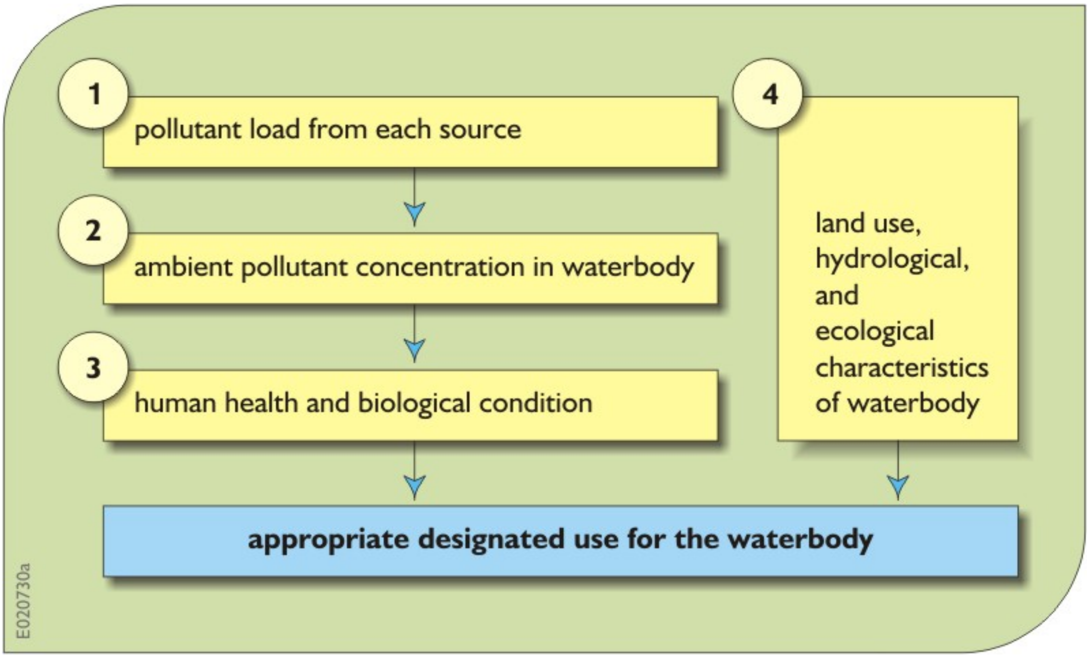
Water Quality Management

- Critical component of overall IWRM
- Water bodies serve many uses
  - transport and assimilation of wastes
  - assimilative capacities can be exceeded WRT intended uses
- Water quality management measures
  - Standards
    - minimum acceptable levels of ambient water quality
  - Actions
    - insure pollutant load does not exceed assimilative capacity while maintaining quality standards
  - Treatment
    - not an option for irrigation water or maintaining natural aquatic ecosystems (bathing, fish and other organisms)

Water Quality Management Process

- Identify Problem
- Identify Indicators and Target Values
- Assess Source(s)
- Determine Linkages Between Targets and Sources
- Allocate Loads
- Monitor and Evaluate
- Implement



Water Uses

Use	Typical quality parameters
Public Water Supply	Turbidity, total dissolved solids, health-related inorganiz and organic compounds, microbial quality
Water contact recreation	Turbidity, bacterial quality, toxic compounds
Fish propagation and wildlife	Dissolved oxygen, chlorinated organic compounds
Industrial water supply	Suspended and dissolved constituents
Agricultural water supply	Sodium content, total dissolved solids
Shellfish harvesting	Dissolved oxygen, bacterial quality

Irrigation Use

Total dissolved Solids

Classification	TDS (mg/L)	Elec Cond. (micromhos/cm)
No adverse effects	< 500	< 750
Adverse effect on sensitive crops	500 – 1000	750 – 1,500
Adverse effects on most crops	1,000-2,000	1,500-7,000
No effect on tolerant crops	2,000-5,000	3,000-7,000
Little value for irrigation	> 5000	

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Municipal Wastewater

	Before		After 2ndary
Variable	Average	Range	Average
Daily flow (Gal/cap)	125	100-200	
Solids (mg/L)	800	450-1200	530
Volatile	400	250-800	220
Disolved	500	300-800	20
Suspended	300	100-400	30
BOD5 (mg/L)	180	100-450	30
Nitrogen (mg/L)	50	15-100	
Organic N	20	5-35	
Ammonia N	28	10-60	
Nitrite+Nitrate	2	0-6	
Phosphate (mg/L)	50	10-50	
Coliform (mln/100 ml)	30	2-50	
Fecal Coliforms	4	0.3-17	

Fish and Aquatic Life

Characteristic	Warm Water Biota	Cold Water Biota	Marine and Estuary
pH	6-9	6-9	6.7-8.5
Alkalinity	<20 mg/L	< 20 mg/L	
Incr. in T (ºF)	< 5	< 5	< 1.5
Turbidity	50	10	
Change in Salinity			10%
Coliform			< 70/ 100 ml

Models

- Advection + dispersion

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(D\frac{\partial C}{\partial x}) - \frac{\partial VC}{\partial x}$$

- Where

C = concentration (M/L^3)
V = Average velocity in reach (L/T)
D = Longitudinal dispersion (L2/T)

