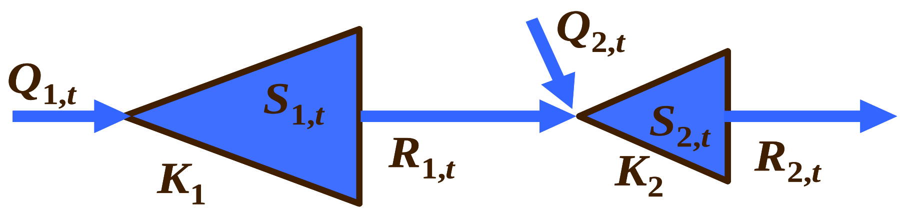
Multipurpose Water Resource Systems

Reservoirs in Series



<u>Objective</u> $$Maximize \sum_{t=1}^{T} \left(B_{1,t} + B_{2,t}\right)$$

Subject to

<u>Continuity</u>

$$S_{1,t+1} = S_{1,t} + Q_{1,t} - R_{1,t} - L_{1,t}$$

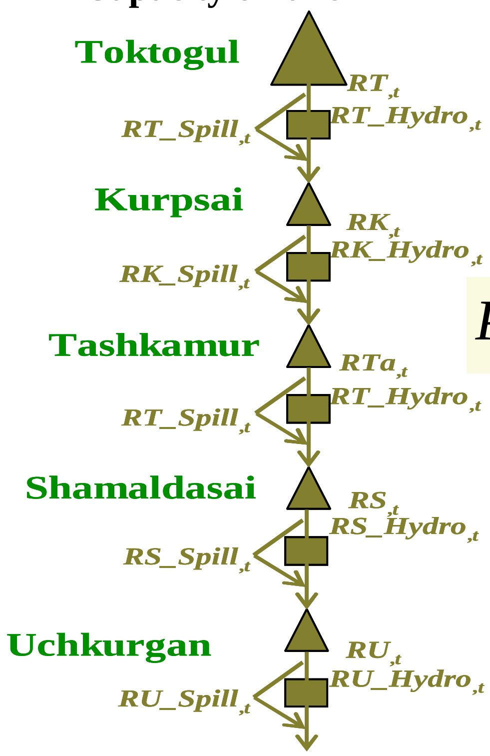
$$S_{2,t+1} = S_{2,t} + Q_{2,t} + R_{1,t} - R_{2,t} - L_{2,t}$$

<u>Capacity</u>

$$S_{1,t} < K_1$$

$$S_{2,t} < K_2$$

t **Time period (month)**
St,l **Storage in lake *i* in period *t***
Qt,l **Inflow to lake *i* in period *t***
Ll **Loss from lake *i* in period *t***
Rt,l **Release from lake *i* in period *t***
Ki **Capacity of lake *i***



$$RT_t = RT\_Hydro_t + RT\_Spill_t$$

$$P_t = (9.81)\varepsilon \overline{H}_t (R_t * 10^6 / time_t) \le P^{Max} \quad \text{kW}$$

$$RT\_Hydro_t \le \frac{P_T^{Max} * 10^6 / time_t}{(9.81)\varepsilon \overline{H} T_t}$$

Example:

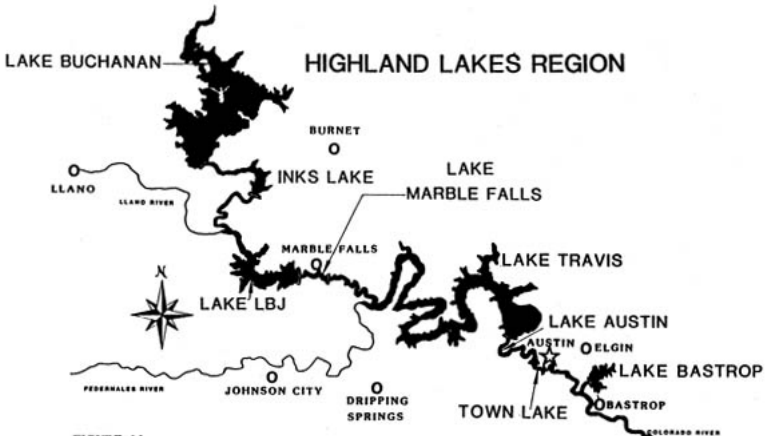
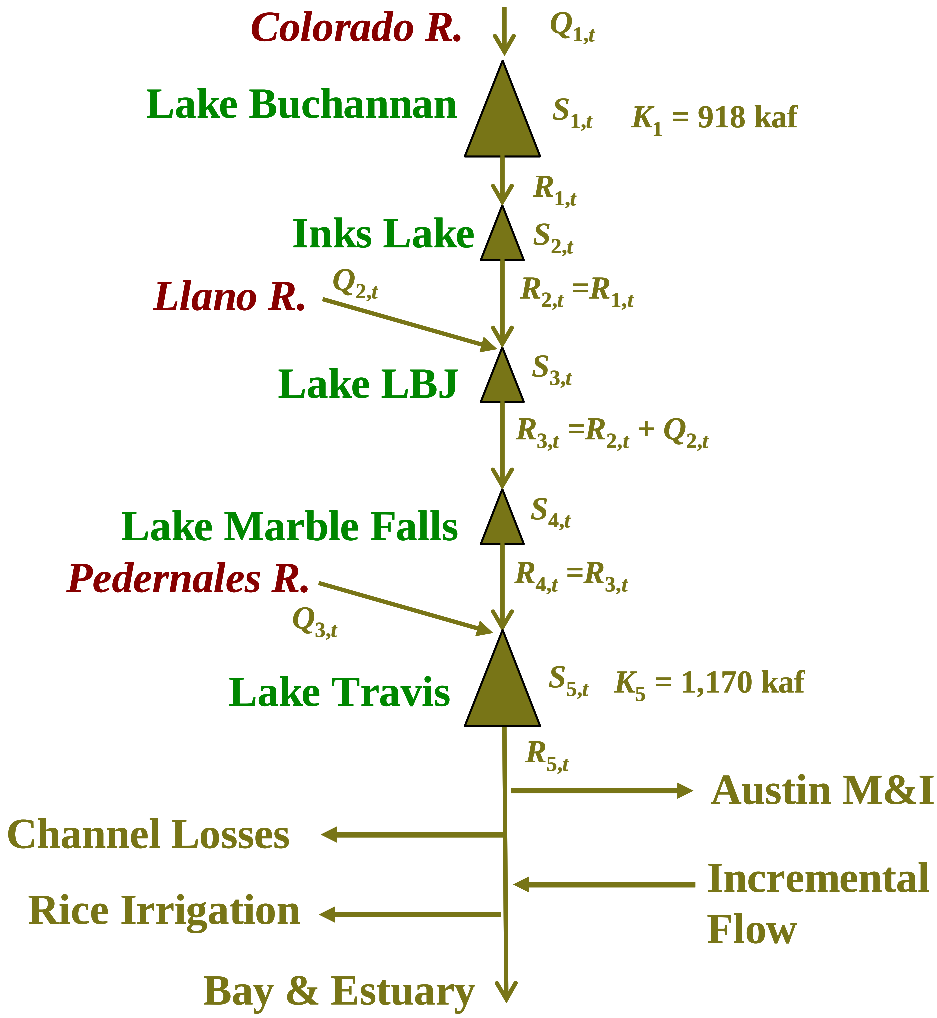


