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Optimization Modeling

Water Users:

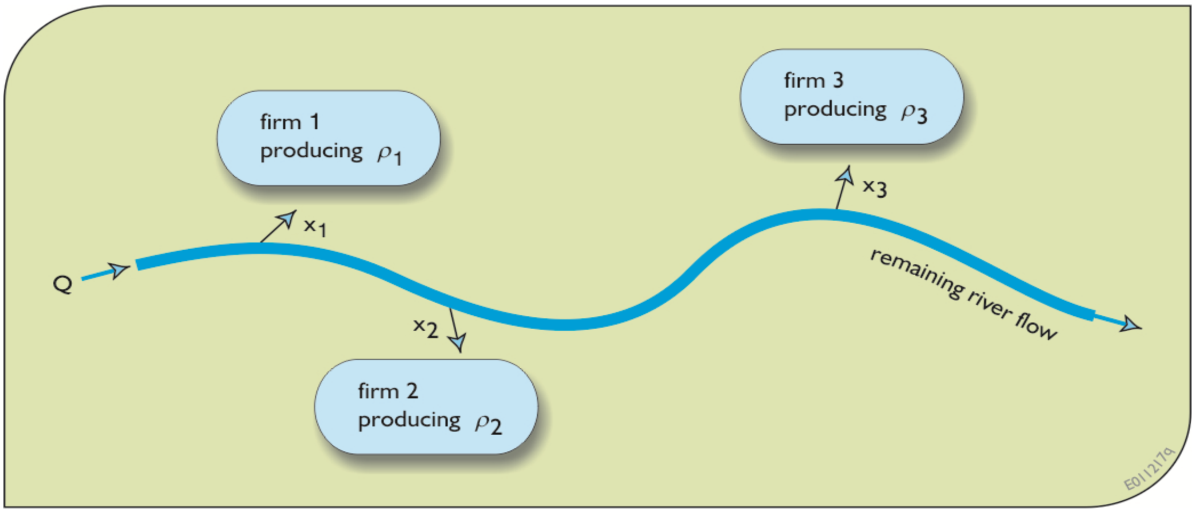
- Allocate release Q to 3 users and provide instream flow S
- Net-benefit from allocation of x_i

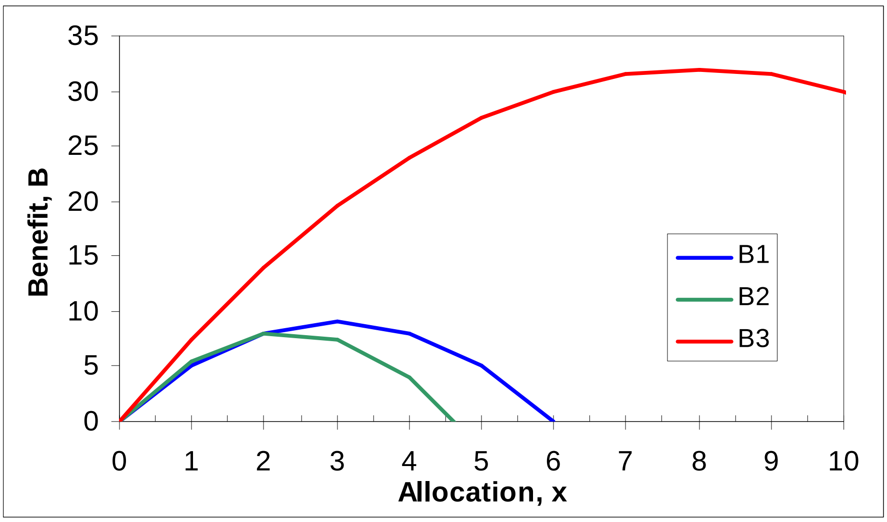
$$B_i(x_i) = a_i x_i - b_i x_i^2 \qquad i = 1,2,3$$

$$B_1(x_1) = 6x_1 - x_1^2$$

$$B_2(x_2) = 7x_1 - 1.5x_2^2$$

$$B_3(x_3) = 8x_3 - 0.5x_3^2$$





- Decision variables: x_i, i=1,2,3
- Objective:

$$maximize \sum_{i=1}^{3}(a_i x_i - b_i x_i^2)$$

Note: if sufficient water is available the allocations are independent and equal to $x_1^* = 3,\ x_2^* = 2.33,\ x_3^* = 8$

- Constraint: $x_1 + x_2 + x_3 + R = Q$

- Optimization model:

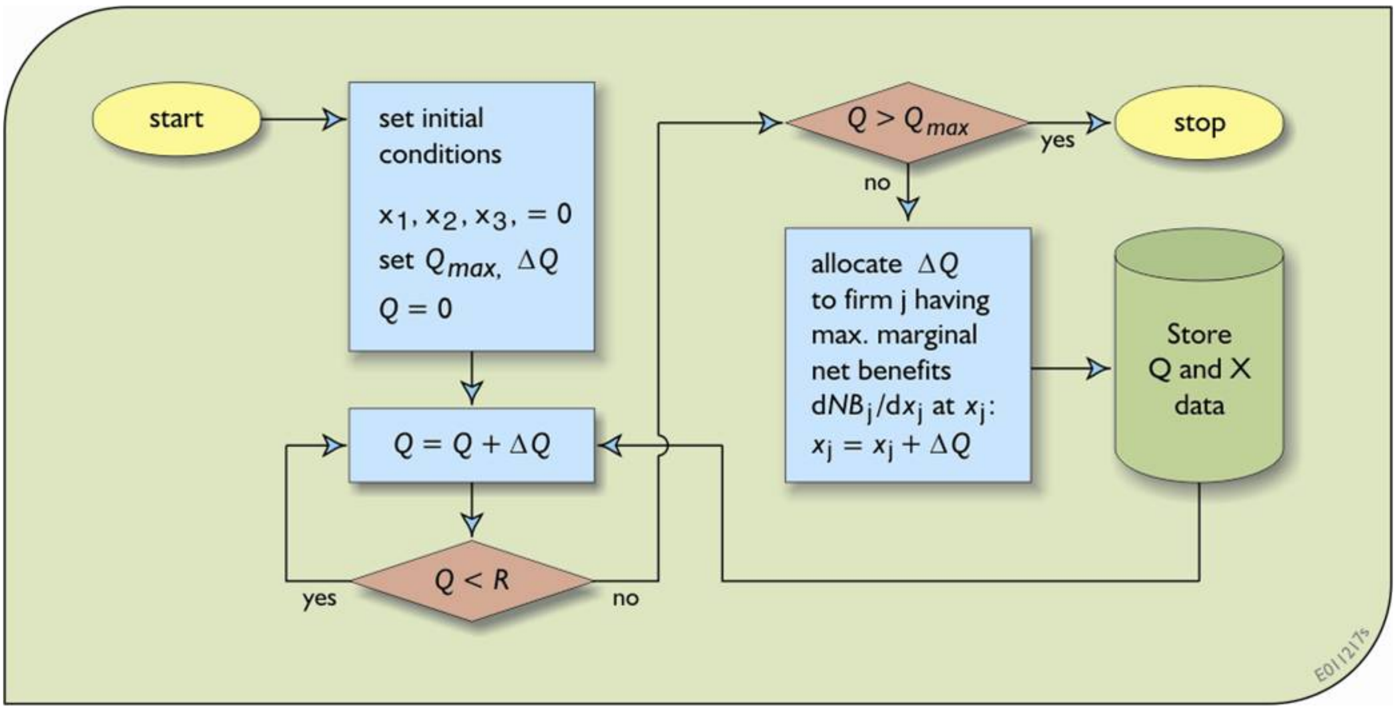
$$\underset{\mathbf{x}}{maximize} \sum_{i=1}^{3}(a_i x_i - b_i x_i^2)$$

$$subject\ to$$

$$\sum_{i=1}^{3} x_i + R = Q$$

Flowcharting Details

- Divide available flow Q into increments and allocate each increment to get maximum additional (marginal) benefit from incremental amount of water