Steady-state Model

- Steady-state
  - Where k = decay rate (1/T)

$$0 = D\frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x} - kC$$

  - *Solution is*

$$C(x) = \begin{cases} \frac{W}{Qm} e^{\frac{V}{2D}(1+m)x} & x < 0 \ (backward) \\ \frac{W}{Qm} e^{\frac{V}{2D}(1-m)x} & x \geq 0 \ (forward) \end{cases}$$

where

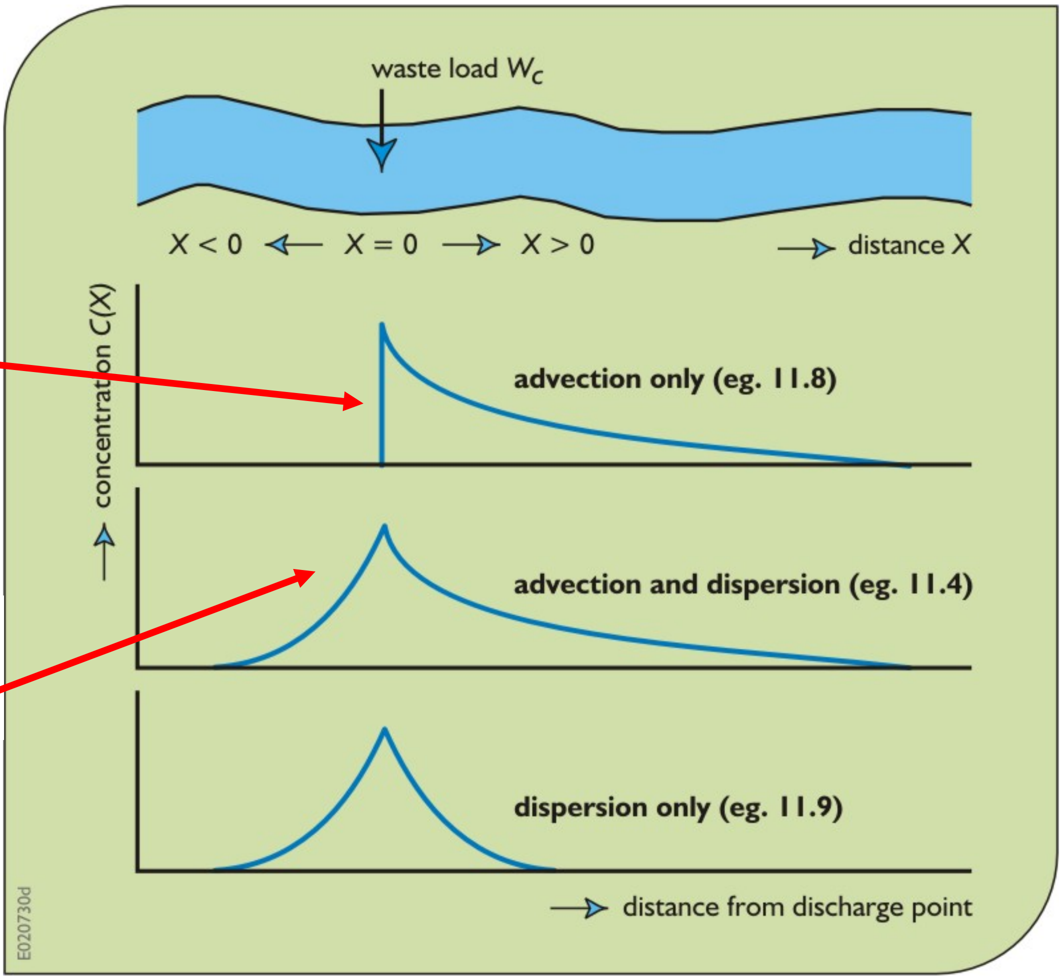
  - W = loading (M/T) at x = 0

$$C(0) = \frac{W}{Qm} \qquad m = \sqrt{1 + \frac{4kD}{V^2}}$$

Steady-state w/o Dispersion

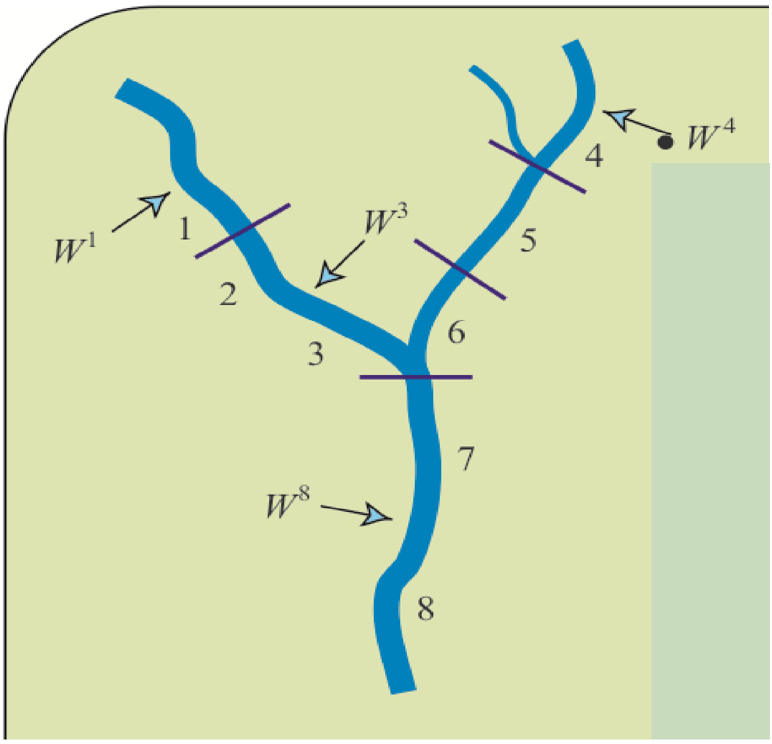
$$C(x) = \frac{W}{Q} e^{\frac{-kx}{V}}$$