FIGURE 14
HIGHLAND LAKES REGION



Continuity

$$S_{i,t+1} = S_{i,t} + Q_{i,t} + R_{i-1,t} - R_{i,t} - L_{i,t}, \quad \forall i, \forall t$$

- t Time period (month)
- i Lake (1 = Buchannan, 2 = Inks, 3 = LBJ, 4 = Marble Falls, 5 = Travis)
- $S_{t,I}$ Storage in lake i in period t (AF)
- $Q_{t,I}$ Inflow to lake i in period t (AF)
- L_I Loss from lake i in period t (AF)
- $R_{t,I}$ Release from lake i in period t (AF)

Release

$$R_{5,t} + I_t > X_{A,t} + X_{I,t} + CL_t + X_{B,t} \quad \forall t$$

- $R_{5,t}$ Release from Lake Travis in period t (AF)
- $X_{A,t}$ Diversion to Austin (AF/month)
- $X_{I,t}$ Diversion to irrigation (AF/month)
- CL_t Channel losses in period t (AF/month)
- $X_{B,t}$ Bay & Estuary flow requirement (AF/month)

$$X_{A,t} < f_{A,t} T_A \quad \forall t$$
$$X_{I,t} < f_{I,t} T_I \quad \forall t$$

- T_A target for Austin water demand (AF/year)
- T_I target for irrigation water demand (AF/year)
- $f_{A,t}$ monthly Austin water demand (%)
- $f_{I,t}$ monthly irrigation water demand (%)

Capacity

$$S_{i,t} < K_i \quad \forall i \ \forall t$$

- K_i Capacity of lake i

Head vs Storage

$$\overline{H}_{i,t} = \frac{H_i(S_{i,t}) + H_i(S_{i,t+1})}{2}$$

- $H_{i,t}$ elevation of lake i

Energy

$$E_{i,t} = 2370 \varepsilon_i \overline{H}_{i,t} R_{i,t}$$

- $E_{,t}$ Energy (kWh)
- ε_i efficiency (%)

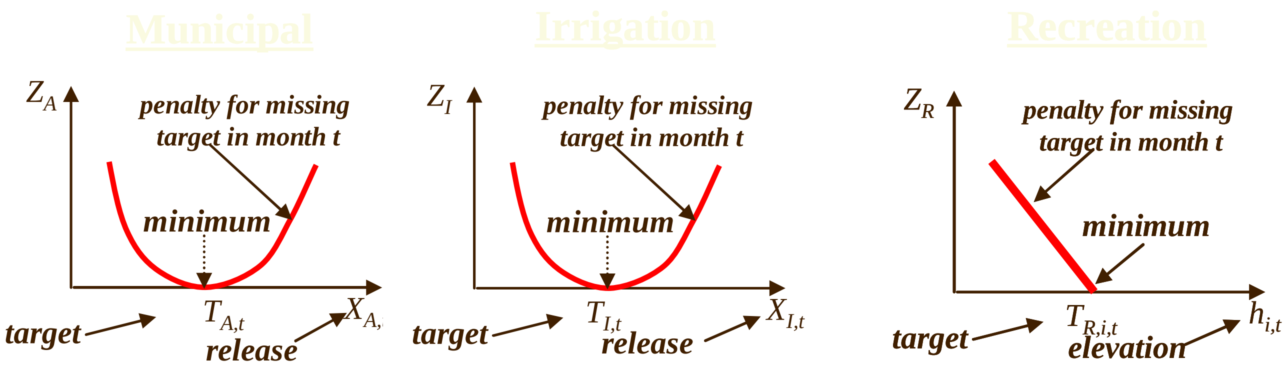
Objective

$$Minimize \sum_{t=1}^{T}\left[w_A\left(T_{A,t}-X_{A,t}\right)^2 + w_I\left(T_{I,t}-X_{I,t}\right)^2 + w_R \sum_{i=1\&5}\left(T_{R,i,t}-h_{i,t}\right)^2 \right]$$