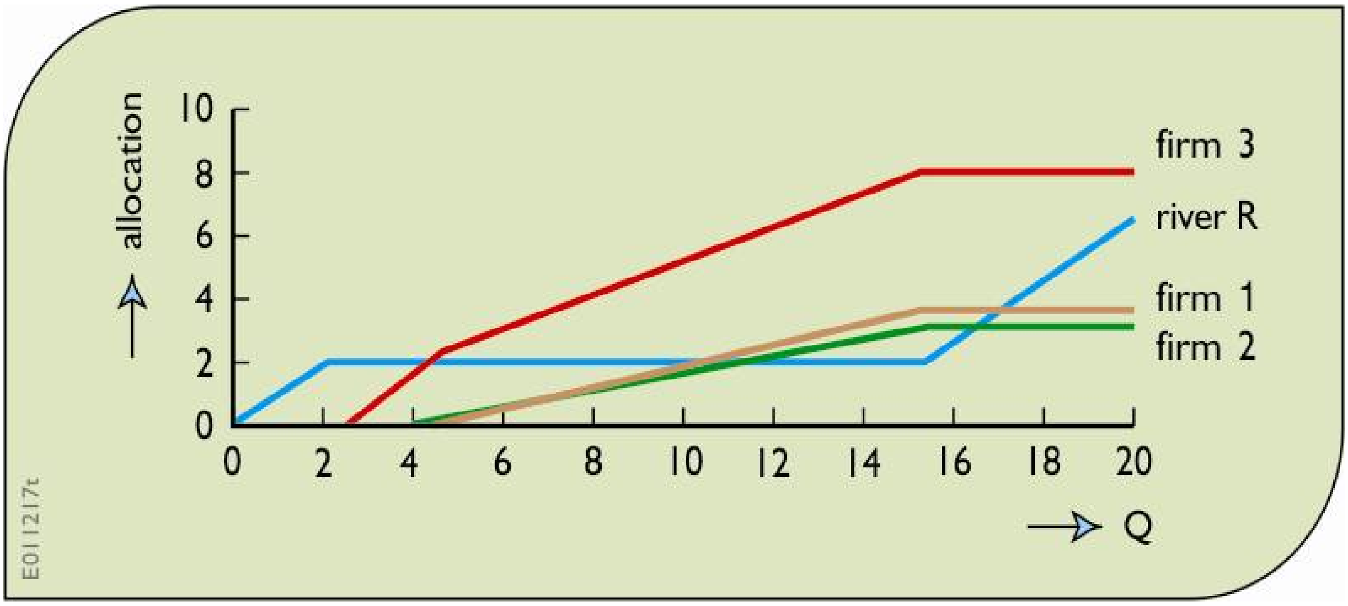


$Q_{max} = 8$; $Q_i = 0$; $\Delta Q = 1$; river flow $R \geq \min\{Q, 2\}$

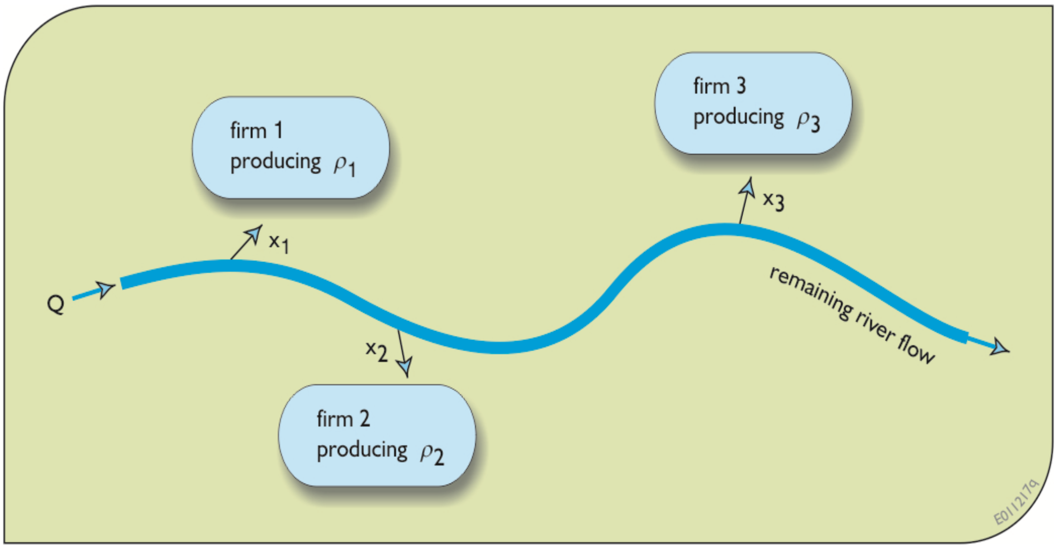
iteration i	Q_i	allocations. R, x_j				marginal net benefits			new allocations					total net benefits
		R	x_1	x_2	x_3	$6-2x_1$	$7-3x_2$	$8-x_3$	$Q_i+\Delta Q$	R	x_1	x_2	x_3	$\Sigma_j NB_j(x_1)$
1–3	0-2	0–2	0	0	0	6	7	8	3	2	0	0	1	7.5
4	3	2	0	0	1	6	7	7	4	2	0	0	2	14.0
5	4	2	0	0	2	6	7	6	5	2	0	1	2	19.5
6	5	2	0	1	2	6	4	6	6	2	0	1	3	25.0
7	6	2	0	1	3	6	4	5	7	2	1	1	3	30.0
8	7	2	1	1	3	4	4	5	8	2	1	1	4	34.5
9	8	2	1	1	4	4	4	4	–	–	–	–	–	---

$Q_{max} = 20$; $\Delta Q \rightarrow 0$; river flow $R \geq \min\{Q, 2\}$ selected values of Q

Q	allocations. R, x_j				marginal net benefits			total net benefits
	R	x_1	x_2	x_3	$6-2x_1$	$7-3x_2$	$8-x_3$	$\Sigma_j NB_j(x_1)$
0–2	0–2	0	0	0	6	7	8	0.0
4	2	0	0.25	1.75	6.00	6.25	6.25	14.1
5	2	0.18	0.46	2.36	5.64	5.64	5.64	20.0
8	2	1.00	1.00	4.00	4.00	4.00	4.00	34.5
10	2	1.55	1.36	5.09	2.91	2.91	2.91	41.4
15	2	2.91	2.27	7.82	0.18	0.18	0.18	49.1
20	6.67	3.00	2.33	8.00	0	0	0	49.2



Example:



- Lagrangean

$$L(\vec{x},\lambda) = \sum_{i=1}^{3} (a_i x_i - b_i x_i^2) - \lambda\left(\sum_{i=1}^{3} x_i + R - Q\right)$$

- First – order conditions

$$\frac{\partial L}{\partial x_1} = a_1 - 2b_1x_1 - \lambda = 0$$

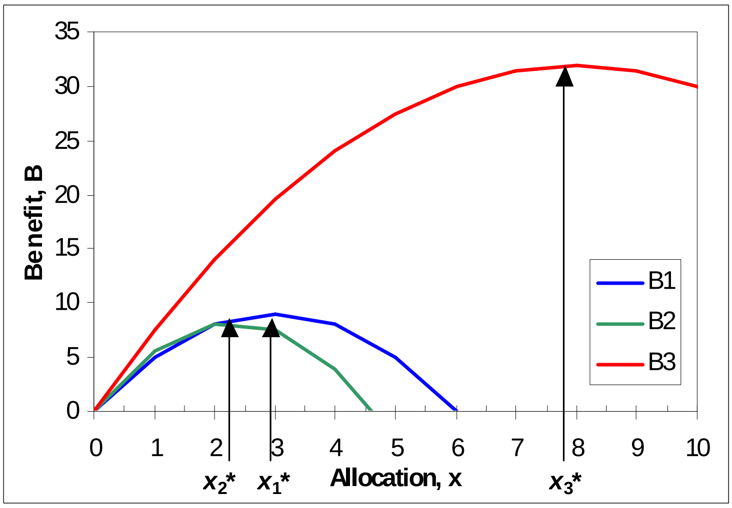
$$\frac{\partial L}{\partial x_2} = a_2 - 2b_2x_2 - \lambda = 0$$

$$\frac{\partial L}{\partial x_3} = a_3 - 2b_3x_3 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 + R - Q = 0$$

$$x_i^* = \frac{a_i - \lambda}{2b_i} \qquad i = 1,2,3$$

$$\lambda^* = \frac{2\left(\sum_{i=1}^{3}\frac{a_i}{2b_i} + R - Q\right)}{\sum_{i=1}^{3}\frac{1}{b_i}}$$



$$B_i(x_i) = a_i x_i - b_i x_i^2 \qquad i = 1,2,3$$

$a_1 = 6;\ a_2 = 7;\ a_3 = 8$

$b_1 = 1.0;\ b_2 = 1.5;\ b_3 = 0.5$

.

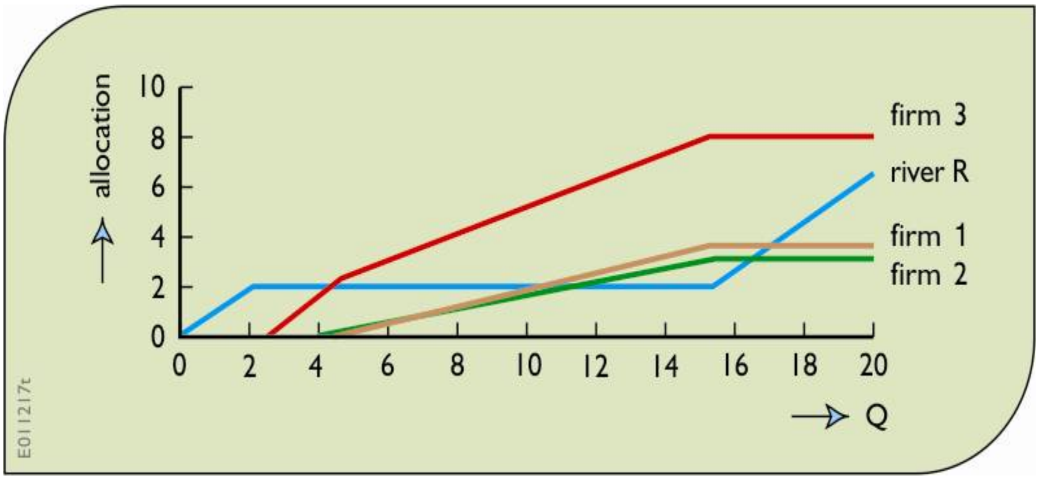
Equal marginal benefits (slopes) for all users

$$\lambda = \frac{\partial NB}{\partial Q} = \frac{\partial B_i}{\partial x_i}$$

Q	x_1	x_2	x_3	λ	R	Total Downstream Flow
5.00	0.18	0.45	2.36	5.64	2.00	2.00
8.00	1.00	1.00	4.00	4.00	2.00	2.00
10.00	1.55	1.36	5.09	2.91	2.00	2.00
15.00	2.91	2.27	7.82	0.18	2.00	2.00
16.00	3.00	2.33	8.00	0.00	2.00	2.67
20.00	3.00	2.33	8.00	0.00	2.00	6.67