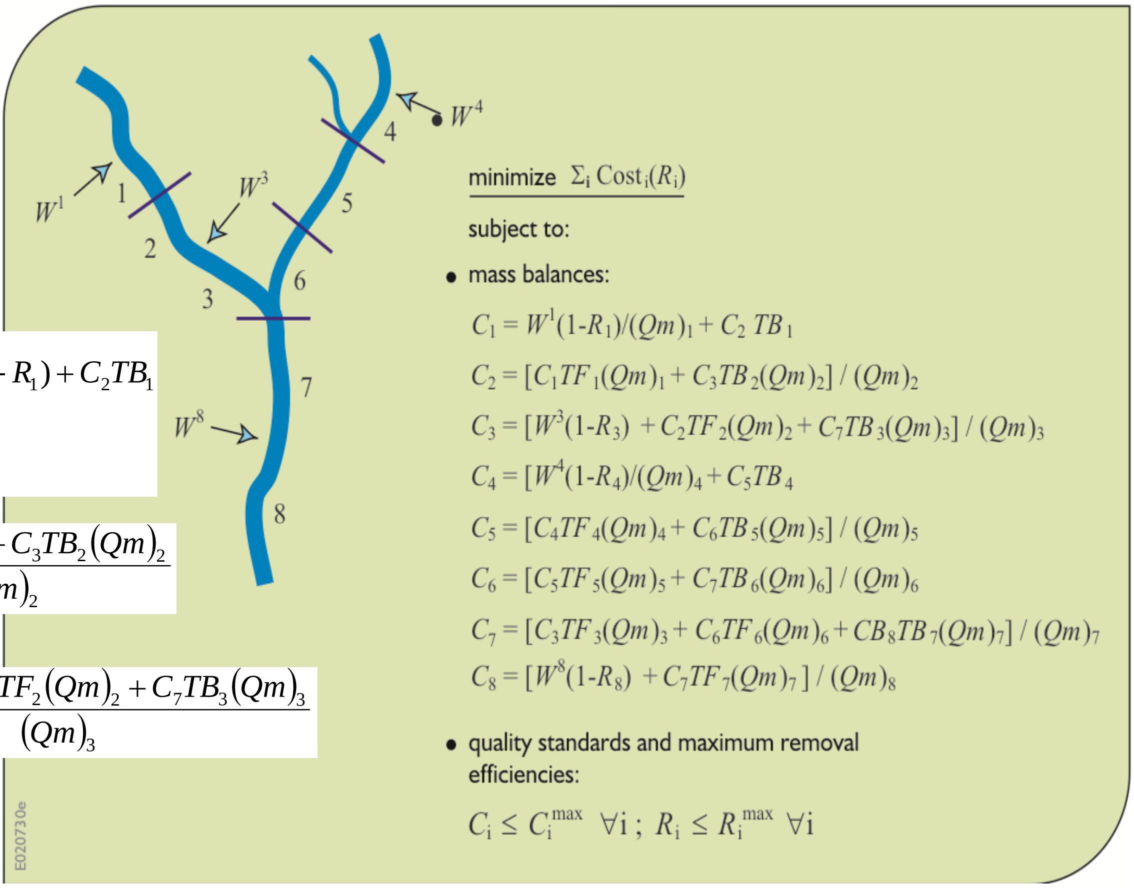
$$C(x) = \frac{W}{Qm} e^{\frac{V}{2D}(1 \pm m)x}$$



Example1:

- Wastewater containing constituent *C* is being discharged at four sites
- maximum allowable concentrations of the constituent *C* are specified at each of those discharge sites
- estimate the necessary reduction in these discharges
- Divide the river into reaches
- Each reach: constant *A*, *D*, *k*, and *v*



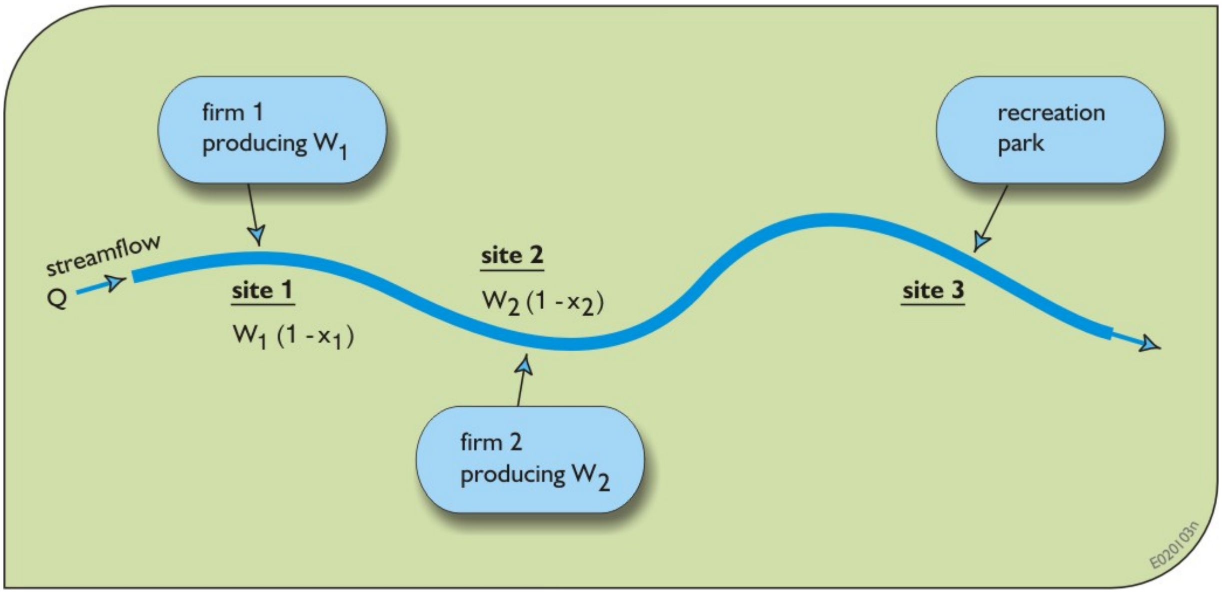


$$C_1 = \frac{W_1}{(Qm)_1}(1-R_1) + C_2 TB_1$$

$$TB_1 = e^{\frac{V}{2D}(1+m)x}$$

$$C_2 = \frac{C_1TF_1(Qm)_1 + C_3TB_2(Qm)_2}{(Qm)_2}$$

$$C_3 = \frac{W_3(1-R_3) + C_2TF_2(Qm)_2 + C_7TB_3(Qm)_3}{(Qm)_3}$$



- Find the waste removal efficiencies ($x1$, $x2$) that meet the stream quality standards at sites 2 and 3 at minimum total cost. $W1$ and $W2$ are the amounts of pollutant prior to treatment at sites 1 and 2.
- Without treatment, the concentration of pollutant, Pj mg/l, at sites j = 2 and 3 will continue to exceed the maximum desired concentration Pjmax