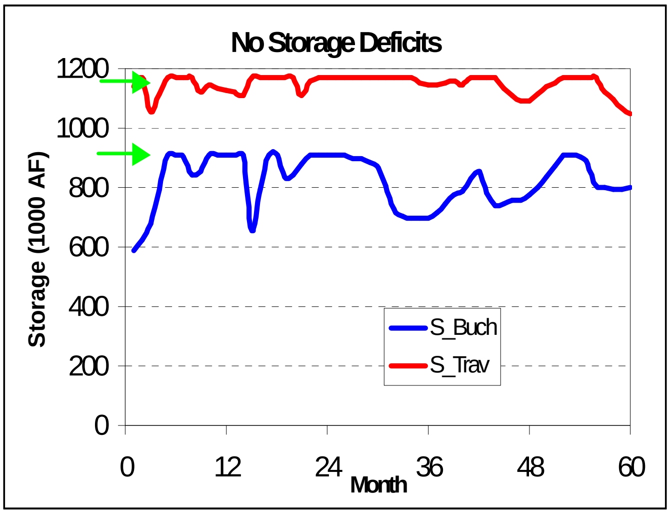
wA weight for Austin demand
wT weight for Austin demand
wR weight for Lake levels
TA,t monthly target for Austin demand
TI,t monthly target for irrigation demand
TR,i,t monthly target for lake levels,
i = Buchanan, Travis

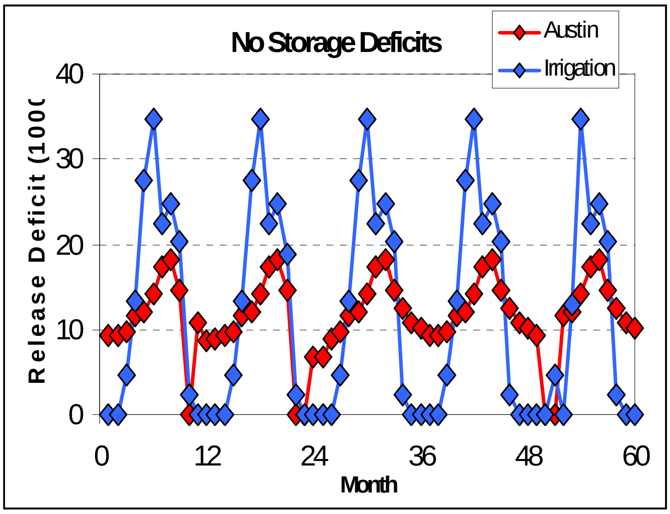
- Municipal Water Supply
  - Benefits: Try to meet targets
- Irrigation Water Supply
  - Benefits: Try to meet targets
- Recreation (Buchanan & Travis)
  - Benefits: Try to meet targets

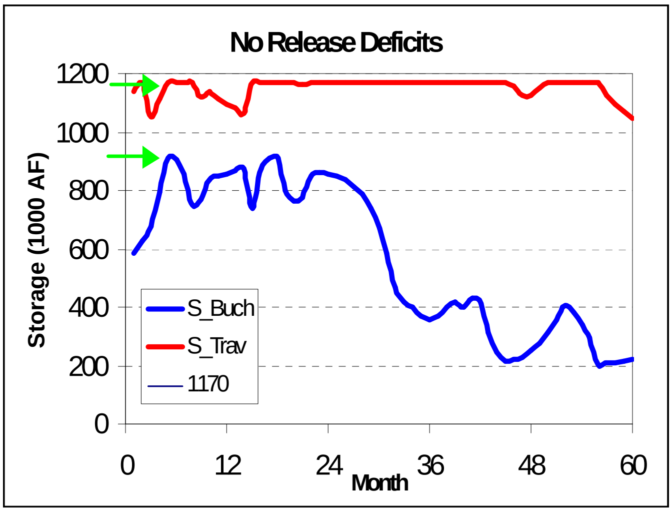
Municipal **Irrigation** **Recreation**

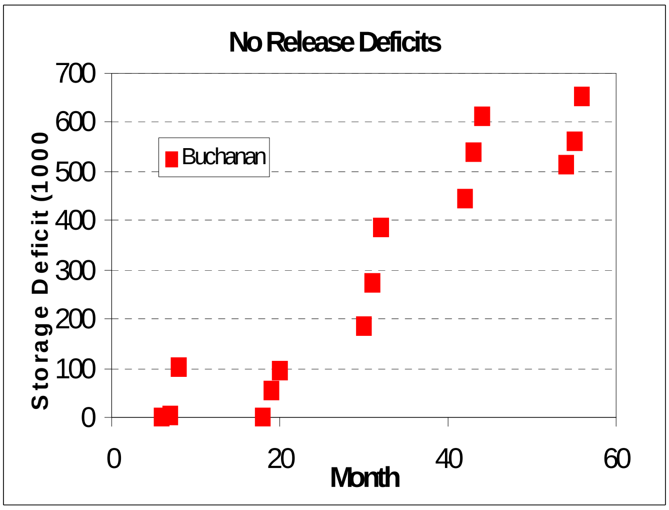


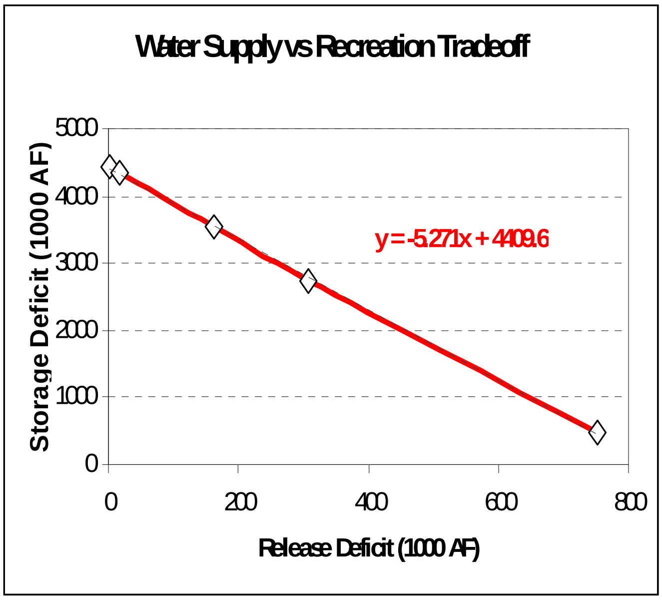
Results
***K*1 = 918 kaf**
***K*5 = 1,170 kaf**