Release Allocation Rule

Rule tells you the amount of released water allocated to each use



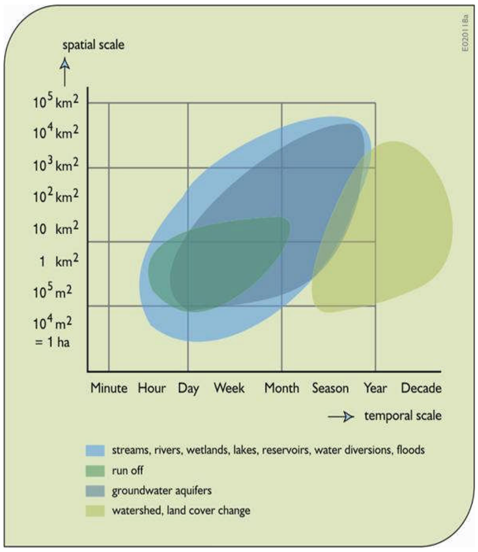
River Basin Modeling

Water Resources

- Water at:
  - Wrong place, wrong quantity, wrong time
- What to do?
- Manipulate the hydrologic cycle
  - Build facilities? Remove facilities? Reoperate facilities?
    - Reservoirs
    - Canals
    - Other infrastructure

Scales of Processes

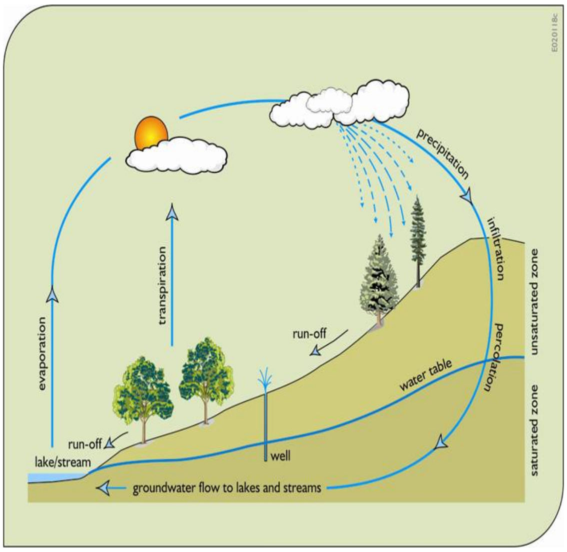
Time Scales



- Water management plans
  - Consider average conditions within discrete time periods
    - Weekly, monthly or seasonal
  - Over a long time horizon
    - Year, decade, century
  - Shortest time period
    - No less than travel time from the upper basin to mouth
    - For shorter time periods some kind of flow routing required
- Flood management
  - Conditions over much shorter periods
    - Hours, Days, Week

Hydrologic Processes

- Precipitation
- Runoff
- Infiltration
- Percolation
- Evapotranspiration
- Chemical concentration
- Groundwater

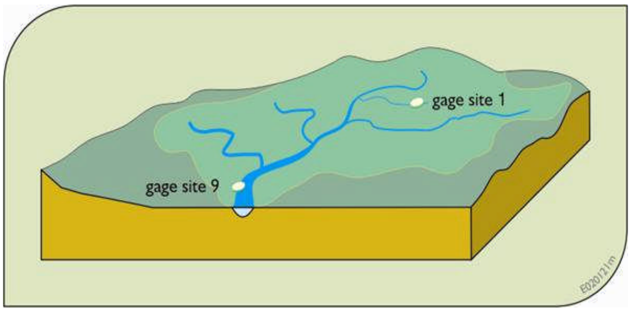


Hydrologic Models

- Precipitation - runoff models
  - Predict runoff from land areas, infiltration into soils, and percolation into aquifers
  - Used when stream gage data are not available, not reliable, or not representative of natural flow conditions (including climate change conditions)
  - Estimate impacts of changing land uses on temporal and spatial distribution of runoff
- Model characteristics
  - Theoretical or Empirical
  - Deterministic or Stochastic
  - Event-based or Continuous-time
  - Lumped or Distributed

Streamflow Data

- Measurement
- Data sources
- Flow conditions
  - Natural
  - Present
  - Unregulated
  - Regulated
  - Future
- Reservoir losses
- Missing data
  - Precipitation-runoff models
  - Stochastic streamflow models
  - Extending and filling in historic records



Flows at Ungaged Sites

- Have streamflows qtg at gauge sites g = 1 and 9
- Use these to estimate flows at other sites
- When runoff per unit land area can be assumed constant over space
  - For a gauge site g, runoff per unit area can be calculated by dividing the gauge flow by the upstream drainage area,

$$q_t^g = \frac{Q_t^g}{A_g}$$

  - the estimated streamflow at site *s* will be

$$Q_t^s = \left(\frac{Q_t^g}{A_g}\right) A_s$$

- Multiple gauge sites
  - estimated streamflow at ungauged site *s* is weighted linear combination of unit runoffs times contributing area at site *s*
- *where*
  - *Qst* flow at ungaged site
  - *As* upstream drainage area
  - *g* gage site
  - *Qgt* gage flow
  - *Ag* gage drainage area
  - *wg* weights
- *Incremental flow* - difference between the natural streamflows at any two sites is called the

$$Q_t^s = \sum_g w_g \frac{Q_t^g}{A_g} A_s$$

Yield of a System

- Yield - amount of water that can be supplied during some time interval

- Firm yield - amount of water that can be supplied in a *critical period*
  - Without storage: firm yield is lowest streamflow on record,
  - With storage: firm yield can be increased to approximately the mean annual flow of stream

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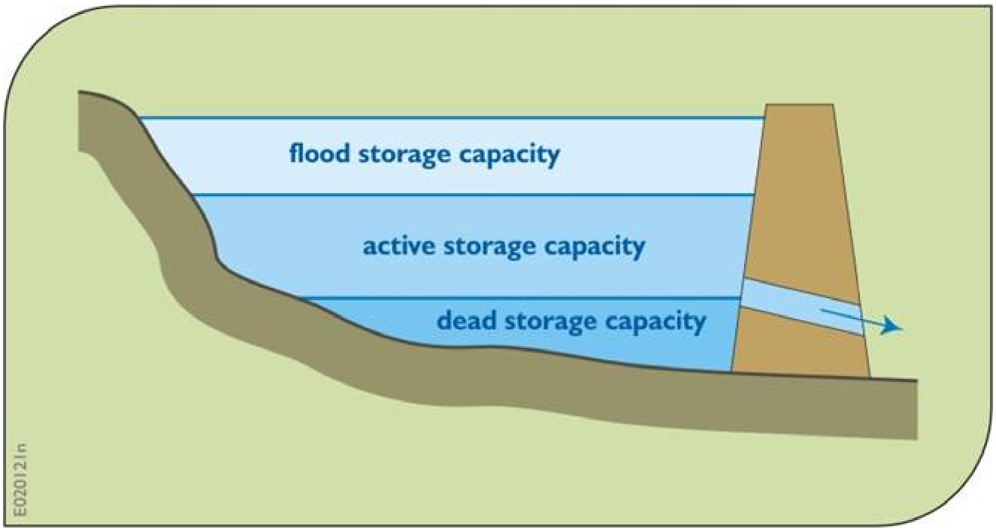
Increase Firm Yield Add storage

- To increase the firm yield of a stream, impoundments are built. Need to develop the storage-yield relationship for a river
- Simplified methods
  - Mass curve (Rippl) method
  - Sequent peak method
- More complex methods
  - Math Programming
  - Simulation



Simplified Methods

- Mass curve (Rippl) method
  - Graphical estimate of storage required to supply given yield
  - Constructed by summing inflows over period of record and plotting these versus time and comparing to demands
- Time interval includes “critical period”
  - Time over which flows reached a minimum
  - Causes the greatest drawdown of reservoir

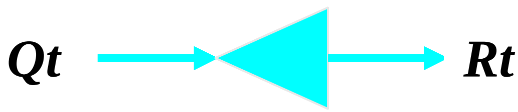
Rippl method

- Find the maximum cumulative difference between reservoir releases R_t and inflows Q_t
- Assuming constant reservoir release, R_t, in each period t
  - A line with slope R_t is placed tangent to the
  - cumulative inflow curve.
  - To the right the release R_t exceeds the inflow Q_t

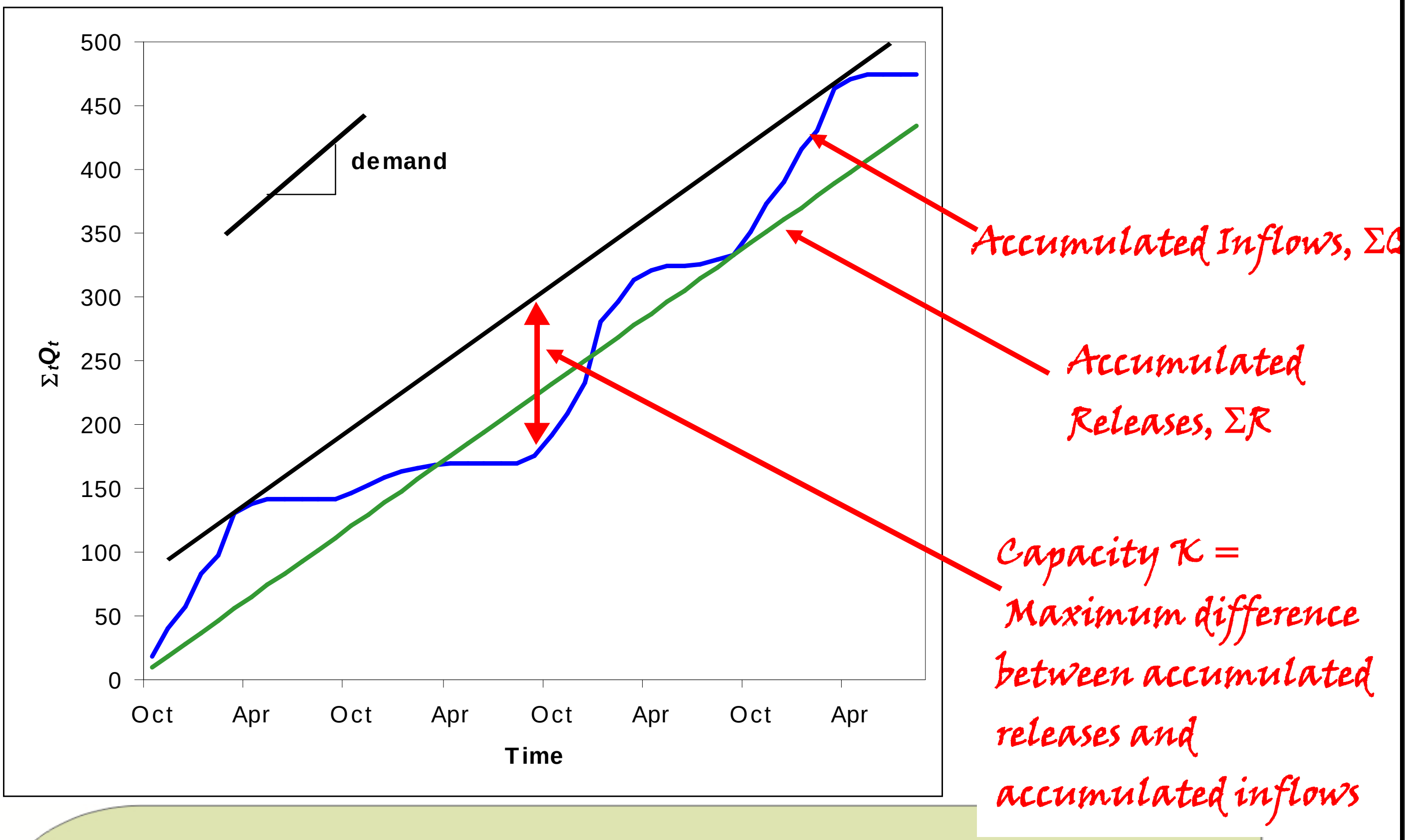
- The maximum vertical distance between the cumulative inflow curve and the release line equals the maximum water deficit
- This is the required active storage capacity