- *P*j = pollutant concentration in the stream at site *j*
- Total mass per unit time of the pollutant (M/T) in the stream at site j will be its concentration *P*j (M/L3) times the streamflow *Q*j (L3/T)
- Each unit of pollutant mass at site 1 will decrease as it travels downstream to site 2. Similarly each unit of pollutant mass at site 2 will decrease as it travels downstream to site 3.
- The fraction *a*12 of mass at site 1 that reaches site 2 is assumed to be:

$$a_{1,2} = \frac{1}{mQ} e^{\frac{V}{2E}(x_2 - x_1)(1-m)}$$

- Similarly for mass at site 2 traveling to site 3

$$a_{2,3} = \frac{1}{mQ} e^{\frac{V}{2E}(x_3 - x_2)(1-m)} \qquad a_{1,3} = a_{1,2} a_{2,3}$$

Determine Transfer Coefficients

- To determine the transfer coefficients *a*12 and *a*23 requires concentration measurements at sites 1, 2 and 3 during design streamflow conditions
- Concentrations must be measured just downstream of the site 1 discharge, just upstream and downstream of the site 2 discharge , and at site 3

$$P_2^- = a_{1,2} P_1^+ \qquad P_3^- = a_{2,3} P_2^+$$

- Assume five pairs of sample pollutant concentration measurements have been taken in the two stream reaches during design flow conditions
- assume that the design streamflow just downstream of site 1 and just upstream of site 2 are the same and equal to 12 m3/s
- SP1s = sample measurement s in the 1st reach (downstream of site 1)
- SP2s = sample measurement s in the 2nd reach (upstream of site 2)
- Es = error

$$SP_{2,s}^- + E_s = a_{1,2} SP_{1,s}^+ \left(\frac{Q_1}{Q_2} \right)$$

$$\text{Minimize } \sum_{s} |E_s|$$

$$\text{Subject to}$$

$$E_s = a_{1,2} SP_{1,s}^{+} \left(\frac{Q_1}{Q_2} \right) - SP_{2,s}^{-} \qquad \forall s$$

- If the absolute value signs can be removed, we will have an LP model
- Write each error term as the difference between two non-negative variables

$$E_s = PE_s - NE_s$$
$$PE_s \geq 0, \text{and} NE_s \geq 0$$

$$\text{If } E_s \begin{cases} < 0 & PE_s = 0, \text{and} - NE_s = E_s \\ > 0 & PE_s = E_s, \text{and} NE_s = 0 \end{cases}$$

- Objective function - minimize the sum of the positive and negative components of *Es*

$$\text{Minimize } \sum_{s} \left(PE_s + NE_s \right)$$

$$\text{Subject to}$$

$$PE_s - NE_s = a_{1,2} SP_{1,s}^{+} \left(\frac{Q_1}{Q_2} \right) - SP_{2,s}^{-} \qquad \forall s$$

$$PE_s \geq 0, \text{and } NE_s \geq 0 \qquad \forall s$$

GAMS Code for Example

```
Scalar Q1 /12/;
Scalar Q2 /12/;

SETS s /1*5/;

Parameter SP1(s) /1 232, 2 256, 3 220, 4 192, 5 204/;
Parameter SP2(s) /1 55, 2 67, 3 53, 4 50, 5 51/;

POSITIVE VARIABLES
PE(s), NE(s);

VARIABLES
obj, a12;

EQUATIONS  objective, error(s);
objective.. obj =E= SUM(s,( PE(s)+NE(s) ) );
error(s)..  PE(s)-NE(s) =E= a12*SP1(s)*(Q1/Q2)-SP2(s);

MODEL Calibrate / ALL /;
SOLVE Calibrate USING LP MINIMIZING obj;

FILE res /Calibrate.txt/;
PUT res
PUT "obj = ", PUT obj.l,PUT /;
PUT "a12 = ", PUT a12.l,PUT /;
*
```

$a_{12} = 0.25$, $a_{23} = 0.60$, and
$a_{12}\, a_{23} = a_{13} = 0.15$

Minimize $C_1(x_1) + C_2(x_2)$

Subject to

$$P_2 = \frac{[P_1 Q_1 + W_1(1 - x_1)]a_{12}}{Q_2}$$

$$P_3 = \frac{[P_2 Q_2 + W_2(1 - x_2)]a_{23}}{Q_3}$$

$$P_j \le P_j^{\max} \qquad j = 2, 3$$

$$0 \le x_i \le 1 \qquad i = 1, 2$$

Minimize $C_1(x_1) + C_2(x_2)$

Subject to

$$\frac{[P_1 Q_1 + W_1(1 - x_1)]a_{12}}{Q_2} \le P_2^{\max}$$

$$\frac{[P_1 Q_1 + W_1(1 - x_1)]a_{13} + [W_2(1 - x_2)]a_{23}}{Q_3} \le P_3^{\max}$$

$$0 \le x_i \le 1 \qquad i = 1, 2$$

	parameter	unit	value	remark
flow	Q_1	m^3/s	10	flow just upstream of site 1
	Q_2	m^3/s	12	flow just upstream of site 2
	Q_3	m^3/s	13	flow at park
waste	W_1	kg/day	250,000	pollutant mass produced at site 1
	W_2	kg/day	80,000	pollutant mass produced at site 2
pollutant conc.	P_1	mg/l	32	concentration just upstream of site 1
	P_2	mg/l	20	maximum allowable concentration upstream of 2
	P_3	mg/l	20	maximum allowable concentration at site 3
decay fraction	a_{12}	--	0.25	fraction of site 1 pollutant mass at site 2
	a_{13}	--	0.15	fraction of site 1 pollutant mass at site 3
	a_{23}	--	0.60	fraction of site 2 pollutant mass at site 2