Weights	Release	Storage
100-100-1	1.61	4423.79
10-10-1	16.41	4341.56
1-1-1	162.45	3531.82
1-1-2	307.62	2747.64
1-1-10	752.12	465.86

Multiobjective Programming

- Multipurpose system
  - Conflicting objectives
  - Tradeoffs between uses: Recreation vs. irrigation
  - Often there is no unique "optimal" solution
- Let each use *j* have an objective $Zj(\boldsymbol{x})$
- We want to

$$\text{Mazimize} \quad \vec{\mathbf{Z}}(\vec{\mathbf{x}}) = [Z_1(\vec{\mathbf{x}}), Z_2(\vec{\mathbf{x}}), \ldots, Z_p(\vec{\mathbf{x}})]$$

subject to

$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \in X$$

- <u>Single objective problem</u>:
  - Identify optimal solution, e.g., feasible solution that gives best objective value. That is, we obtain a full ordering of the alternative solutions.
- <u>Multiobjective problem</u>
  - We obtain only a partial ordering of the alternative solutions. Solution which optimizes one objective may not optimize the others
  - Noninferiority replaces optimality

$$\text{Mazimize} \quad Z_1(\vec{\mathbf{x}})$$

$$\text{subject to}$$

$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

$$\text{Mazimize}\, Z(\vec{\mathbf{x}}) = w_1 Z_1(\vec{\mathbf{x}}) + w_2 Z_2(\vec{\mathbf{x}}) + \ldots + w_p Z_p(\vec{\mathbf{x}})$$

$$\text{subjectto}$$

$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

Example

- Flood control project for historic city with scenic waterfront

Alternative	Net Benefit	Method	Effects
1	$120k	Increase channel capacity	Change riverfront, remove historic bldg's
2	$700k	Construct flood bypass	Create greenbelt
3	$650k	Construct detention pond	Destroy recreation area
4	$800k	Construct levee	Isolate riverfront

Objective 1	Objective 2
Maximize Net Benefit	Maximize Scenic Beauty
Alternative	Alternative
4	2
2	3
3	1
1	4

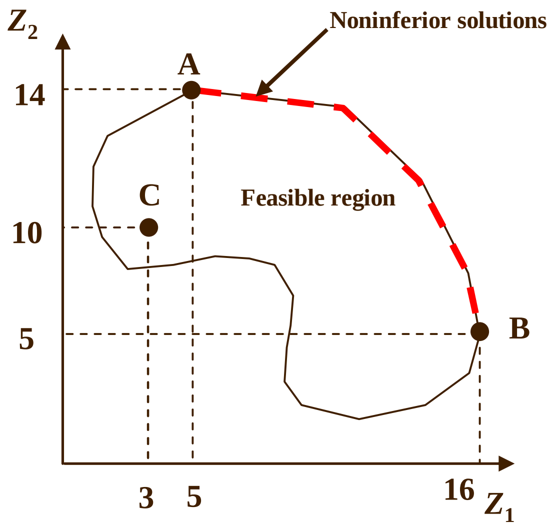
- Does gain in scenic beauty outweigh $100k loss in NB?
- Alternative 2 is better than Alternatives 1 and 3 with respect to both objectives. Never choose 1 or 3. They are inferior solutions.
- Alternatives 2 and 4 are not dominated by other alternatives. They are noninferior solutions

Noninferior Solutions

- A feasible solution is <u>noninferior</u>
  - if there exists no other feasible solution that will yield an improvement in one objective w/o causing a decrease in at least one other objective (A & B are noninferior, C is inferior)
- All interior solutions are inferior

- move to the boundary by increasing one objective w/o decreasing another

- Northeast rule:
  - A feasible solution is noninferior is there are no feasible solutions lying to the northeast (when maximizing)



Alternative	Z_1	Z_2
A	5	14
B	16	5
C	3	10

$\vec{\mathbf{x}}$ is noninferior if there exists no feasible $\vec{\mathbf{y}}$ such that

$$Z_k(\vec{\mathbf{y}}) \geq Z_k(\vec{\mathbf{x}}) \qquad k = 1, \dots, p$$

Maximize $\vec{\mathbf{Z}}(\vec{\mathbf{x}}) = [Z_1(\vec{\mathbf{x}}), Z_2(\vec{\mathbf{x}})]$

subject to

$$Z_1(\vec{\mathbf{x}}) = 5x_1 - 2x_2$$

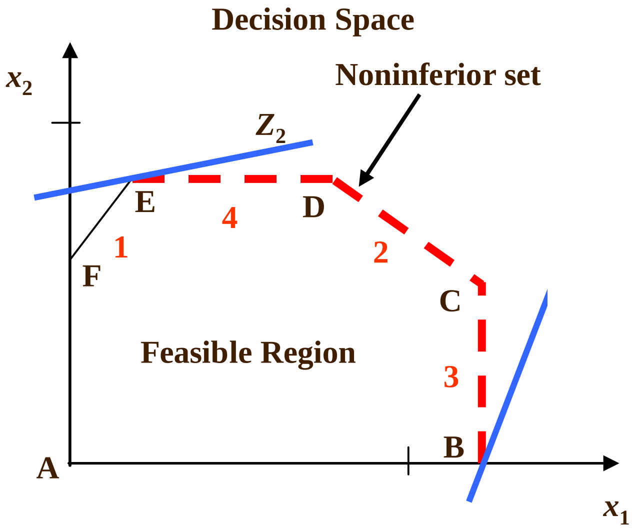
$$Z_2(\vec{\mathbf{x}}) = -x_1 + 4x_2$$

(1) $-x_1 + x_2 \leq 3$

(2) $x_1 + x_2 \leq 8$

(3) $x_1 \leq 6$

(4) $x_2 \leq 4$



	x_1	x_2	Z_1	Z_2
A	0	0	0	0
B	6	0	30	-6
C	6	2	26	2
D	4	4	12	12
E	1	4	-3	15
F	0	3	-6	12

