failure

We have at least one failure & one success in each direction S_1°, S_2°

Note: New base point $x=\begin{bmatrix}-0.4\\0.8\end{bmatrix}\&f\ (x)=-0.88$

$$A_1=\sum\lambda_1@Success=-0.4$$

$$A_2=\sum\lambda_2@Success=0.8$$

Stage 2: New set of search direction S^1

$$P^1_{2x\,2}=\begin{bmatrix}P_1^1 & P_2^1\end{bmatrix}=\begin{bmatrix}S_1^{\circ} & S_2^{\circ}\end{bmatrix}\begin{bmatrix}A_1 & 0\\A_2 & A_2\end{bmatrix}$$

$$P^1_{2x\,2}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}-0.4 & 0\\0.8 & 0.8\end{bmatrix}=\begin{bmatrix}-0.4 & 0\\0.8 & 0.8\end{bmatrix}$$

$$\therefore P_1^1=\begin{bmatrix}-0.4\\0.8\end{bmatrix}\&P_2^1=\begin{bmatrix}0\\0.8\end{bmatrix}$$

$$D_1^1=P_1^1=\begin{bmatrix}-0.4\\0.8\end{bmatrix}$$

$$S_1^1=\frac{D_1^1}{\sqrt{D_1^{1T}.D_1^1}}=\frac{\begin{bmatrix}-0.4\\0.8\end{bmatrix}}{\sqrt{(0.4)^2+(0.8)^2}}=\begin{bmatrix}-0.447\\0.894\end{bmatrix}$$

$$D_i^j=P_i^j-\sum_{m=1}^{i-1}\left[\left\{P_{m+1}^{(j)T}.S_m^j\right\}S_m^j\right]$$

$$D_2^1=P_2^1-\sum_{m=1}^{2-1}\left[\left\{P_2^{1T}.S_m^1\right\}S_m^1\right]=\begin{bmatrix}0\\0.8\end{bmatrix}-\left(\begin{bmatrix}0 & 0.8\end{bmatrix}\begin{bmatrix}-0.447\\0.894\end{bmatrix}\right)\begin{bmatrix}-0.447\\0.894\end{bmatrix}$$

$$=\begin{bmatrix}0\\0.8\end{bmatrix}-\left[0*(-0.447)+0.8*(0.894)\right]\begin{bmatrix}-0.447\\0.894\end{bmatrix}$$

$$=\begin{bmatrix}0\\0.8\end{bmatrix}-\begin{bmatrix}-0.320\\0.639\end{bmatrix}=\begin{bmatrix}0.320\\0.160\end{bmatrix}$$

$$S_2^1=\frac{D_2^1}{\sqrt{D_2^{1T}.D_2^1}}=\frac{\begin{bmatrix}0.32\\0.16\end{bmatrix}}{\sqrt{(0.32)^2+(0.16)^2}}=\begin{bmatrix}0.894\\0.447\end{bmatrix}$$

Due to λ_1 $\begin{bmatrix}x_1\\x_2\end{bmatrix}_{new}=\begin{bmatrix}x_1\\x_2\end{bmatrix}_{old}+\lambda_1\underbrace{\begin{bmatrix}-0.447\\0.894\end{bmatrix}}_{S_1^1}$

Due to λ_2 $\begin{bmatrix}x_1\\x_2\end{bmatrix}_{new}=\begin{bmatrix}x_1\\x_2\end{bmatrix}_{old}+\lambda_2\underbrace{\begin{bmatrix}0.894\\0.447\end{bmatrix}}_{S_2^1}$

point	***λ_1***	***λ_2***	***X_1***	***X_2***	***f(x)***	***Remark***
0	***-***	***-***	***-0.4***	***0.8***	***-0.88***	***Base Point***
1	***0.8***	***-***	***-0.758***	***1.515***	***-1.125***	***success***
2	***-***	***0.8***	***-0.042***	***1.873***	***1.439***	***failure***
3	***2.4***	***-***	***-7.831***	***3.662***	***1.212***	***failure***
4	***-***	***-0.4***	***-1.116***	***1.337***	***-1.159***	***success***
5	***-1.2***	***-***	***-0.579***	***0.263***	***0.0499***	***failure***
6	***-***	***-1.2***	***-2.189***	***0.8***	***3.731***	***failure***