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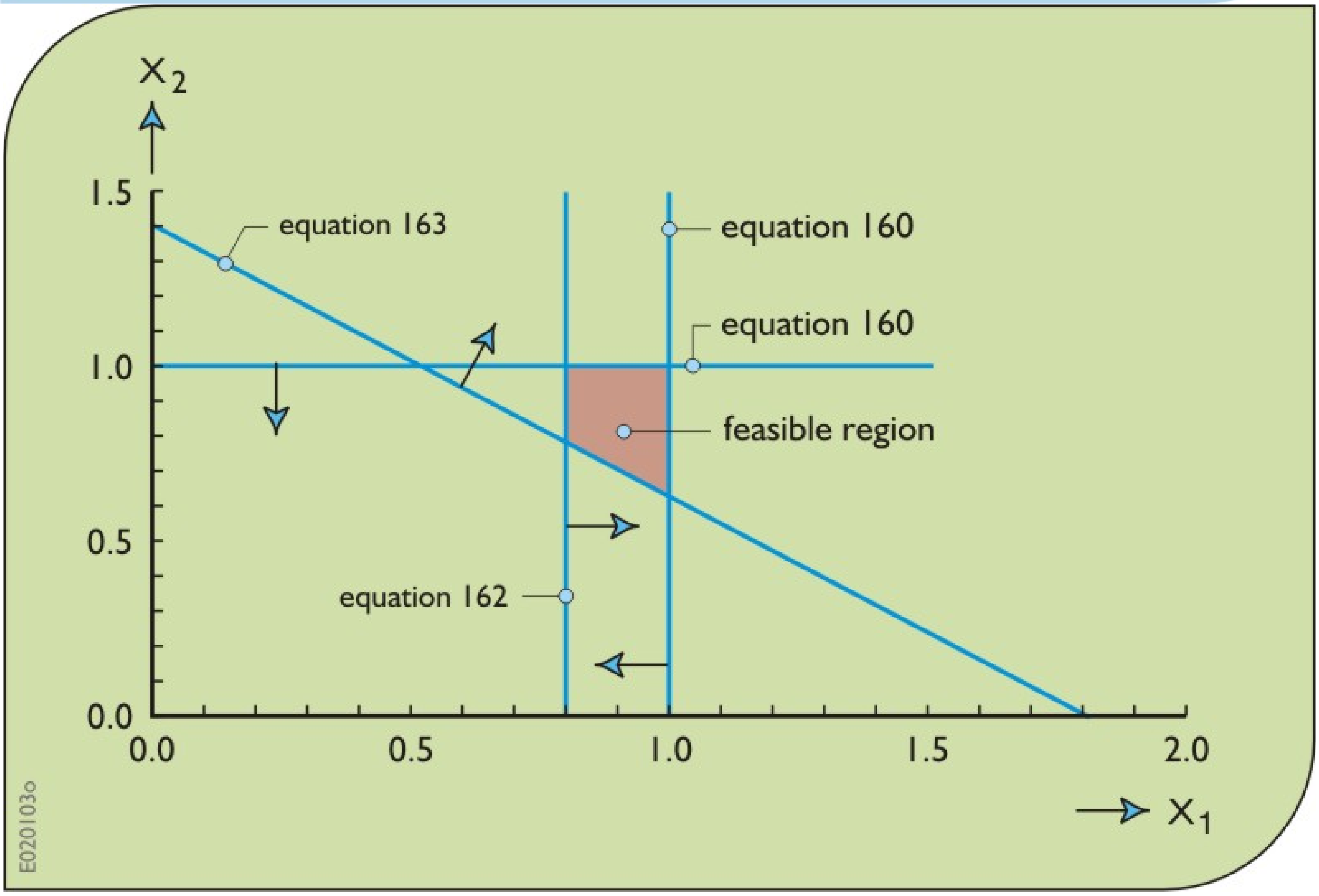