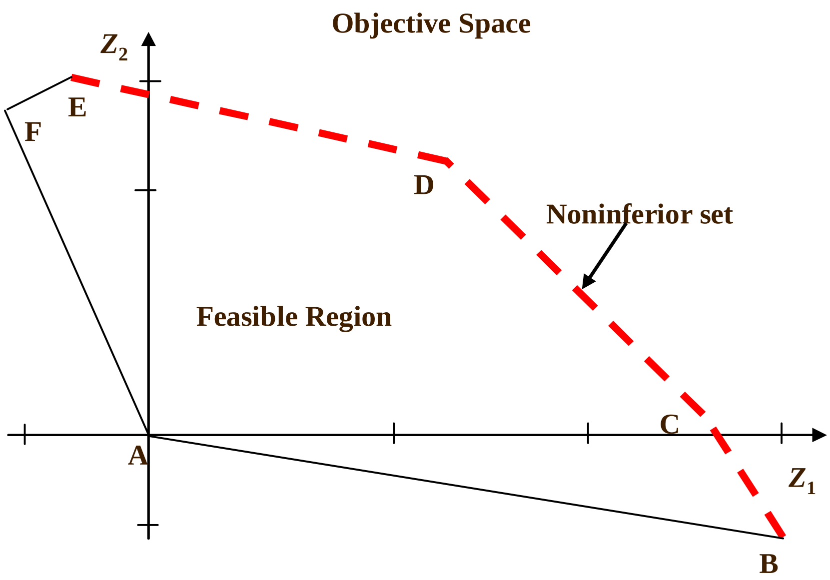
Evaluate the extreme points in decision space (*x*1, *x*2) and get objective function values in objective space (*Z*1, *Z*2)

- Noninferior set contains solutions that are not dominated by other feasible solutions.

- Noninferior solutions are not comparable:

C: 26 units *Z*1; 2 units *Z*2
D: 12 units *Z*1; 12 units *Z*2

- Which is better?
  Is it worth giving up 14 units of *Z*2 to gain 10 units of *Z*1 to move from D to C?



Tradeoffs

- Tradeoff = Amount of one objective sacrificed to gain an increase in another objective, i.e., to move from one noninferior solution to another
- Example: Tradeoff between $Z1$ and $Z2$ in moving from D to C is 14/10, i.e., 7/5 unit of $Z1$ is given up to gain 1 unit of $Z2$ and vice versa

$$\frac{\partial Z_i(\vec{\mathbf{x}})}{\partial Z_j(\vec{\mathbf{x}})}$$

Multiobjective Methods

- Information flow in the decision making process
  - Top down: Decision maker (DM) to analyst (A)
    - ✓ Preferences are sent to A by DM, then best compromise solution is sent by A to DM
    - ✓ Preference methods
  - Bottom up: A to DM
    - ✓ Noninferior set and tradeoffs are sent by A to DM
    - ✓ Generating methods
- Generating methods
  - Present a range of choice and tradeoffs among objectives to DM
    - ✓ Weighting method
    - ✓ Constraint method
    - ✓ Others
- Preference methods
  - DM must articulate preferences to A. The means of articulation distinguishes the methods
    - ✓ Noninterative methods: Articulate preferences in advance
      - Goal programming method, Surrogate Worth Tradeoff method
    - ✓ Iterative methods: Some information about noninferior set is available to DM and preferences are updated
      - Step Method

Generating Techniques

- Weighting method

Mazimize $Z_1(\vec{\mathbf{x}})$

subject to

$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

Mazimize $Z(\vec{\mathbf{x}}) = w_1 Z_1(\vec{\mathbf{x}}) + w_2 Z_2(\vec{\mathbf{x}}) + \ldots + w_p Z_p(\vec{\mathbf{x}})$