We have at least one failure & one success in each direction S_1^1, S_2^1

Note: New base point $x=\begin{bmatrix}-1.116\\1.337\end{bmatrix}\& f(x)=-1.159$

$$A_1=\sum\lambda_1 @Success=0.8$$

$$A_2=\sum\lambda_2 @Success=-0.4$$

Stage 3: New set of search direction S^2

$$P_{2x2}^2=\begin{bmatrix}P_1^2 & P_2^2\end{bmatrix}=\begin{bmatrix}S_1^1 & S_2^1\end{bmatrix}\begin{bmatrix}A_1 & 0\\A_2 & A_2\end{bmatrix}$$

$$P_{2x2}^2=\begin{bmatrix}-0.447 & 0.894\\0.894 & 0.447\end{bmatrix}\begin{bmatrix}0.8 & 0\\-0.4 & -0.4\end{bmatrix}=\begin{bmatrix}-0.7152 & -0.3576\\0.5364 & -0.1788\end{bmatrix}$$

$$\therefore P_1^2=\begin{bmatrix}-0.7152\\0.5364\end{bmatrix}\& P_2^2=\begin{bmatrix}-0.3576\\-0.1788\end{bmatrix}$$

$$D_1^2=P_1^2=\begin{bmatrix}-0.7152\\0.5364\end{bmatrix}$$

$$S_1^2=\frac{D_1^2}{\sqrt{D_1^{2T}.D_1^2}}=\frac{\begin{bmatrix}-0.7152\\0.5364\end{bmatrix}}{\sqrt{(0.7152)^2+(0.5364)^2}}=\begin{bmatrix}-0.8\\0.6\end{bmatrix}$$

$$D_2^2=P_2^2-\left(\{P_2^{2T}.S_1^2\}S_1^2\right)=\begin{bmatrix}-0.3576\\-0.1788\end{bmatrix}-\left(\begin{bmatrix}-0.3576 & -0.1788\end{bmatrix}\begin{bmatrix}-0.8\\0.6\end{bmatrix}\right)\begin{bmatrix}-0.8\\0.6\end{bmatrix}=\begin{bmatrix}-0.2146\\-0.2861\end{bmatrix}$$

$$S_2^2=\frac{D_2^2}{\sqrt{D_2^{2T}.D_2^2}}=\frac{\begin{bmatrix}-0.2146\\-0.2861\end{bmatrix}}{\sqrt{(0.2146)^2+(0.2861)^2}}=\begin{bmatrix}-0.6\\-0.8\end{bmatrix}$$

Due to λ_1 $\begin{bmatrix}x_1\\x_2\end{bmatrix}_{new}=\begin{bmatrix}x_1\\x_2\end{bmatrix}_{old}+\lambda_1\begin{bmatrix}-0.8\\0.6\end{bmatrix}$

Due to λ_2 $\begin{bmatrix}x_1\\x_2\end{bmatrix}_{new}=\begin{bmatrix}x_1\\x_2\end{bmatrix}_{old}+\lambda_2\begin{bmatrix}-0.6\\-0.8\end{bmatrix}$

point	λ_1	λ_2	X_1	X_2	*f(x)*	*Remark*
0	*-*	*-*	*-1.116*	*1.337*	*-1.159*	*Base Point*
1	*0.8*	*-*	*-1.756*	*1.817*	*-0.487*	*failure*
2	*-*	*0.8*	*-1.596*	*0.697*	*1.062*	*failure*
3	*-0.4*	*-*	*-0.796*	*1.097*	*-1.169*	*success*
4	*-*	*-0.4*	*-0.556*	*1.417*	*-0.922*	*failure*
5	*-1.2*	*-*	*0.164*	*0.377*	*0.108*	*failure*
6	*-*	*0.2*	*-0.416*	*0.937*	*-1.014*	*failure*
7	*0.6*	*-*	*-1.276*	*1.457*	*-1.072*	*failure*
8	*-*	*-0.1*	*-0.736*	*1.177*	*-1.177*	*success*
9	*-0.3*	*-*	*-0.496*	*0.997*	*-0.996*	*failure*
10	*-*	*-0.3*	*-0.556*	*1.417*	*-0.992*	*failure*