Minimize $C_1(x_1) + C_2(x_2)$

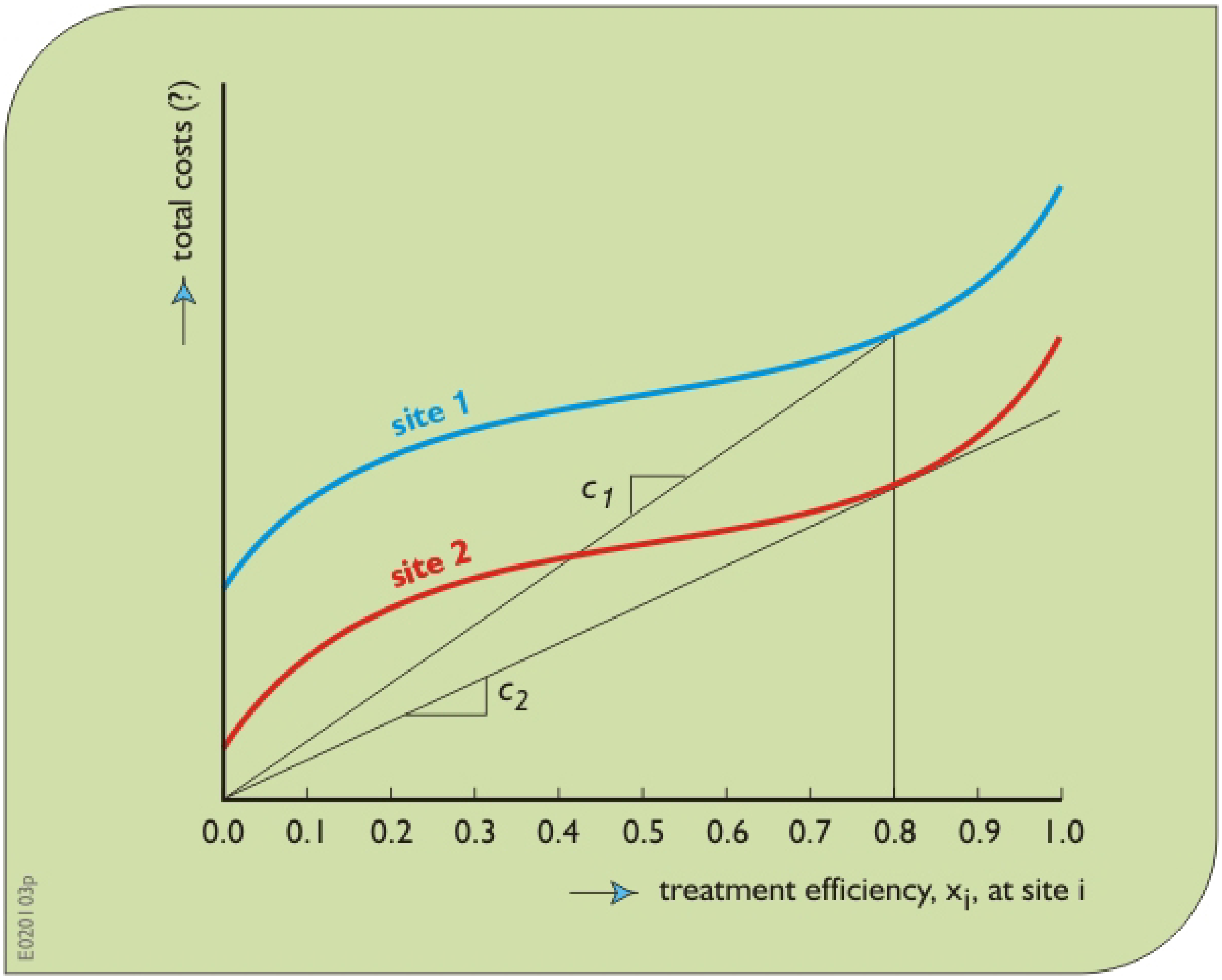
Subject to

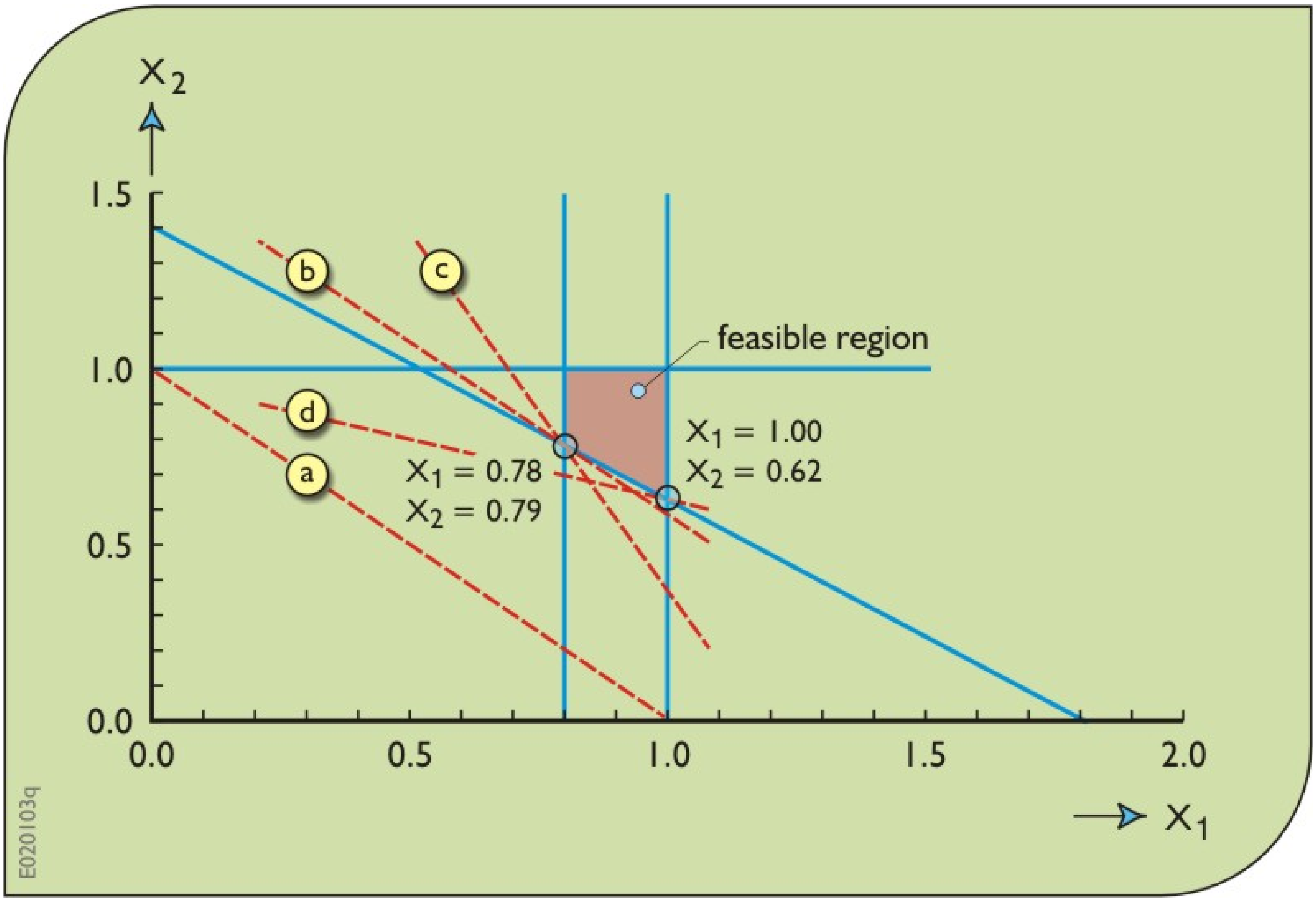
$$x_1 \geq 0.8$$

$$x_1 + 1.28 x_2 \geq 1.79$$

$$0.0 \leq x_i \leq 1.0 \quad i = 1, 2$$

$c1$ and $c2$ are the average cost per unit (percent) removal for 80% treatment





Example2:

- ***Irrigation project***
  - ***1800 acre-feet of water per year***
  - ***Crop A, needs 3 AF/acre, $300/acre, 400 acres max***
  - ***Crop B, needs 2 AF/acre, $500/acre, 600 acres max***
- ***Decision variables***
  - ***x_A = acres of Crop A to plant***
  - ***x_B = acres of Crop B to plant***

$$\text{Maximize} \quad Z = 300\,x_A + 500\,x_B$$

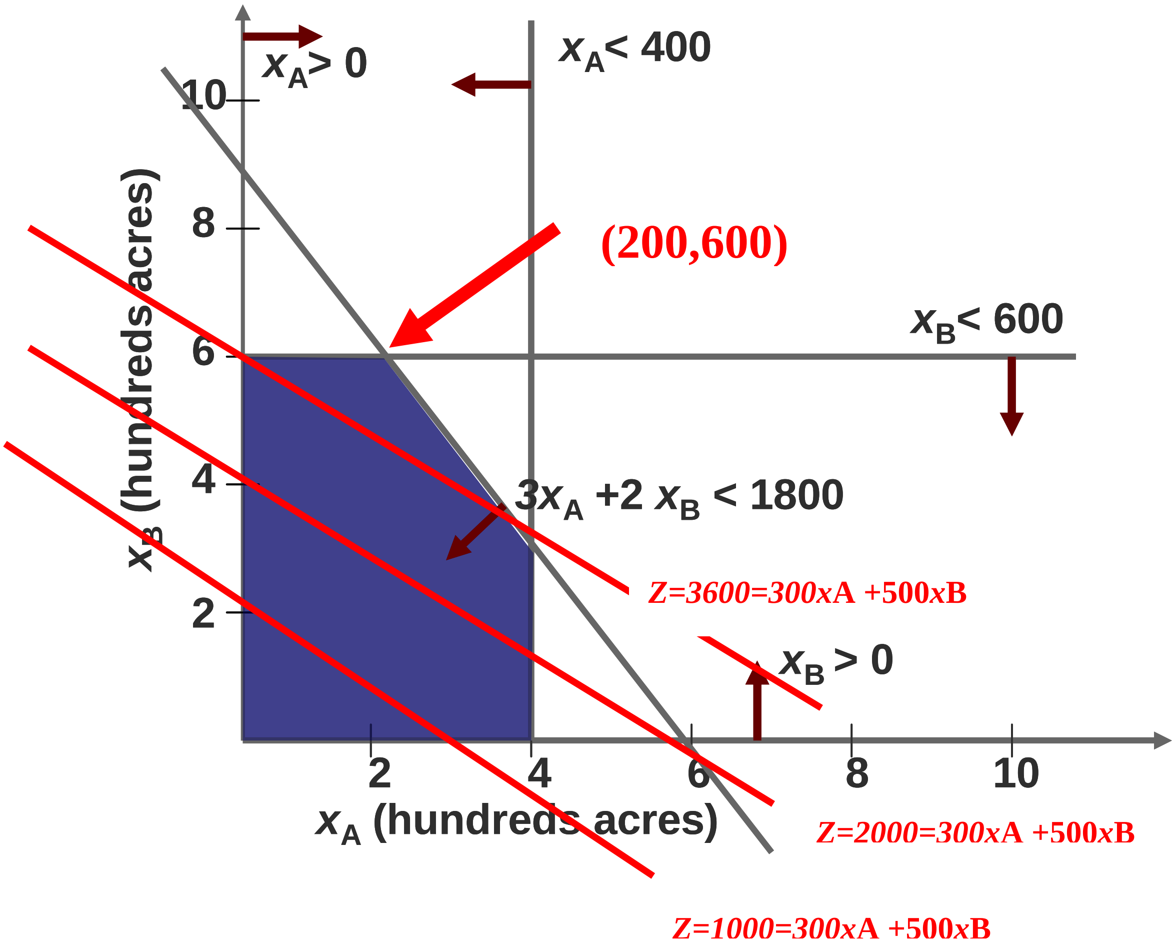
$$\text{Subject to}$$

$$x_A \le 400$$

$$x_B \le 600$$

$$3x_A + 2x_B \le 1800$$

$$x_A \ge 0 \quad x_B \ge 0$$



GAMS Code

```
POSITIVE VARIABLES
xA, xB;

VARIABLES
obj;

EQUATIONS  objective, xAup, xBup, limit;

objective.. obj =E= 300*xA+500*xB;
xAup.. xA =L= 400.;
xBup.. xB =L= 600.;
limit..  3*xA+2*xB =L= 1800;


MODEL Calibrate / ALL /;
SOLVE Calibrate USING LP MAXIMIZING obj;

Display xA.l, xA.m;
Display xB.l, xB.m;
```

GAMS Output

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU objective	.	.	.	1.000
---- EQU xAup	-INF	200.000	400.000	.
---- EQU xBup	-INF	600.000	600.000	300.000
---- EQU limit	-INF	1800.000	1800.000	100.000

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR xA	.	200.000	+INF	.
---- VAR xB	.	600.000	+INF	.
---- VAR obj	-INF	3.6000E+5	+INF	.