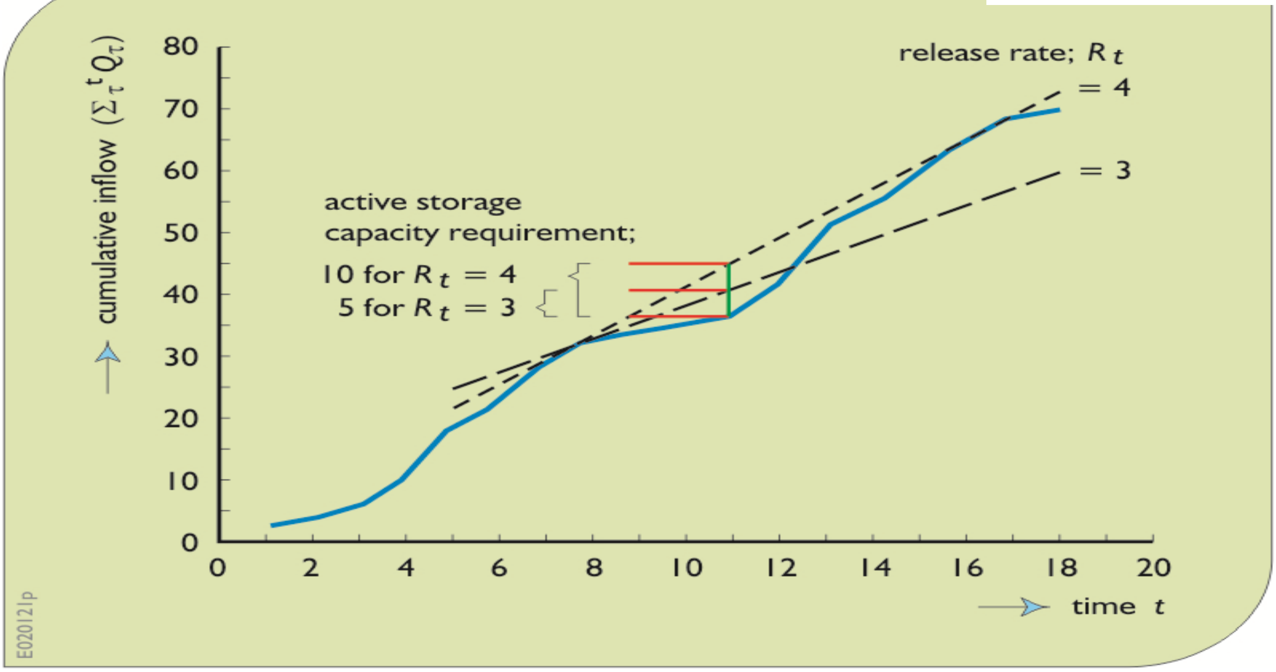
$$K_a = \text{maximum}\left[\sum_{t=i}^{j}(R_t - Q_t)\right]$$

where $1 \leq i \leq j \leq 2T$.



t	Q(t)	ΣQ(t)	R(t)	ΣR(t)	t	Q(t)	ΣQ(t)	R(t)	ΣR(t)
Oct	18	18	9.3	9.3	Apr	1	169	9.3	175.8
Nov	22	40	9.3	18.5	May	0	169	9.3	185.0
Dec	17	57	9.3	27.8	Jun	0	169	9.3	194.3
Jan	26	83	9.3	37.0	Jul	0	169	9.3	203.5
Feb	15	98	9.3	46.3	Aug	0	169	9.3	212.8
Mar	32	130	9.3	55.5	Sep	7	176	9.3	222.0
Apr	8	138	9.3	64.8	Oct	15	191	9.3	231.3
May	3	141	9.3	74.0	Nov	17	208	9.3	240.5
Jun	0	141	9.3	83.3	Dec	25	233	9.3	249.8
Jul	0	141	9.3	92.5	Jan	47	280	9.3	259.0
Aug	0	141	9.3	101.8	Feb	16	296	9.3	268.3
Sep	0	141	9.3	111.0	Mar	18	314	9.3	277.5
Oct	5	146	9.3	120.3	Apr	7	321	9.3	286.8
Nov	6	152	9.3	129.5	May	4	325	9.3	296.0
Dec	6	158	9.3	138.8	Jun	0	325	9.3	305.3
Jan	5	163	9.3	148.0	Jul	1	326	9.3	314.5
Feb	3	166	9.3	157.3	Aug	3	329	9.3	323.8
Mar	2	168	9.3	166.5	Sep	4	333	9.3	333.0



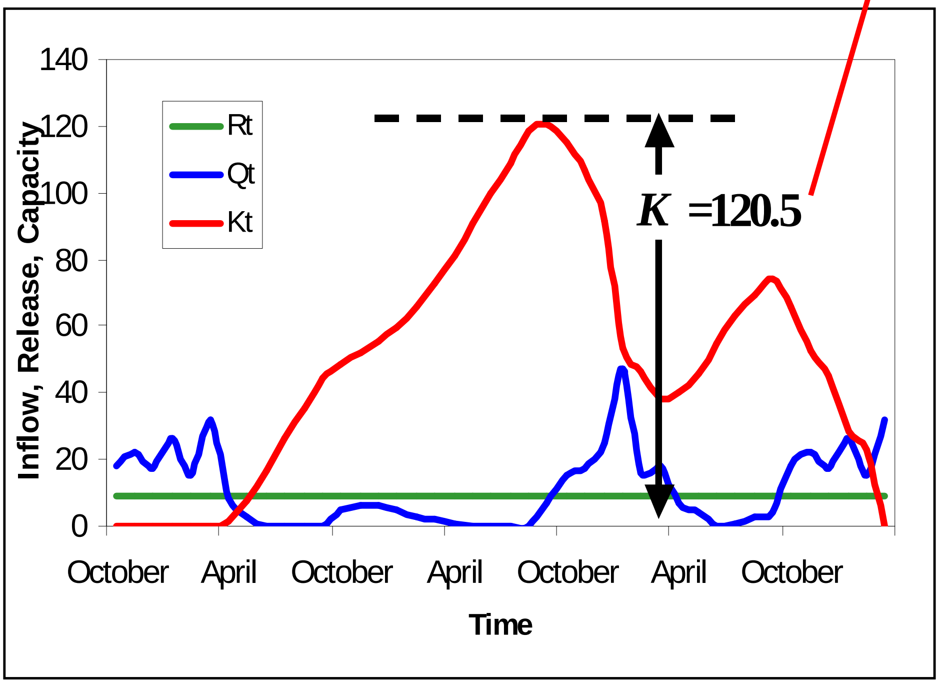


Sequent Peak Method

- Plot cumulative inflows, *Qt*, minus releases, *Rt*
- Note first peak and the sequent peak (next peak that is greater than first)

- Required storage is vertical difference between first peak and low point before sequent peak
- Largest value of storage is design value

t	R_t	Q_t	K_{t-1}	K_t
October	9.25	18	0.0	0.0
November	9.25	22	0.0	0.0
December	9.25	17	0.0	0.0
January	9.25	26	0.0	0.0
February	9.25	15	0.0	0.0
March	9.25	32	0.0	0.0
April	9.25	8	0.0	1.3
May	9.25	3	1.3	7.5
...	...	...	...	...
May	9.25	0	81.3	90.5
June	9.25	0	90.5	99.8
July	9.25	0	99.8	109.0
August	9.25	0	109.0	118.3
September	9.25	7	118.3	**120.5**
October	9.25	15	120.5	114.8
November	9.25	17	114.8	107.0
December	9.25	25	107.0	91.3
...	...	...	...	...
January	9.25	26	45.3	28.5
February	9.25	15	28.5	22.8
March	9.25	32	22.8	0.0