subjectto

$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

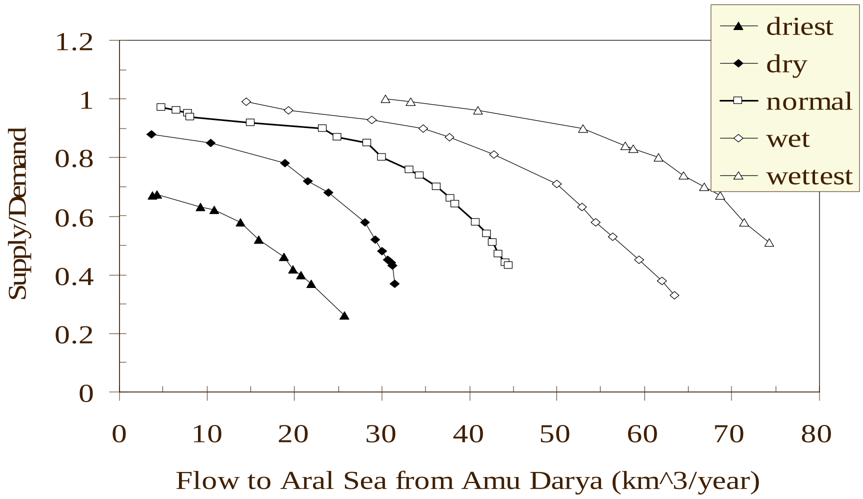
- Tradeoffs are explicit in the weights

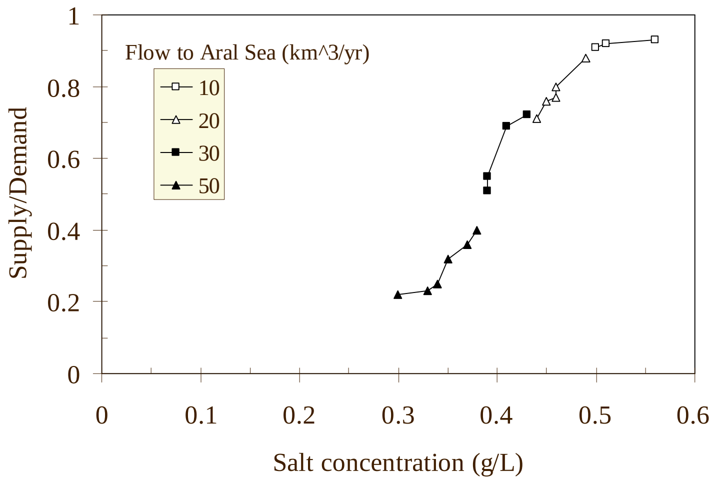
$$-\frac{\partial Z_i(\vec{\mathbf{x}})}{\partial Z_j(\vec{\mathbf{x}})} = \frac{w_j}{w_i}$$

- Vary the weights over reasonable ranges to generate a wide range of alternative solutions reflecting different priorities.

Example – Amu Darya River

- Multiple Objectives
  - Maximize water supply to irrigation
  - Maximize flow to the Aral Sea
  - Minimize salt concentration in the system



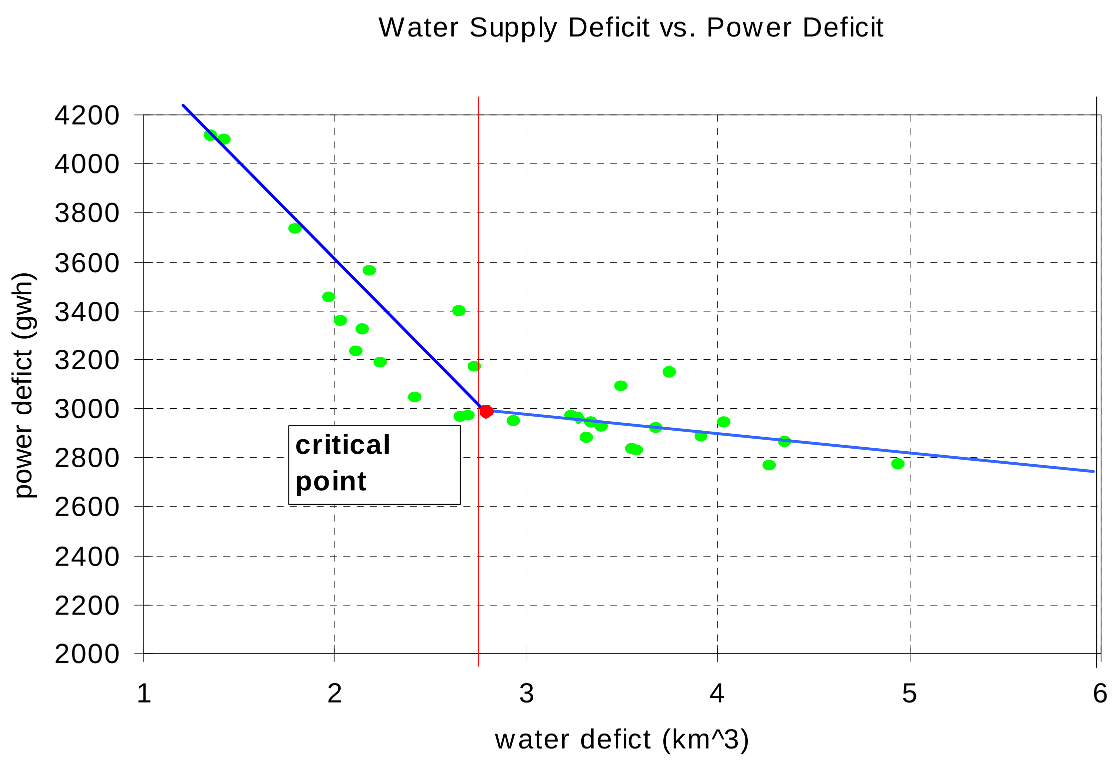


Objective weight matrix

Weight Set	Objective		
	Supply/Demand	Flow to Aral Sea	Salt concentration.
1	1	0	0
2	0	1	0
3	0	0	1

Objective	Weight Set 1	Weight Set 2	Weight Set 3
	Value		
Supply/Demand	0.99	0.32	0.50
Flow to Aral Sea (km^3)	4.71	55.3	46.3
Salt concentration in river (g/L)	0.56	0.36	0.30
Salt concentration in groundwater (g/L)	0.82	0.38	0.31