We have at least one failure & one success in each direction S_1^2, S_2^2

Note: New base point $x=\begin{bmatrix}-0.736\\1.177\end{bmatrix}\&f(x)=-1.177$

$$A_1=\sum\lambda_1 @Success=-0.4$$

$$A_2=\sum\lambda_2 @Success=-0.1$$

Stage 4: New set of search direction S^3

$$P^3_{2x\,2} = \begin{bmatrix} P_1^3 & P_2^3 \end{bmatrix} = \begin{bmatrix} S_1^2 & S_2^2 \end{bmatrix} \begin{bmatrix} A_1 & 0 \\ A_2 & A_2 \end{bmatrix}$$

$$P^3_{2x\,2} = \begin{bmatrix} -0.8 & -0.6 \\ 0.6 & -0.8 \end{bmatrix} \begin{bmatrix} -0.4 & 0 \\ -0.1 & -0.1 \end{bmatrix} = \begin{bmatrix} 0.38 & 0.06 \\ -0.16 & 0.08 \end{bmatrix}$$

$$\therefore P_1^3 = \begin{bmatrix} 0.38 \\ -0.16 \end{bmatrix} \& P_2^3 = \begin{bmatrix} 0.06 \\ 0.08 \end{bmatrix}$$

$$D_1^3 = P_1^3 = \begin{bmatrix} 0.38 \\ -0.16 \end{bmatrix}$$

$$S_1^3 = \frac{D_1^3}{\sqrt{D_1^{3T} . D_1^3}} = \frac{\begin{bmatrix} 0.38 \\ -0.16 \end{bmatrix}}{\sqrt{(0.38)^2 + (0.16)^2}} = \begin{bmatrix} 0.922 \\ -0.388 \end{bmatrix}$$

$$D_2^3 = P_2^3 - \left(\left\{P_2^{3T}\, S_1^3\right\} S_1^3\right) = \begin{bmatrix} 0.06 \\ 0.08 \end{bmatrix} - \left(\begin{bmatrix} 0.06 & 0.08 \end{bmatrix} \begin{bmatrix} 0.922 \\ -0.388 \end{bmatrix}\right) \begin{bmatrix} 0.922 \\ -0.388 \end{bmatrix} = \begin{bmatrix} 0.037 \\ 0.089 \end{bmatrix}$$

$$S_2^3 = \frac{D_2^3}{\sqrt{D_2^{3T} . D_2^3}} = \frac{\begin{bmatrix} 0.037 \\ 0.089 \end{bmatrix}}{\sqrt{(0.037)^2 + (0.089)^2}} = \begin{bmatrix} 0.384 \\ -0.923 \end{bmatrix}$$

Due to λ_1 $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{new} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{old} + \lambda_1 \begin{bmatrix} 0.922 \\ -0.388 \end{bmatrix}$

Due to λ_2 $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{new} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{old} + \lambda_2 \begin{bmatrix} 0.384 \\ -0.923 \end{bmatrix}$

point	***λ_1***	***λ_2***	***X_1***	***X_2***	***f(x)***	***Remark***
0	***-***	***-***	***-0.736***	***1.177***	***-1.177***	***Base Point***
1	***0.8***	***-***	***0.002***	***0.866***		
2	***-***	***0.8***				

Continue !!!!!

خلاصة طريقة (Resonbrock)

١- نبدأ (Initial value) لقيم (x_1&x_2) ونعتبرها (Base) أي قاعدة للحل ثم نجد منها قيم جديدة ل(x_1&x_2) التالية ولكل انحدار (Slop) و(λ) في الخطوة صفر لا تؤخذ قيمة (λ)

ثم نحسب قيمة (x) الجديدة وفق المعادلات التالية $\begin{bmatrix} x_1 \end{bmatrix}_{new} = \begin{bmatrix} x \end{bmatrix}_{old} + \lambda_1 \begin{bmatrix} S_1 \end{bmatrix}$ ونعوض في Z

٢- نعوض قيم (x_1&x_2) الجديدة في المعادلة لإيجاد قيمة Z فإذا نجحت قيمة (x_1&x_2) في الخطوة هذه تأخذ هذه القيم كقاعدة (Base) للتحرك للخطوة التالية .أما قيمة (λ) الجديدة = ($3\lambda_1$). أما إذا فشلت فنرجع إلى القيمة السابقة (التي حققت نجاح سابق) ونعتبرها (Base) أما قيمة (λ) الجديدة = ($-0.5\lambda_1$).