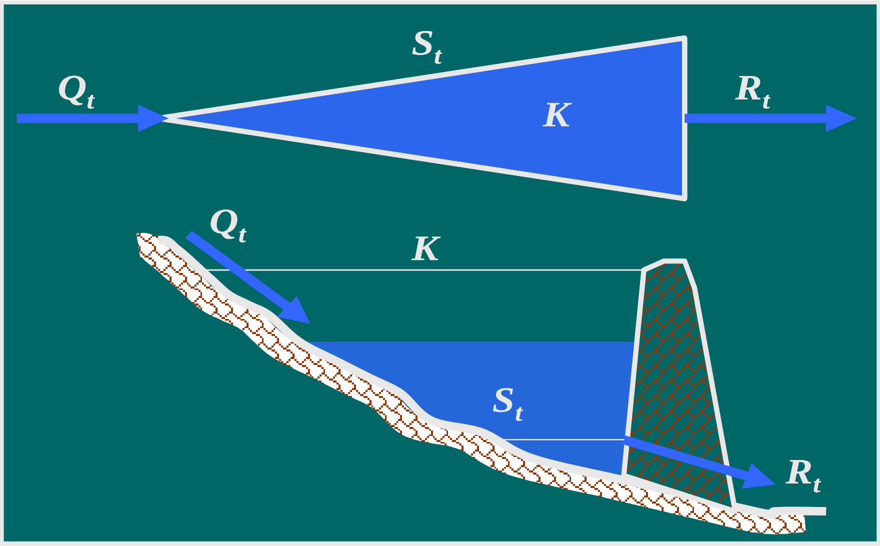


Seque

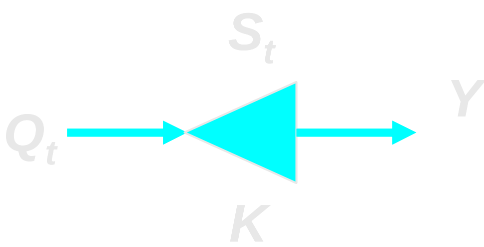
$$K_t = \begin{cases} R_t - Q_t + K_{t-1} & If\ R_t - Q_t + K_{t-1} \geq 0 \\ 0 & If\ R_t - Q_t + K_{t-1} < 0 \end{cases}$$

time t	$(R_t - Q_t + K_{t-1})^+ = K_t$	
1	3.5 − 1.0 + 0.0 = 2.5	
2	3.5 − 3.0 + 2.5 = 3.0	
3	3.5 − 3.0 + 3.0 = 3.5	
4	3.5 − 5.0 + 3.5 = 2.0	
5	3.5 − 8.0 + 2.0 = 0.0	
6	3.5 − 6.0 + 0.0 = 0.0	
7	3.5 − 7.0 + 0.0 = 0.0	
8	3.5 − 2.0 + 0.0 = 1.5	
9	3.5 − 1.0 + 1.5 = 4.0	
1	3.5 − 1.0 + 4.0 = 6.5	
2	3.5 − 3.0 + 6.5 = 7.0	
3	3.5 − 3.0 + 7.0 = 7.5	K_a
4	3.5 − 5.0 + 7.5 = 6.0	
5	3.5 − 8.0 + 6.0 = 1.5	
6	3.5 − 6.0 + 1.5 = 0.0	repetition begins
7	3.5 − 7.0 + 0.0 = 0.0	
8	3.5 − 2.0 + 0.0 = 1.5	
9	3.5 − 1.0 + 1.5 = 4.0	

Reservoir



Maximize yield (Y),Given capacity (K)



- Maximize constant release given capacity and inflows
  - ***Qt*** **Inflows to the reservoir**
  - ***St*** **Storage volumes in the reservoir**
  - ***Y*** **Constant release (yield) from the reservoir**
  - ***K*** **Capacity of the reservoir**

Maximize Y

subject to

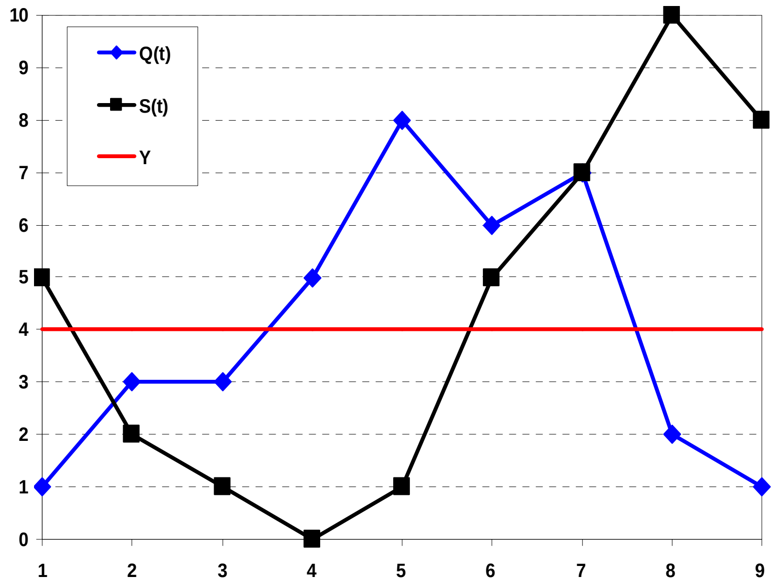
Fined

$$S_t + Q_t - Y = S_{t+1} \qquad t = 1,\ldots,T;\ T+1=1$$

$$S_t \le K \qquad t = 1,\ldots,T$$

Given

Inflow Q(t)	Storage S(t)	Yield Y
1.00	5.00	4.00
3.00	2.00	4.00
3.00	1.00	4.00
5.00	0.00	4.00
8.00	1.00	4.00
6.00	5.00	4.00
7.00	7.00	4.00
2.00	10.00	4.00
1.00	8.00	4.00



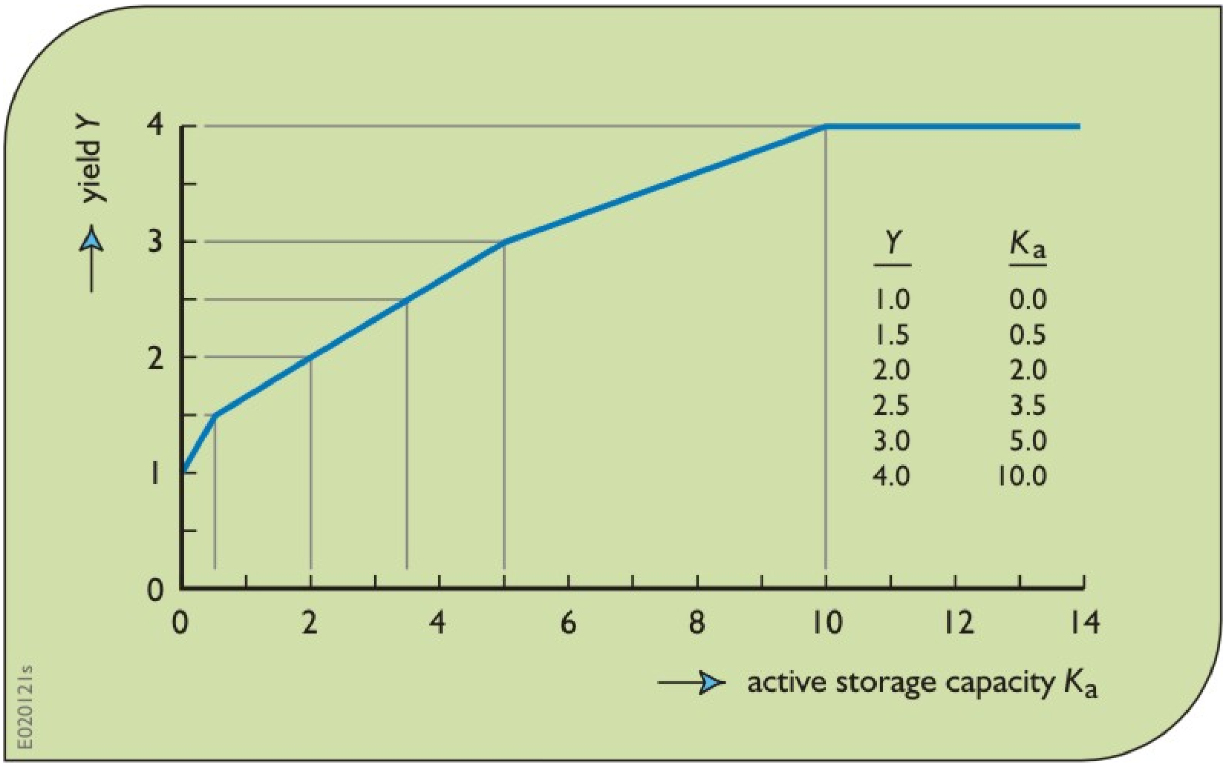
Maximize Y

subject to

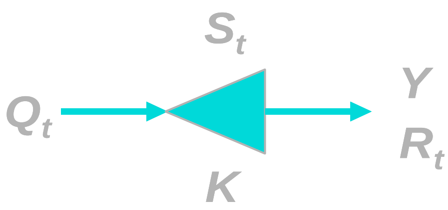
$$S_{t+1} \le S_t + Q_t - Y \qquad t = 1,\ldots,T;\ T+1=1$$

$$S_t \le K \qquad t = 1,\ldots,T$$

Capacity – Yield Function



Max K, Given Y



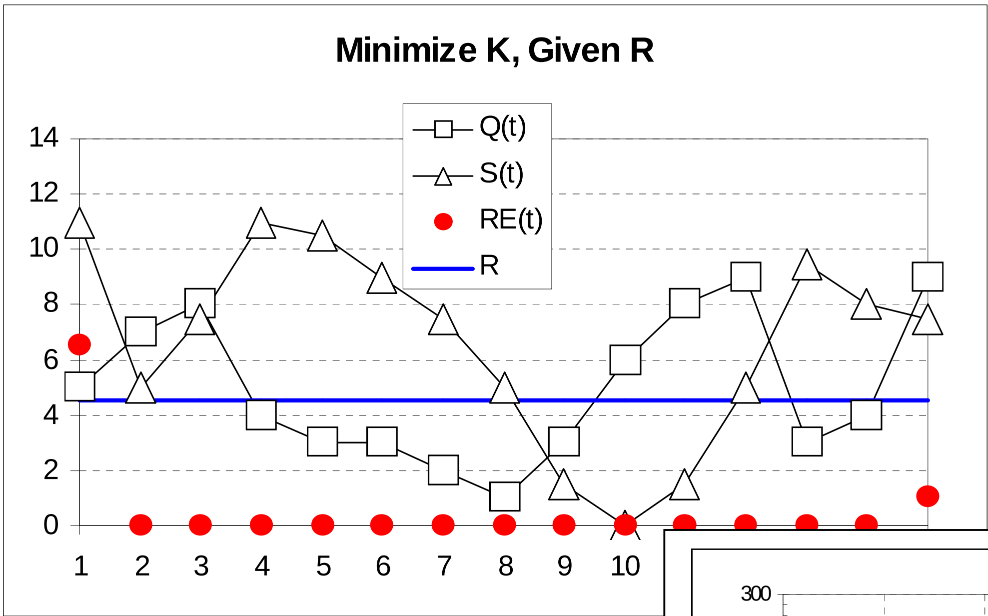
- Maximize constant release given capacity and inflows