Example – Syr Darya River



Generating Techniques

- Constraint method

Mazimize $Z_1(\vec{\mathbf{x}})$

subjectto

$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

Mazimize $Z_k(\vec{\mathbf{x}})$

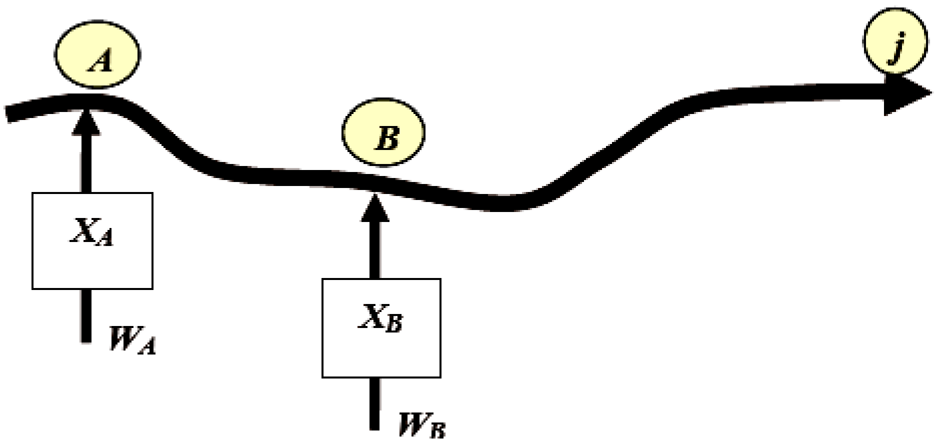
subject to

$$Z_i(\vec{\mathbf{x}}) \ge L_i \qquad i = 1, \ldots, p,\ i \ne k$$
$$g_1(\vec{\mathbf{x}}) \le b_1$$
$$\vdots$$
$$g_m(\vec{\mathbf{x}}) \le b_m$$
$$\vec{\mathbf{x}} = (x_1, x_2, \ldots, x_n) \ge 0$$

- Optimize one objective while all others are constrained to some particular bound
- Solutions are noninferior solutions if correct values of the bounds (Lk) are used, i.e., they must be chosen such that feasible solutions to the single objective problem exist, and the constraints on the objectives must be binding at the solution.
- Dual variables (marginals) associated with the RHS's Li are the tradeoffs of Zk per unit change in $Li = Zi$

Example1: Two waste dischargers, *A* and *B*, are located along a river and they can increase their waste treatment efficiencies, *XA* and *XB*, at costs *CA*(*XA*) and *CB*(*XB*), respectively. Let *WA* and *WB* be the untreated waste produced. The resulting waste discharges into the river at sites *A* and *B* must not exceed effluent standards E_A^{Max} and E_B. The resulting pollution concentration at various locations *j* in the river downstream of the discharge points must not exceed the stream standards S_j^{Max}. Let the pollutant concentration at location *j* be *cj*. Further, let *aij* be a transfer coefficient describing the concentration at location *j* due to a unit input of pollutant upstream at point *i* (*i*= *A* or *B*) , and $\hat{c}_j$ is the unregulated concentration at site *j*.

Assume that (1) the total cost of providing treatment and (2) any treatment cost inequity (the difference in cost between sites *A* and *B*) are water quality management objectives to be minimized simultaneously.



(a) Define a non-inferior (or Pareto efficient) solution, i.e., state what characteristics a non-inferior solution has compared to an inferior solution.

SOLUTION

In principle if one has identified a noninferior solution, to increase the value of one objective, one has to accept less of another objective.

A feasible solution is noninferior if there exist no other feasible solutions that will yield an improvement in one objective without causing a decrease in at least one other objective

$\vec{x}$ noninferio if there exists no feasible $\vec{y}$ such that

$$Z_k(\vec{y}) \geq Z_k(\vec{x}) \qquad k = 1,\ldots,p$$

(b) Show how you would model this multiobjective problem using the weighting approach. Does this formulation lead to a linear programming model? If not, then show how it can be converted to a linear program.

SOLUTION

Minimize $W_1[C_A(X_A)+C_B(X_B)]+W_2|C_A(X_A)-C_B(X_B)|$

Subject to

$$W_A(1-X_A)\le E_A^{\max}$$

$$W_B(1-X_B)\le E_B^{\max}$$

$$c_j=a_{Aj}W_A(1-X_A)+a_{Bj}W_B(1-X_B)\le S_j^{\max}$$

$$0\le X_A\le 1$$

$$0\le X_B\le 1$$

As written, the model is nonlinear because of the absolute value in the objective function. However, this can be linearized by defining some slack and surplus variables

Minimize $W_1[C_A(X_A)+C_B(X_B)]+W_2\left(D^{pos}+D^{neg}\right)$

Subject to

$$C_A(X_A)-C_B(X_B)=D^{pos}-D^{neg}$$

$$W_A(1-X_A)\le E_A^{\max}$$

$$W_B(1-X_B)\le E_B^{\max}$$

$$c_j=a_{Aj}W_A(1-X_A)+a_{Bj}W_B(1-X_B)\le S_j^{\max}$$

$$0\le X_A\le 1$$

$$0\le X_B\le 1$$

Example2: A reservoir is planned for irrigation and low flow augmentation for water quality control. A storage volume of 6 million m3 will be available for these two conflicting uses each year. The maximum irrigation demand is 4 million m3. Let $X1$ be the allocation of water to irrigation and $X2$ the allocation for downstream flow augmentation. Assume that there are two objectives, expressed as

$$Z_1(\boldsymbol{X}) = 4X_1 - X_2$$

$$Z_2(\boldsymbol{X}) - = 2X_1 + 6X_2$$

(a) Write a multiobjective planning model using a weighting approach.

Weighting approach

Maximize $W_1 Z_1 + W_2 Z_2 = W_1(4X_1 - X_2) + W_2(-2X_1 + 6X_2)$

Subject to

$$X_1 \le 4$$

$$X_1 + X_2 \le 6$$

$$X_1, X_2 \ge 0$$

Various values of the weights (W_1 and W_2) are selected and the problem is solved.