٣- لغرض التوقف للمحاولة (Stage) يجب أن يتحقق على الأقل لكل نجاح فشل لنفس (x).

٤- تحسب قيم (A_1) أي المجموع الجبري لـ(A_1) التي تمثل قيم لـ(λ) التي أدت إلى النجاح و(A_2) ونقارن بينهما لتجد أيهما يعطي القيمة المثلى (أمثل قيمة).
عندها تستخدم في إيجاد القيم الجديدة والتي تعتبر (Base) لـ(x_1&x_2) التالية.

٥- في أي مرحلة يجب أن تحسب الانحدارات الجديدة (New Set of Search Direction).

Non-Linear programming with Inequality Constrains

The general nonlinear problems is defined as

$$\begin{pmatrix}\max.\\ \min.\end{pmatrix} Z = f(x), \qquad x = (x_1, x_2, \ldots\ldots\ldots x_n)$$

Subject to:

$$g_i(x) \le 0 \qquad (i = 1, 2, \ldots\ldots\ldots, r)$$
$$g_i(x) \ge 0 \qquad (i = r+1, \ldots\ldots\ldots, p)$$
$$g_i(x) = 0 \qquad (i = p+1, \ldots\ldots\ldots, m)$$

The inequality constraints may be converted into equation by adding nonlinear slack variable.

Now, the Lagrange function may be written as:

$$L(x, \lambda, s) = f(x) - \sum_{i=1}^{r} \lambda_i \left(g_i(x) + s_i^2\right) - \sum_{i=r+1}^{p} \lambda_i \left(g_i(x) - s_i^2\right) - \sum_{i=p+1}^{m} \lambda_i \left(g_i(x)\right)$$

A necessary condition for optimality, that be non-negative (non-positive) for max (min) problems.

- For direct search method:
  The success & failure depend not only on the value of the objective function but on the constraints.

Example: $Max.Z = x_1 + x_2$

Subject to: $x_1^2 + x_2^2 \le 20$

($x_1 = x_2 = \Delta x_1 = \Delta x_2 = 1$)

Using Hooks & Jeeves method?

Solution.

Max. $Z = x_1 + x_2$, subject to $g(x_1,x_2) = x_1^2 + x_2^2 - 20 \le 0$

Step	***Computation***	***X_1***	***X_2***	***Z***	***g (X_1, X_2)***	***Remark***
0	*Initial*	*1*	*1*	*2*	*-18*	*Initial value*
1	*Exploration*	*2*	*1*	*3*	*-15*	*Success*
		2	*2*	*4*	*-13*	*Success*

Base 1 (2 , 2) , Set 1 (1 , 1)

2	*Pattern move*	*3*	*3*	*6*	*-2*	*Success*

Base 2 (3 , 3) , Set 1 (1 , 1)

3	*Exploration*	*4*	*3*	*7*	*5*	*failure*
		2	*3*	*5*	*-7*	*failure*
		3	*4*	*7*	*5*	*failure*
		3	*2*	*5*	*-7*	*failure*

Use $\Delta x_1 = \Delta x_2 = 0.5$

4	*Exploration*	*3.5*	*3*	*6.5*	*1.25*	*failure*
		2.5	*3*	*5.5*	*-4.75*	*failure*
		3	*3.5*	*6.5*	*1.25*	*failure*
		3	*2.5*	*5.5*	*-4.75*	*failure*

Use $\Delta x_1 = \Delta x_2 = 0.25$

5	*Exploration*	*3.25*	*3*	*6.25*	*-0.4375*	*Success*
		3.25	*3.25*	*6.5*	*1.125*	*failure*
		3.25	*2.75*	*6*	*-1.875*	*failure*

Base 3 (3.25 , 3) , Set 2 (2 , 2)

6	*Pattern move*	*4.5*	*4*	*8.5*	*16.25*	*failure*

Use $\Delta x_1 = \Delta x_2 = 0.125$

7	*Exploration*	*3.375*	*3*	*6.375*	*0.391*	*failure*
		3.125	*3*	*6.125*	*-1.234*	*failure*
		3.25	*3.125*	*6.375*	*0.328*	*failure*
		3.25	*2.875*	*6.125*	*-1.172*	*failure*

Use $\Delta x_1 = \Delta x_2 = 0.0625$

8	*Exploration*	*3.3125*	*3*	*6.3125*	*-0.027*	*Success*