Minimize K

subject to

$$S_{t+1} = S_t + Q_t - R_t \qquad t = 1,\dots,T;\ T+1 = 1$$
$$S_t \le K \qquad t = 1,\dots,T$$
$$R_t \ge Y$$

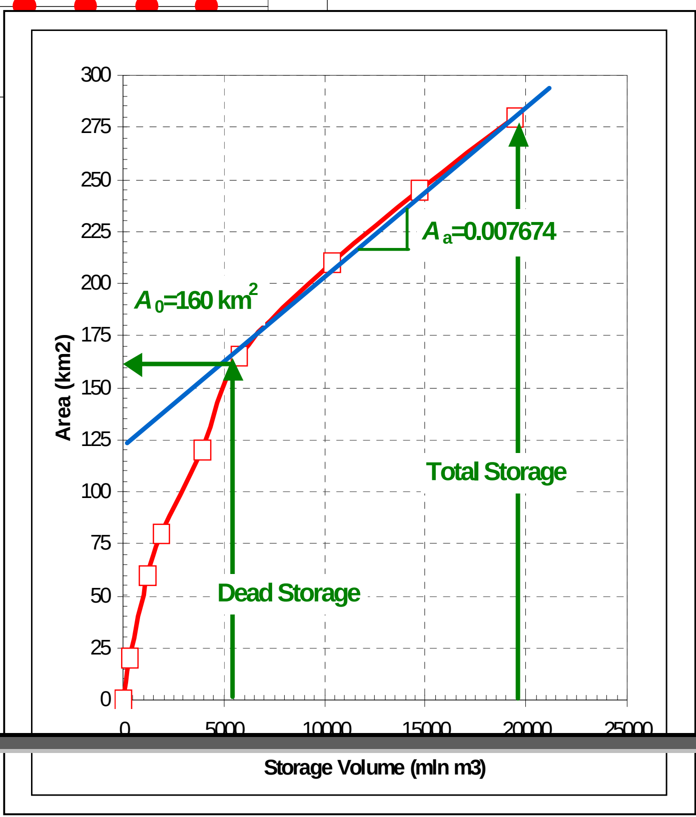
	Inflow	Storage	Release	Yield
t	Q(t)	S(t)	RE(t)	R
1	5	11	6.5	4.5
2	7	5	0	4.5
3	8	7.5	0	4.5
4	4	11	0	4.5
5	3	10.5	0	4.5
6	3	9	0	4.5
7	2	7.5	0	4.5
8	1	5	0	4.5
9	3	1.5	0	4.5
10	6	0	0	4.5
11	8	1.5	0	4.5
12	9	5	0	4.5
13	3	9.5	0	4.5
14	4	8	0	4.5
15	9	7.5	1	4.5



Evaporation

$$S_{t+1} = S_t + Q_t - R_t - L_t$$

- *Lt* Losses from reservoir
- A Surface area of reservoir
- *et* ave. evaporation rate



$$\begin{aligned} L_t &= \left[A_a \bar{S}_t + A_0\right] e_t \\ &= \left[A_a\left(\frac{S_t + S_{t+1}}{2}\right) + A_0\right] e_t \\ &= A_a e_t\left(\frac{S_t + S_{t+1}}{2}\right) + A_0 e_t \\ &= 0.5 A_a e_t S_t + 0.5 A_a e_t S_{t+1} + A_0 e_t \\ &= a_t S_t + a_t S_{t+1} + b_t \end{aligned}$$

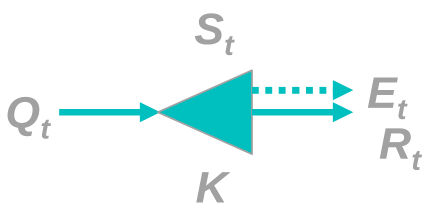
$$S_{t+1} = S_t + Q_t - R_t - L_t$$

$$L_t = a_t S_t + a_t S_{t+1} + b_t$$

$$S_{t+1} = S_t + Q_t - R_t - (a_t S_t + a_t S_{t+1} + b_t)$$

$$(1 + a_t) S_{t+1} = (1 - a_t) S_t + Q_t - R_t - b_t$$

Reservoir with Power Plant
Power Production



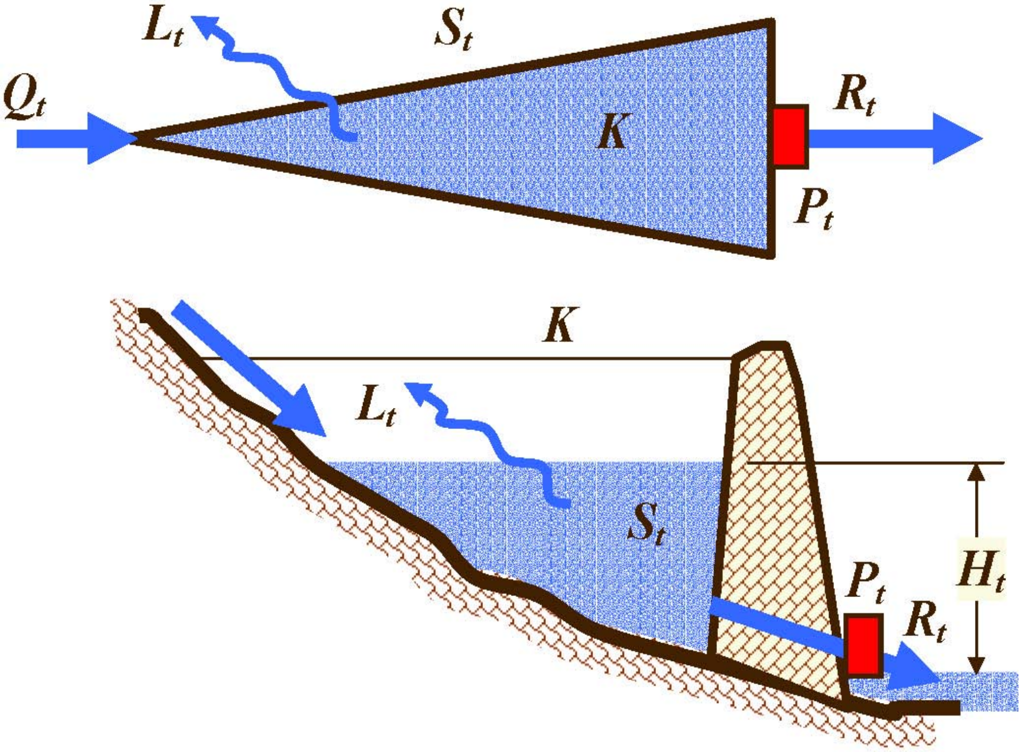


Figure 3.1.1. Diagram of reservoir system.

Qt **= Inflows (mln.m^3/period)**
St **= Storage volume (mln.m^3)**
K **= Capacity (mln.m^3)**
Rt **= Release (mln.m^3/period)**
Et **= Energy (kWh)**
Ht **= Head (m)**
k **= coefficient (efficiency, units)**