Constraint approach

Maximize $Z_1 = 4X_1 - X_2$

Subject to

$$Z_2 = -2X_1 + 6X_2 \ge L_2$$

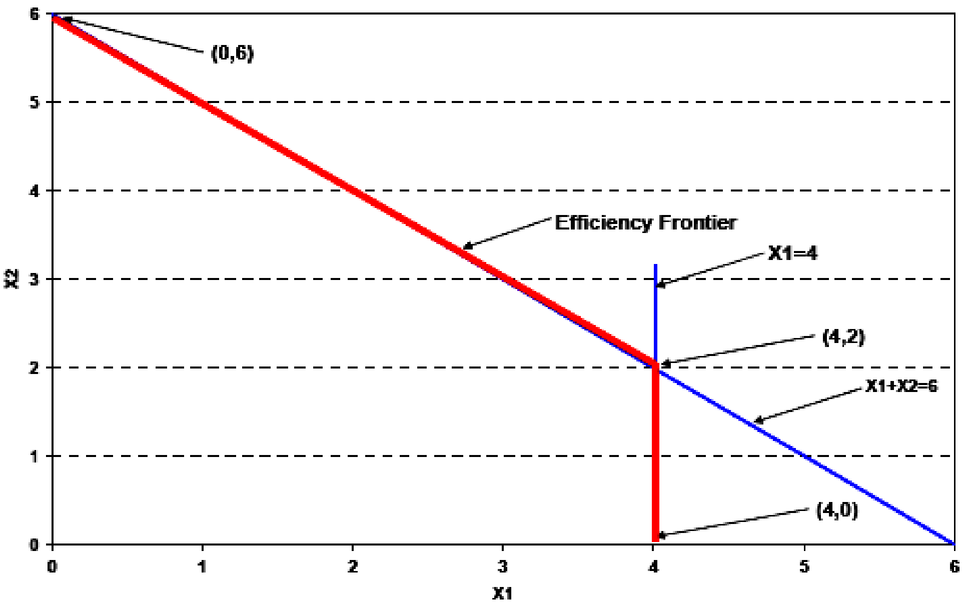
$$X_1 \le 4$$

$$X_1 + X_2 \le 6$$

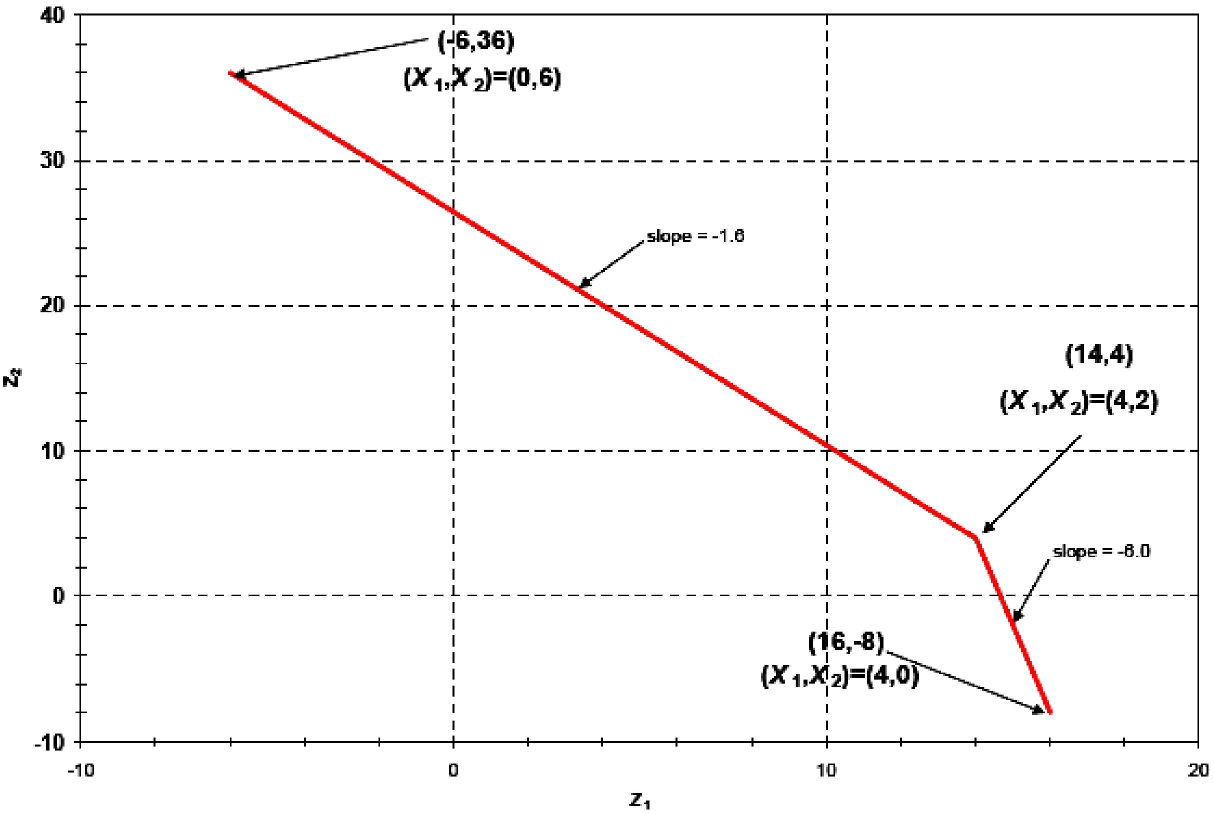
$$X_1, X_2 \ge 0$$

Various values of the lower bound (L_2) are selected and the problem is solved.

(b) Write a multiobjective planning model using a constraint approach.



(c) Define the efficiency frontier. This requires a plot of the feasible combinations of X_1 and X_2.



(d) Assuming that various values are assigned to the weight W_1 (W_2 is constant and equal to 1), verify the following solutions to the weighting approach model

W_1	X_1	X_2	Z_1	X_2
> 6	4	0	16	-8
6	4	0 to 2	16 to 14	-8 to 4
< 6 to > 1.6	4	2	14	4
1.6	4 to 0	2 to 6	14 to -6	4 to 36
< 1.6	0	6	-6	36

SOLUTION: See the graph in Part (c).