Marginals

- Marginal value, Dual price, Shadow price, Lagrange multiplier
  - Marginal for a constraint = the change in the objective per unit increase in RHS of that constraint.
  - i.e., change *xB* $x_B \le 600$ $x_B \le 601$
  - Marginal for that equation is 300, so expect objective to increase to 360,300

New Solution

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU objective	.	.	.	1.000
---- EQU xAup	-INF	199.333	400.000	.
---- EQU xBup	-INF	601.000	601.000	300.000
---- EQU limit	-INF	1800.000	1800.000	100.000

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR xA	.	199.333	+INF	.
---- VAR xB	.	601.000	+INF	.
---- VAR obj	-INF	3.6030E+5	+INF	.

Note: Adding 1 unit to *xB* adds 300 to the objective, but constraint 3 says

$$3x_A + 2x_B \le 1800$$

and this constraint is "tight" (no slack) so it holds as an equality, therefore *xA* must decrease by 1/3 unit for *xB* to increase by a unit.

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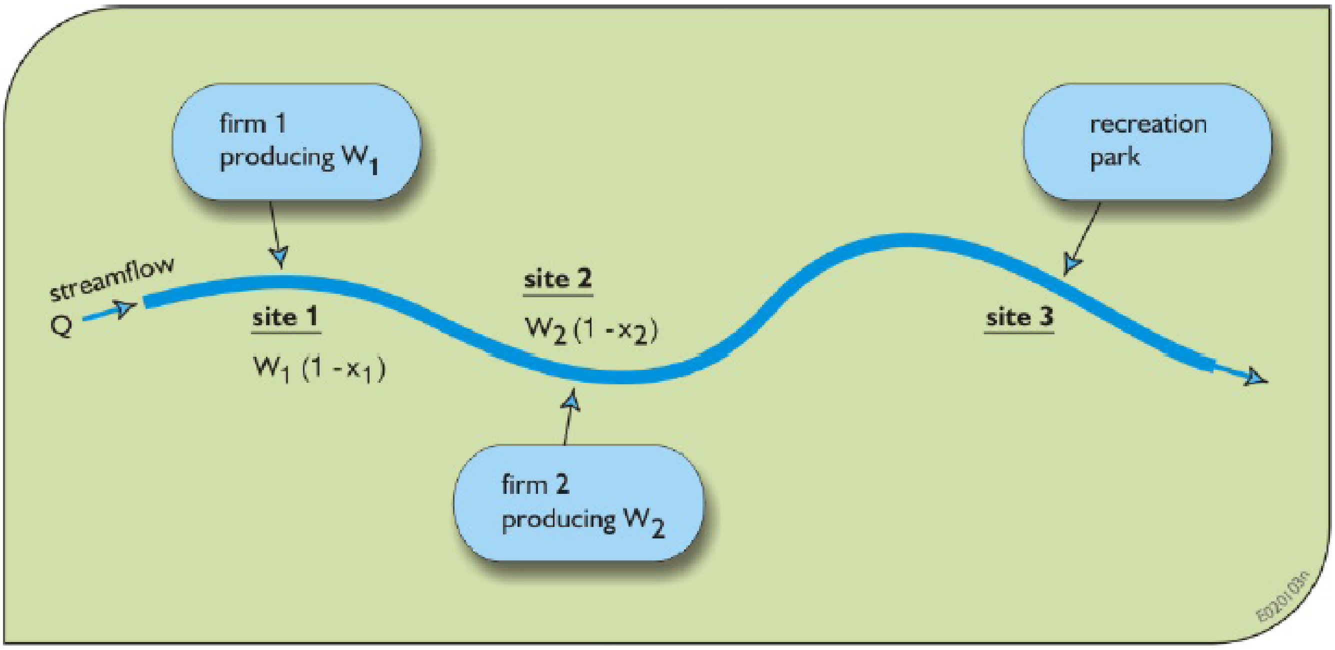
Example3: Assume that here are two sites along a stream, $i = 1, 2$, at which waste (BOD) is discharged. Currently, without any wastewater treatment, the quality (DO), $P2$ and $P3$, at sites 2 and 3 is less than the minimum desired, P_2^{max} and P_3^{max}, respectively. For each unit of waste removed at site i upstream of site j, the quality improves by aij. How much treatment is required at sites 1 and 2 that meets the standards at a minimum cost? Following are the necessary data:



Ci = cost per unit of waste treatment at site i (both $C1$ and $C2$ are unknown but for the same amount of treatment, whatever that amount, $C1 > C2$)

Wi = the amount of waste to be treated at sites 1 and 2

xi = decision variables, unknown waste removal fractions at sites 1 and 2

$$a_{12} = 1/20 \qquad W_1 = 100 \qquad P_2^{max} = 6$$
$$a_{13} = 1/40 \qquad W_2 = 75 \qquad P_3^{max} = 4$$
$$a_{23} = 1/30 \qquad P_2 = 3 \qquad P_3 = 1$$

$$Minimize \sum_{i=1}^{2} C_i x_i$$

subject to

$$q_j - \sum_{i=1}^{2} a_{ij} W_i x_i \geq Q_j \qquad j = 2,3$$
$$0 \leq x_i \leq 1 \qquad i = 1, 2, 3$$

Solution

$$Minimize\ C_1x_1 + C_2x_2$$

subject to

$$3 + \frac{1}{20}(100)x_1 \geq 6 \quad or \quad x_1 \geq 0.6$$

$$3 + \frac{1}{40}(100)x_1 + \frac{1}{30}(75)x_2 \geq 4 \quad or \quad x_1 + x_2 \geq 1.2$$

$$0 \leq x_1 \leq 1$$

$$0 \leq x_2 \leq 1$$