$$\overline{H}_t = \frac{H(S_t)+H(S_{t+1})}{2} \quad \text{m}$$

$$E_t = 2730 * \varepsilon \overline{H}_t R_t \qquad \text{kWh}$$

$$E_t = 1.024 * \varepsilon \overline{H}_t R_t \qquad \text{kWh}$$

H in ft

R in acre - feet

$$q_t = R_t * 10^6 / time_t \quad \text{m}^3/\text{sec}$$

$$P_t = (9.81)\varepsilon \overline{H}_t q_t \qquad \text{kW}$$

$$E_t = P_t * time(t) / 3600 \qquad \text{kWh}$$

$$\text{Maximize} \quad \sum_{t=1}^{T} E_t$$

subject to

$$S_t + Q_t - R_t = S_{t+1}$$

$$S_t \le K$$

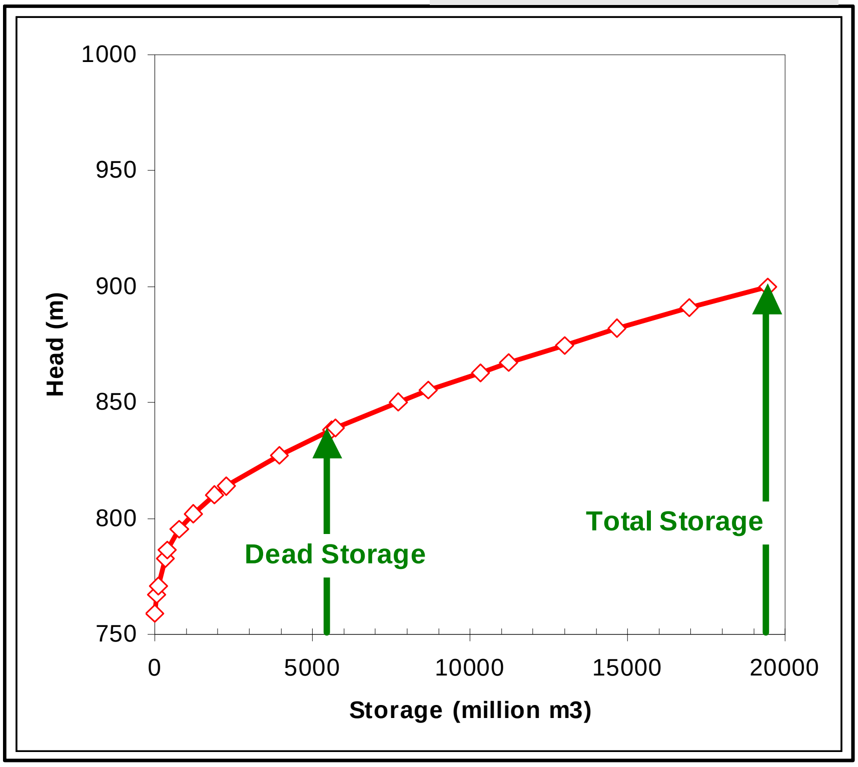
$$\overline{H}_t = \frac{H(S_t) + H(S_{t+1})}{2}$$

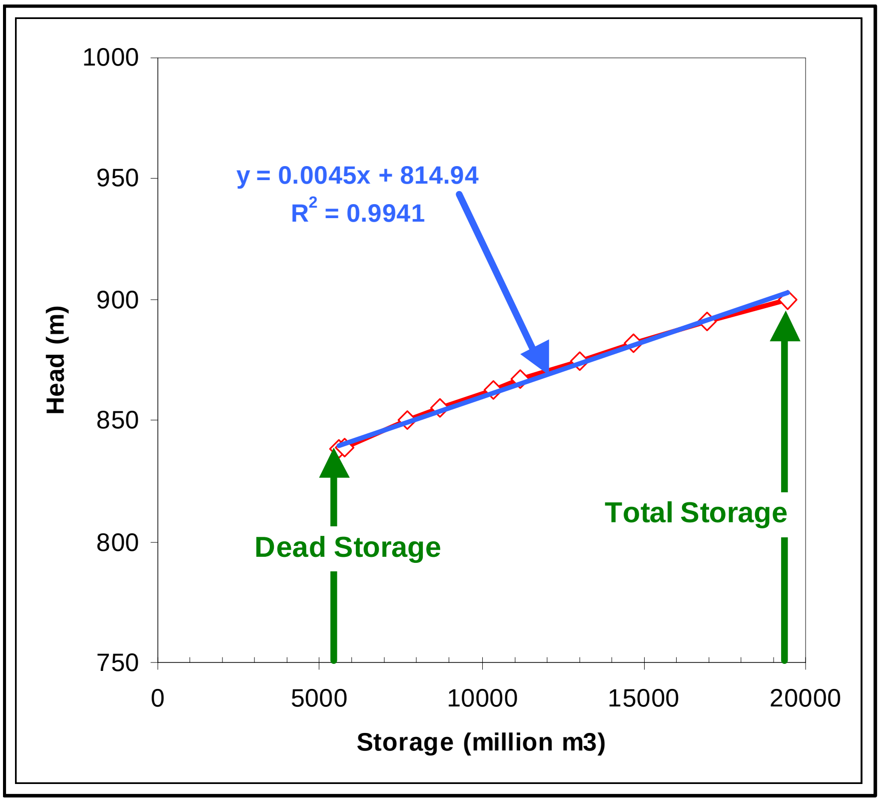
$$E_t = k\overline{H}_t R_t$$

$$t = 1, \ldots, T$$

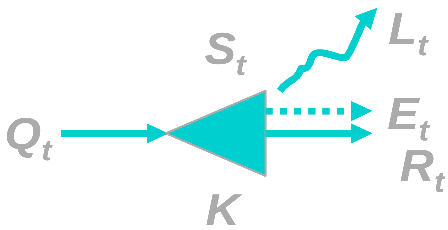
Head vs Storage Relation

$$\overline{H}_t = \frac{H(S_t) + H(S_{t+1})}{2}$$





Model



Minimize $$\sum_{t=1}^{T}\left(R_t - D_t\right)^2$$

subject to

$$(1+a_t)S_{t+1} = (1-a_t)S_t + Q_t - R_t - b_t$$

$$S_t \le K \qquad t = 1,\ldots,T$$

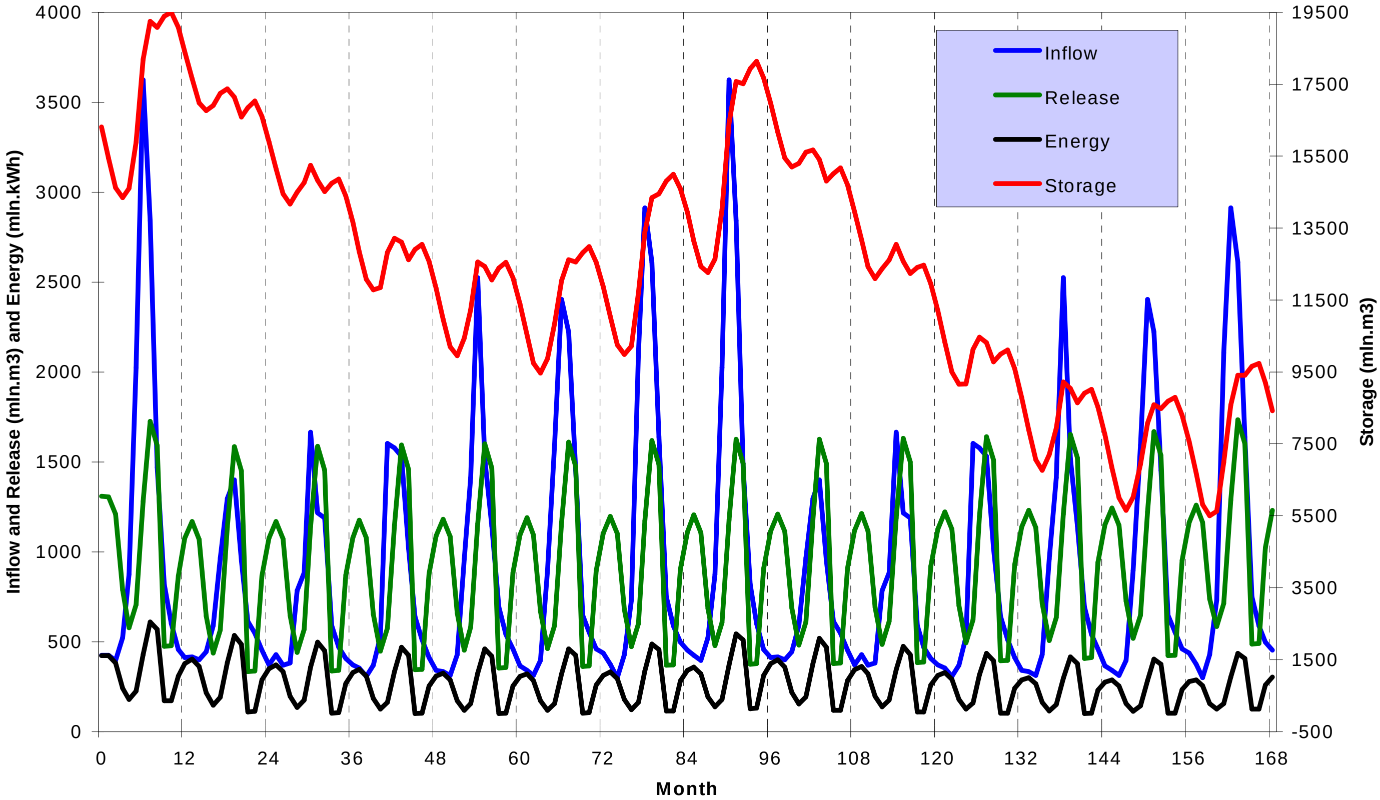
$$H_t = H_o + cS_t$$

$$\overline{H}_t = \frac{\left(H_t - H_o\right) + \left(H_{t+1} - H_o\right)}{2}$$

$$q_t = R_t * 10^6 / time_t \le q_{\max}$$

$$P_t = (9.81)\varepsilon \overline{H}_t q_t \le P_{\max}$$

$$E_t = P_t * time(t) / 3600$$



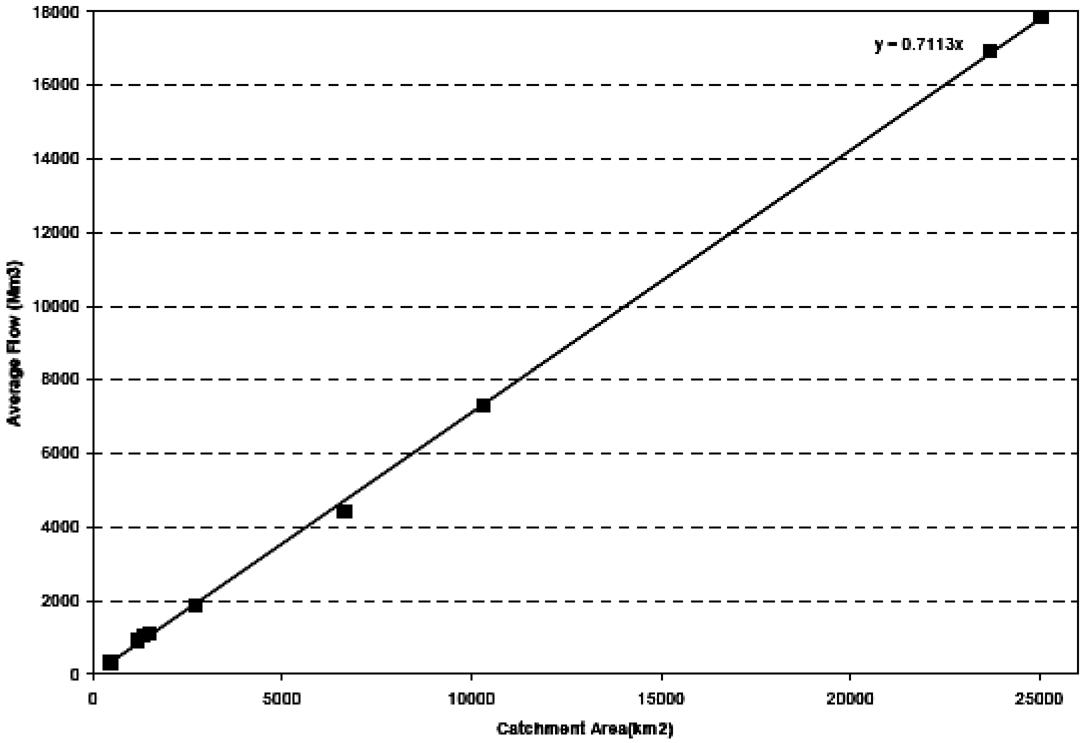
Example1: Using the following information pertaining to the drainage area and discharge in the Han River in South Korea, verify the equation

$$Q_t^s = \sum_g w_g \frac{Q_t^g}{A_g} A_s$$

for predicting the unregulated flow at any site in the river, by plotting the average flow as a function of catchments area. What does the slope of the line equal?

Gage Point	Catchment Area (km2)	Average Flow (million m3/yr)
First Bridge	25047	17860
Pal Dang	23713	16916
So Yang	2703	1856
Chung Ju	6648	4428
Yo Ju	10319	7300
Hong Chun	1473	1094
Dal Chung	1348	1058
Kan Yun	1180	926
Im Jae	461	316

ANSWER--A plot of the average flow Q^s as a function of the catchments area A^s upstream of each site *s* shows an essentially constant slope. This indicates that the flow runoff per unit catchments area is approximately constant throughout the watershed, as assumed by the equation.



Example2: Compute the storage-yield function for a single reservoir by the sequent peak method given the following sequence of inflows: (7,3,5,1,2,5,6,3,4). Next assume that each year has two distinct hydrologic seasons, one wet and one dry, and that 80% of the inflow occurs in the wet season and 80% of the yield is desired in the dry season. Using the sequent peak method, show the increase in storage capacity required for the same annual yield resulting from the within-year

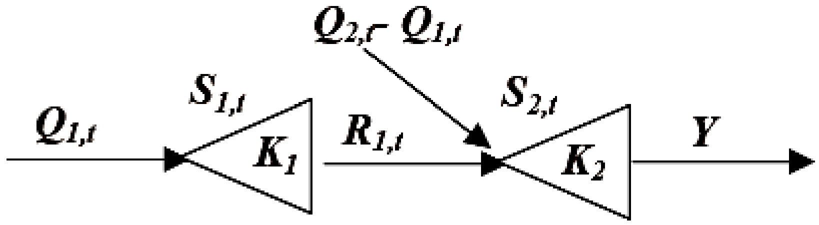
redistribution requirements.

ANSWER

Single Season					Dual Season				
t	R	Q	K-1	K	t	R	Q	K-1	K
1	3.5	7	0	0	11	0.7	5.6	0	0
2	3.5	3	0.5	0	12	2.8	1.4	0	1.4
3	3.5	5	0	0.5	21	0.7	2.4	1.4	0
4	3.5	1	2.5	0	22	2.8	0.6	0	2.2
5	3.5	2	4	2.5	31	0.7	4.0	2.2	0
6	3.5	5	2.5	4	32	2.8	1.0	0	1.8
7	3.5	6	0	2.5	41	0.7	0.8	1.8	1.7
8	3.5	3	0.5	0	42	2.8	0.2	1.7	4.3
9	3.5	4	0	0.5	51	0.7	1.6	4.3	3.4
					52	2.8	0.4	3.4	5.8
					61	0.7	4.0	5.8	2.5
					62	2.8	1.0	2.5	4.3
					71	0.7	4.8	4.3	0.2
					72	2.8	1.2	0.2	1.8
					81	0.7	2.4	1.8	0.1
					82	2.8	0.6	0.1	2.3
					91	0.7	3.2	2.3	0
					92	2.8	0.8	0	2
Capacity	K=			4					5.8

Single Season		Dual Season			Increase
Yield	Capacity	Yield	Capacity	Yields	
1	0	1	0.6	0.2 0.8	0.6
2	1	2	1.6	0.4 1.6	0.6
3	3	3	4.4	0.6 2.4	1.4
3.5	4	3.5	5.8	0.7 2.8	1.8
4	5	4	7.2	0.8 3.2	2.2

Example3: Construct an optimization model for estimating the least-cost combination of active storage capacities, $K1$ and $K2$, of two reservoirs located on a single stream, used to produce a constant flow or yield downstream of the two reservoirs. Assume that the cost functions $Cs(Ks)$ at each reservoir site s are known and there is no dead storage and no evaporation. Assume that 10 years of monthly unregulated flows are available at site s. The system diagram is as shown in Figure



where $Q1,t$ and $Q2,t$ are the unregulated streamflows at sites 1 and 2, respectively, $Q2,t$ - $Q1,t$ is the incremental inflow between site 1 and site 2, $R1,t$ is the release from the upstream reservoir, and Y is the constant yield from the downstream reservoir, $S1,t$., and $S1,t$ are the reservoir storages, and $K1$ and $K2$ are the reservoir capacities.

The model can be formulated as:

$$\textit{Minimize } C_1(K_1) + C_2(K_2)$$

subject to

$$S^1_{t,y} + Q^1_{t,y} - R^1_{t,y} = S^1_{t+1,y} \qquad \forall t, y$$

$$S^1_{t,y} \le K_1 \qquad \forall t, y$$

$$S^2_{t,y} + (Q^2_{t,y} - Q^1_{t,y}) + R^1_{t,y} - Y = S^2_{t+1,y} \qquad \forall t, y$$

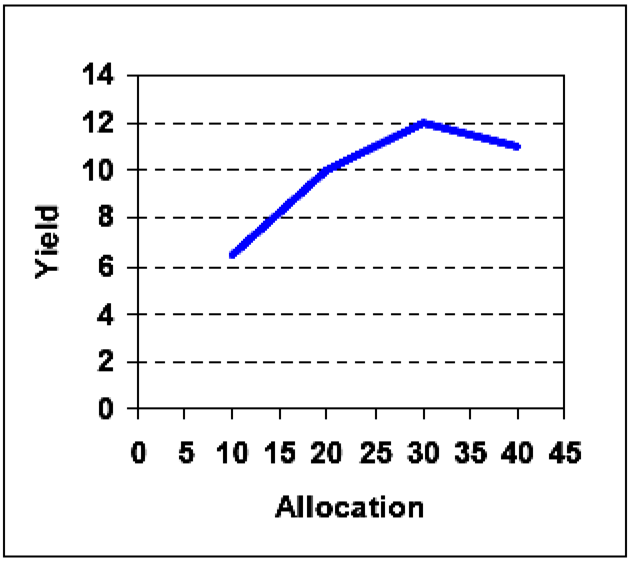
$$S^2_{t,y} \le K_2 \qquad \forall t, y$$

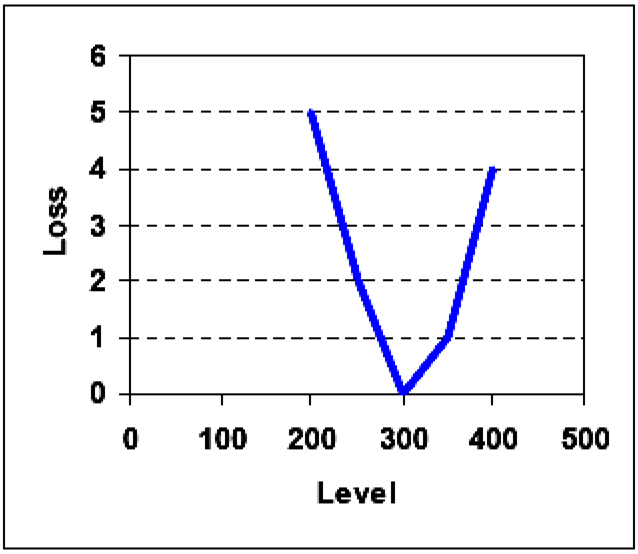
The constant yield Y is fixed and all the inflows are known. When $t = 12$, $t+1 = 1$ and y is $y+1$. If $y = 10$, $y+1 = 1$.

Example4: Plot the following data on possible recreation losses and irrigated agricultural yields. Show that use of the expected storage level or expected allocation underestimates the expected value of reservoir losses while it overestimates the expected value of crop yield.

Irrigation Water Allocation	Crop Yield/Hectare	Probability of Allocation
10	6.5	0.20
20	10.0	0.30
30	12.0	0.30
40	11.0	0.20

Summer storage level	Decrease in recreation benefits	Probability of storage level
200	5	0.10
250	2	0.20
300	0	0.40
350	1	0.20
400	4	0.10





Expected Allocation = ABar = 25; Yield(ABar) = 11
Expected Yield = 10.1 < 11

Expected Level = LBar = 300; Loss(LBar) = 0
Expected Loss = 1.5 > 0